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The Bakhshālī Manuscript: A Response to the Bodleian Library's Radiocarbon Dating

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1. INTRODUCTION

Popular attention has recently been captured by the results of the Bodleian Library's 2017 project of radiocarbon dating portions of the birch-bark fragments constituting what is known as the Bakhshālī Manuscript.[1] As the Government Palaeographer of India, A. F. R. Hoernle, reported in 1882, this artefact was

> … found in a ruined enclosure, near Bakhshálí, a village of the Yusufzai District, in the Panjáb, by a man who was digging for stones … unfortunately, by far the largest portion of the MS. was destroyed when the finder took it up; and even the small portion that now remains is in a very mutilated state.[2]

Hoernle studied the manuscript in detail, and later passed it to the Bodleian Library, where it arrived in 1902. Due to the subsequent deterioration of the manuscript, the modern study of its surviving content is primarily based on the facsimiles published by G. R. Kaye with his edition of the text in 1927.[3]

This content consists of a Hybrid Sanskrit compendium in 70 folios of mathematical formulas and examples, in the form of verse rules or *sūtras* and sample problems mixed with a prose commentary. The manuscript's colophon states that it was written (whether compiled, copied or both is not certain) by an otherwise unknown *brāhmaṇa* identified as the "son of Chajaka" for the use of one

1 Bodleian Library 2017; Devlin 2017; Howell, du Sautoy, and Formigatti 2017.

2 Hoernle 1882: 108.

3 Kaye 1927.

equally unknown "Hasika son of Vasiṣṭha" and his descendants, in a locality probably called Mārtikāvatī in the Gandhāra region of northwest South Asia. The definitive study on this artefact is the edition, translation and commentary published by Takao Hayashi as *The Bakhshālī Manuscript: An Ancient Indian Mathematical Treatise* (Groningen, 1995) based on his 1985 doctoral dissertation in the History of Mathematics Department at Brown University.[4]

In his study, Professor Hayashi examined various earlier hypotheses about the date of the Bakhshālī Manuscript.[5] In 1888, Hoernle himself originally suggested the third or fourth century of the Common Era, based on the assumption that the "North-Western Prâkṛit" language of the text preceded the use of Classical Sanskrit "in secular composition".[6] This assumption has not been borne out by subsequent scholarship. In a 1912 article and in his 1927 edition, Kaye argued for a date around the twelfth century, based on comparisons of the manuscript's distinctive script (a north-western descendant of Gupta script known as Śāradā) with epigraphic evidence.[7] Kaye was partly influenced by a presupposition that the mathematical content of the work was ultimately derived from Greek sources, a view that is now recognized to be untenable.[8]

Hayashi considered these hypotheses along with various intermediate dates proposed by later researchers, in the light of paleographic evidence and more recent information about other works in the Sanskrit genre of arithmetic calculation (*pāṭī-gaṇita*). As a consequence, he tentatively assigned the date of the commentary's composition to the seventh century, and of the manuscript itself to somewhere between the eighth and the twelfth centuries.[9] The verses may be somewhat older than the date assigned to the commentary, and many of the rules, technical terminology, and metrology they contain are older still, as attested by numerous extant earlier Sanskrit texts.[10] Note that although the vast majority of surviving Indic manuscripts were written comparatively recently, the texts they contain are in many cases much older, preserved in an unbroken chain of scribal copying extending over centuries or millennia.[11]

The Bodleian's radiocarbon dating project took three samples of bark, avoiding areas containing ink, from fragments identified as belonging to folios 16, 17 and 33 of the Bakhshālī Manuscript. The date ranges found for these samples are 224–383, 680–779, and 885–993 respectively (all Common Era). These findings were reported by the Bodleian in a research statement internally dated 3 July

4 Hayashi 1995.
5 Hayashi 1995: 4–5.
6 Hoernle 1888.
7 Kaye 1912, 1927.
8 Kaye's views on the Bakhshālī Manuscript were decisively refuted by Datta (1929) and by Clark (1929).
9 Hayashi 1995: 148–149.
10 Hayashi 1995: § I.6, I.12, I.13.
11 Plofker 2009: 304–7; Wujastyk 2014: 161 f.

2017 and distributed in an electronic document with filename dated 13 September 2017; in an article by the *Guardian*'s science correspondent Hannah Devlin that appeared on 14 September (BST); and in a YouTube video released by Oxford University at the same time.[12] These reports discuss the findings on the assumption that the proposed date ranges pertain to the actual writing of the manuscript sections sampled, even though no tests were performed on the written (inked) portions of the leaves.

The level of popular interest and excitement provoked by these discoveries and speculations reflects a thoroughly understandable, and in our view entirely commendable, fascination with the history of mathematics and the many remarkable discoveries made by ancient Indian mathematicians. Regrettably, however, the popular reports have somewhat garbled or obscured several important facts about the Bakhshālī Manuscript and about Indian mathematics in general.

2. MISCONCEPTIONS ABOUT THE MANUSCRIPT

Although we are not qualified to challenge the assigned radiocarbon dates on technical grounds, we think it important to point out several ways in which the inferences drawn from these results seem implausible to us in light of the features of the manuscript.

General palaeographic consistency

The script, handwriting and layout format seen on all the leaves of the Bakhshālī Manuscript show far more uniformity than we would expect to find in manuscripts differing in date by hundreds of years. The manuscript's consistency of appearance has produced the generally accepted (though not entirely definitive) conclusion that it is a single work written by the same hand in most parts, with a second hand seen in one portion.[13] It must be strongly emphasized that ancient and medieval Indic cultures, despite some popular preconceptions to the contrary, were not intrinsically any more "timeless" or "stagnant" than other contemporaneous societies. Any researcher wishing to maintain that all these sampled portions of the Bakhshālī Manuscript were actually written separately at intervals of several centuries will have to supply a plausible explanation of why this script remained so remarkably unchanged, in defiance of all that we

12 Howell, du Sautoy, and Formigatti 2017; Devlin 2017; University of Oxford 2017.
13 The second hand appears in the part designated by Hayashi section X, nearly the same as Kaye's section M. See Hayashi 1995: 8–10, 23–25, 85.

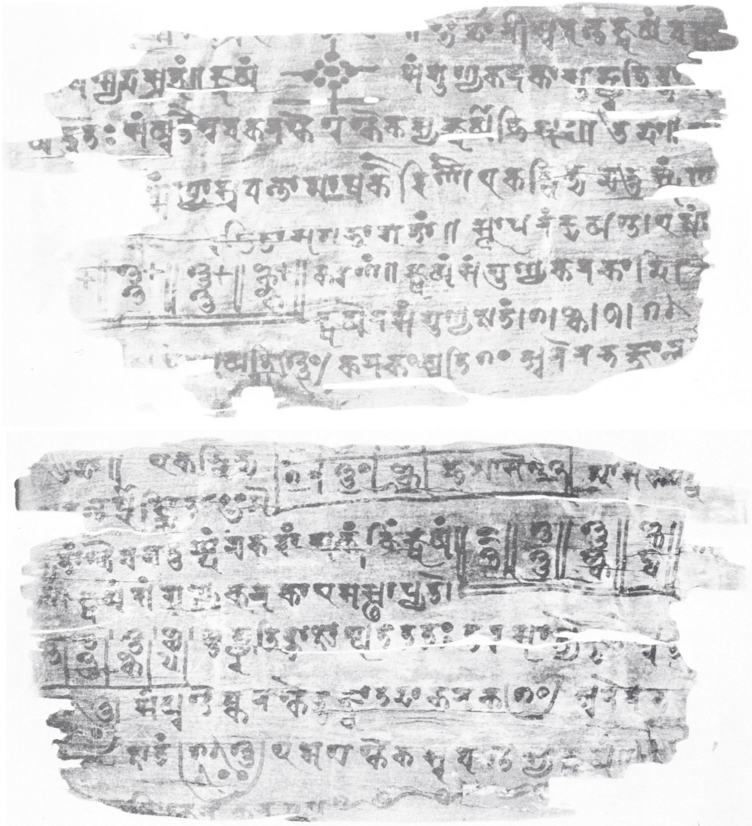

Figure 1: Folios 16v and 17r of the Bakhshālī MS. from Hayashi 1995: 555–556, reproducing Kaye's original facsimiles.

currently know about the palaeographic evolution of Indic scripts, or else why its later writers chose to simulate so faithfully the writing of a long-dead predecessor.

Continuity of content between samples

As Professor Hayashi has pointed out in a personal communication of 15 September 2017, "there is no evidence at all to show that the scribe of fol. 17 is different from that of fol. 16," and furthermore, "the content of the reverse of fol. 16 (Example 1 for Sutra 27) continues on the obverse of fol. 17".[14] These two written

14 Hayashi 2017a. See Hayashi 1995: 203, 310–311, and 555–556 for transcription, translation and facsimile respectively. These leaves are reproduced here in Fig. 1.

leaves are reproduced in Fig. 1. The Bodleian researchers claim that they are separated by at least three and possibly as much as five and a half centuries, but the leaves treat exactly the same problem on the topic of "mixture," specifically a mixture of gold alloys of different impurities, in what appears to be a continuous and logically consistent expository stream of *sūtra*, example problem, and displayed calculations. For example, the figures "30" and "10" of the worked example are clearly visible on both pages. Whatever may be the age difference between the physical birch bark supports constituting leaves 16 and 17, the transition in handwriting and textual content from the former to the latter appears to be seamless.

The combination of palaeographic consistency and continuity of content suggests that the manuscript was written out as a single unified work. In that case, it follows that the date of the written zeros is the date of the scribe, which is the date of the latest of the folia, not the earliest.

LONGEVITY OF A WORKING DOCUMENT

There exist many examples of carefully preserved (often interred) written birch-bark fragments from Central Asia and northern South Asia dated as far back as the early Common Era.[15] But manuscripts in active use tend to have much shorter life expectancies.[16] If the proposed dating is to be maintained, it will require a plausible explanation of how the different parts of this instructional text were maintained in comparable states of preservation over such widely varying time-spans up to the point of their common burial some time in or after the ninth century.

PHYSICAL CHARACTERISTICS OF BIRCH BARK AS A WRITING MEDIUM

Although we do not know what techniques were used by the ancient artisan(s) in the original preparation of the Bakhshālī Manuscript's birch bark for scribal use, it seems possible the resulting leaves may be quite different in chemical composition from the natural bark. B. N. Goswamy describes the traditional production of such leaves as follows:

> … after being peeled off the tree, the bark was dried; oil was then applied over it and it was polished; layers were joined together by a natural gum; and finally it was cut to appropriate size and kept between wooden covers.[17]

15 Sathaye 2017: 56 f.
16 Walser 2014: n. 19.
17 Goswamy and Bhattacharya 2007: 20.

The Bodleian researchers say nothing about their assumptions concerning the physical makeup of the samples they took from the leaves, except to note that the samples

> … were treated with a mild chemical pretreatment, which included washes using organic solvents to remove any oils or grease on the manuscript and a sequential acid-base-acid wash to remove any potential carbonates or humics.[18]

We cannot tell from this description whether the pretreatment would have been sufficient to remove all possible organic contaminants that may have been introduced during the initial production of the leaves, as opposed to oils and dirt that may have accumulated on them later. If not, then the question of sample contamination will have to be reconsidered.

In this context it is noteworthy that at least one other experiment in carbon-dating Asian manuscripts has also produced results that are "at variance with the codicological, art historical and historical contexts as hitherto understood".[19] In this experiment, two Tibetan manuscript folia are unified by style and content, but the C-14 dating separates them by six hundred years.[20] Citing the opinion of Prof. Richard Ernst, Nobel Prize for Chemistry in 1991 and a specialist in Tibetan art and pigment analysis, the study concluded:

> The discrepancy between the two sets of radiocarbon results remains unresolved. It is possible that there was an external contamination….[21]

This apparently anomalous result points to the desirability of re-examining the Bodleian's own carbon-dating procedures to see if they may hold a clue to the similarly puzzling discrepancies in the results for the Bakhshālī Manuscript.

3. MISCONCEPTIONS ABOUT THE MATHEMATICAL CONTENT AND CONTEXT

The initial speculations about the mathematical content of the manuscript have included some statements that need to be clarified or corrected.

18 Howell, du Sautoy, and Formigatti 2017: 2.

19 Heller 2016: 129.

20 The manuscript folios are MS Rome IsIAO 1329 E and MS Los Angeles LACMA M 81.90.13. On the unification of these folia, see Harrison 2007. On the present location of the IsIAO folio, see Barrera 2016.

21 Heller 2016: 129. We are grateful to Matthew Kapstein for this reference.

"number in its own right"

The central point of the Bodleian Library's announcement is its claim that the Bakhshālī Manuscript contains

> one of the earliest uses of zero … as a placeholder, i.e. the use of zero to indicate orders of magnitude in a number system.[22]

Later in the same document Professor du Sautoy asserts that "The zero used in the Bakhshali Manuscript is not yet a number in its own right. It is a place holder," and suggests in a general way that the "real" zero was a later Indian development.[23]

By "number in its own right," a qualification that may not be entirely clear to a general audience, he is referring to the distinction between zero as a place-holder and zero as an arithmetical operator.[24] In the first case, the zero functions as a simple "empty" marker in written numerals; in the second case, the zero is treated as a number in arithmetical operations just like the other nine digits (although obeying certain special rules). Place-holder zeros appeared in sexagesimal and vigesimal place-value number systems in Mesopotamian, Greek and Mayan texts, some dating from before the Common Era.[25] The Bakhshālī zero, in Prof. du Sautay's account, is not functionally different from any of these.

But if we consider the manuscript as a unified work compiled by a single commentator—and so far we have seen no persuasive reason not to do so, whatever the age variations in its constituent bark pieces may be—then distinguishing within it between uses of zero as a mere "place-holder" and as a "number" becomes more difficult. To what extent do techniques for carrying out arithmetic operations on decimal place-value numbers intrinsically imply some concept of zero as a "number in its own right"?[26]

For example, instances of arithmetic operations prescribed for very large decimal integers strongly suggest, although they do not conclusively demonstrate, the existence of some sort of long-multiplication method performed digit by digit, which requires arithmetic operations with zero. One such instance is

22 Howell, du Sautoy, and Formigatti 2017: 1.
23 Howell, du Sautoy, and Formigatti 2017: 5 and in the YouTube video announcement (University of Oxford 2017).
24 This distinction is well explained in Gupta 1995: 45–47. This topic was raised in relation to the Bakhshālī zero over a century ago by Hoernle (1888: 38).
25 Gupta 1995; Pingree 2003: 137–58.
26 Note that the Bakhshālī Manuscript also employs the zero-dot as a place-holder in a different sense, i.e., as an algebra-like symbol standing for an unknown quantity.

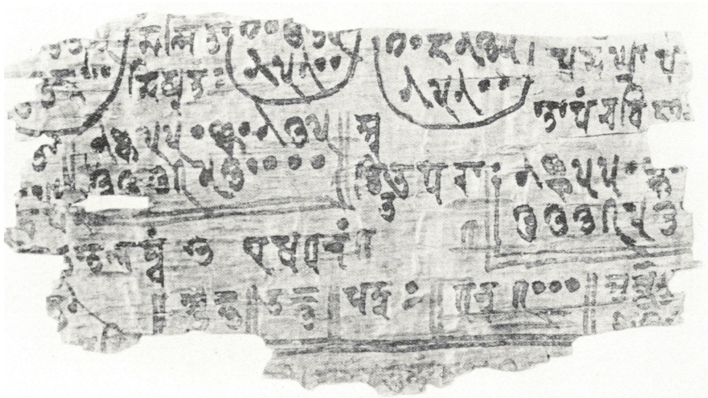

Figure 2: Folio 47r, large integer multiplication.

described in the following worked sample problem appearing on f. 47r, shown in Fig. 2.[27] The fraction

$$\frac{108625}{65600}$$

is multiplied by

$$\frac{59425}{49200}$$

to produce

$$\frac{6455040625}{3227520000}$$

This long-multiplication procedure that includes zeros is hard to conceive of without the inclusion of those zeros in the arithmetic operations.

Furthermore, another procedure in the Bakhshālī Manuscript apparently combines the arithmetic use of zero as a number with the above-mentioned "algebra-like" practice of using the zero-dot as a symbol for an unknown quantity.[28] That is, in a problem solution on folio 22v,[29] a zero-dot standing for the unknown quantity is replaced by the nonzero number 1. This quasi-algebraic

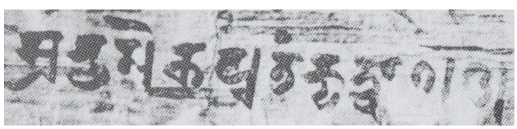

Figure 3: Folio 22v, text of the substitution/addition rule *śūnyam ekayutaṃ kṛtvā*.

27 Hayashi 1995: 329: problem § VII 8, example 3 for *sūtra* N18.

28 See footnote 26 above.

29 Hayashi 1995: §IV 4, pp. 210, 315.

substitution for a zero symbol is indicated by quoting a rule referencing the arithmetic operation of *addition* to the number zero: namely, "having added unity to zero" (*śūnyam ekayutaṃ kṛtvā*; see Figure 3). In other words, the author of the commentary seems to understand quite well that replacing a zero-dot by some number is equivalent to adding zero to that number.[30]

THE ANTIQUITY OF THE INDIAN DECIMAL PLACE-VALUE ZERO

Based on the proposed C-14 dating, Professor du Sautoy claims that "we now know that it was mathematicians in India in 200–400 CE who planted the seed of the idea" of the modern zero. This reflects a widespread misconception about the historical information already available concerning the Indian "seed of the idea" of zero. It is often supposed that the decimal place-value zero dates back only as far as the earliest surviving physical example of it in epigraphical sources from the second half of the first millennium CE. Sometimes the "invention" of the zero is linked instead to the algebra rules whose first surviving formulation (which, *pace* Professor du Sautoy,[31] may be much later than their initial discovery) appears in the work of Brahmagupta in the early seventh century.[32] But in fact, it has long been known from textual rather than physical evidence that ideas of zero in South Asia stretch back at least to the early centuries of the Common Era.

For example, a symbol (or blank) called *śūnya*, a word meaning "empty, void" and later established as one of the standard technical terms for zero, served some notational purpose(s) in Sanskrit combinatorial calculations in prosody, a subject whose founding text by Piṅgala was written before the Common Era. It is also known to historians that several early Common Era Indic texts contain explicit statements describing a decimal place-value system using counting rods or tokens, with locations for units, tens, hundreds, and thousands, that demonstrate a clear understanding of how the same digit may have a quite different function and value according to its position.[33]

30 The false-position technique employing such substitutions is treated also in, e.g., the *Gaṇitasārasaṅgraha* of Mahāvīra (ca. 850 CE) (Raṅgācārya 1912: 40, 62; #107–108) and the *Līlāvatī* of Bhāskara (1149/1150) (Colebrooke 1817: 23; #50–51). We thank Hayashi (2017b) for these references and for his guidance concerning the functional equivalence of *śūnyam ekayutaṃ kṛtvā* ("having added unity to zero," Bakhshālī *sūtra* N8) and *śūnyasthāne rūpaṃ dattvā* ("having put unity in the place of zero," Bakhshālī *sūtra* N12) in this context.

31 Howell, du Sautoy, and Formigatti 2017: 5.

32 Plofker 2009: 140 ff.

33 Ruegg 1978: 173–4; Gupta 1995: 58; Bag and Sarma 2003; Staal 2010: 42. Recent discoveries on the transmission history of these accounts are described in Wujastyk 2017: 19–20.

Exactly when a fully arithmetic system of decimal place-value digits including zero as a numerical operand came into being is hard to pinpoint.[34] The earliest definite evidence for such a complete system now apparently dates from around 550 CE, in an astronomical work (described by its author as a synopsis of several earlier treatises) that defines a particular constant as "sixty minus zero," i.e., just sixty.[35] But as the foregoing remarks make clear, the decimal place value notation was probably in use in South Asia centuries earlier. Consequently, even if part of the Bakhshālī Manuscript text does turn out—against all expectation—to be as old as the third or fourth century, its evidence for the use of zero in South Asia at that time will indeed be interesting, but as a confirmation rather than a revelation or refutation.

relation of the proposed dating to the language(s) of the manuscript

Dr. Formigatti speculates that various linguistic irregularities in the Sanskrit text, which Hayashi noted bear some resemblance to features of "a dialect or dialects" including Buddhist Hybrid Sanskrit, Prakrit and Apabhraṃśa, and Old Kāshmīrī,[36] "can be explained by the fact that the manuscript in its present state is composed of at least three different manuscripts with different dates." That is, he suggests that the three date ranges found for the samples "correspond to different stages of linguistic development".[37] This suggestion seems very difficult to test, given that so many of the features pointed out by Hayashi overlap in some or all of these languages. Moreover, there appears to be no distinct pattern in the linguistic identity of the irregularities found in folios 16, 17 and 33.[38] In fact, all of them appear to be indicated in various places as exhibiting various characteristics associated with every one of the languages in question.[39]

In our view, nothing currently known about the manuscript suggests different stages of linguistic development in its different parts, especially not in contiguous individual folia presenting a continuous discussion of a particular problem. Rather, the work as a whole appears to exhibit throughout a form of language that shares features with many other Indian dialects of the time.

34 This is especially true since previous inferences of a definite *terminus ante quem* in 269/270 CE were recently revealed to be erroneous; see Mak 2013.
35 Hayashi 2003: 368.
36 Hayashi 1995: 27.
37 Howell, du Sautoy, and Formigatti 2017: 6.
38 These folios are assigned in Hayashi's Revised Order to sections § III.31–32, III.33–34, and X.7–8 respectively (Hayashi 1995: 11–14).
39 Hayashi 1995: § I.4.

buddhist merchants?

Dr. Formigatti also states:

> The content of the Bakhshali manuscript is similar to the type of texts that Buddhist merchants [in the Gandhāra region] would have needed to study (and possibly use as reference) for their daily trading activities.[40]

And this point was given emphasis in the accompanying YouTube video.[41] Yet the category of technical works that Dr. Formigatti suggests that the ancient "Buddhist merchants would have needed to study" is not known from historical studies of Buddhist merchant culture in the Gandhāra region.[42] Furthermore, the author of the colophon of the manuscript identifies himself as a *brāhmaṇa*: he is neither a merchant nor a Buddhist.[43]

Despite the charm of the "Buddhist merchant" scenario, we have no warrant for drawing any definite conclusions about the social milieu in which the Bakhshālī Manuscript was produced or used, beyond the bare fact that the colophon author intended the work for the family of one Vasiṣṭha. Its mathematical content foreshadows some rules and problems found in later compositions dealing with arithmetic (*pāṭīgaṇita*), a technical genre characterized by significant variety in the religious communities and social contexts of its authors. For example, some of these treatises were composed by *brāhmaṇas* and others by Jain scholars.[44] What few clues we have about the text of the Bakhshālī Manuscript point to an origin in a brahmanic environment.[45]

40 Howell, du Sautoy, and Formigatti 2017: 5–6.
41 University of Oxford 2017: 0:10-0:17.
42 For example, the study by Neelis (2011) refers to many manuscripts that were carried by Buddhist merchants, but all are religious texts, and arithmetical handbooks do not figure amongst them.
43 Hayashi 1995: 271, 360. In pre-modern South Asia, an author identifying as a *brāhmaṇa* was typically a scholar or priest within the socioreligious traditions associated with the worship of Indic deities such as Śiva and Viṣṇu. These traditions, consciously differentiated from Buddhist beliefs and practices, gave rise to the modern religious identities now designated "Hindu."
44 See, e.g., SaKHYa 2009: xx ff. The eleventh-century *Gaṇitatilaka* by the Śaiva author Śrīpati was the subject of a commentary in the thirteenth century by the Jain author Siṃhatila Sūri (Kāpadīā 1937). The similarities between the Bakhshālī Manuscript's arithmetical problems and procedures and others known elsewhere in Indian mathematical literature are certainly worthy of further research. The circulation of such problems is discussed by, for example, Dold-Samplonius et al. (2002) and Høyrup (2004).
45 For instance, the *brāhmaṇa* writer of the manuscript colophon describes mathematics as generated by Śiva (or Brahmā) following the universal creation (*sṛṣṭi*). Other references to *brāhmaṇas* can be found in the text, along with story problems about events from Sanskrit epics and the mythology of Indic deities.

4. CONCLUSIONS

Our main views may be summarized in the following statements:

- The proposed division of the Bakhshālī Manuscript text into three chronologically distinct sections corresponding to the three radiocarbon date ranges is contradicted by the unified appearance of its content and writing. If its birch-bark leaves do indeed differ widely in age, the date of the youngest folio is logically the (approximate) date of the scribal activity. This fits well with past estimates of the date of the Bakhshālī Manuscript based on historical, philological and palaeographic arguments.

- Bakhshālī Manuscript, considered as the carrier of a unified text, includes a concept of written zeros that function as arithmetical operators, i.e., as numbers in their own right, and not merely as place-holder digits. This too fits well with the manuscript's generally-accepted dating to the second half of the first millennium CE.

- Attempting to trace the historical development of mathematical concepts such as the zero and decimal place value solely or primarily through ancient physical evidence is a fundamentally unreliable enterprise. The historical significance of the Bakhshālī Manuscript and its mathematical content cannot be understood by isolated speculative inferences based on the apparent physical age(s) of the bark it was written on: it requires careful comparison with related ideas in a long sequence of other Indic texts treating various concepts associated with calculation (*gaṇita*).

While investigating another very famous mathematical artefact, the cuneiform tablet known as Plimpton 322, the Assyriologist Eleanor Robson formulated six criteria against which interpretations of its content should be tested: namely, historical sensitivity, cultural consistency, calculational plausibility, physical reality, textual completeness, and tabular order.[46] These criteria have since provided useful benchmarks for historians of mathematics when proposing interpretations of mathematical documents. As Robson noted, the criterion of physical reality concerning the material characteristics of artefacts has indeed been too often neglected in favor of discussing their disembodied "text." Technological advances such as the C-14 analysis used by the Bodleian researchers are potentially of great value in remedying this imbalance. However, the conclusions drawn from physical testing do not automatically override

46 Robson 2001: 174–176.

the requirements of other criteria such as historical sensitivity and cultural consistency. A plausible historical hypothesis needs to account successfully for all the known data about a given phenomenon or artefact, rather than selectively disregarding the data that contradict it.

With regard to the physical testing procedures, we would greatly appreciate some supplementary explanations suitable for the nonspecialist reader concerning the following issues:

- The possibility and desirability of cross-checking the findings by testing more than one sample from each sampled folios.
- The possibility and desirability of sampling the actual written characters rather than un-inked portions of the birch bark leaves, since the former would directly attest to the actual writing of the text.

More consideration should also be given to the historical techniques of birch-bark manufacture, storage and use, and to the dating of other early manuscript folios, to assess the possible scenarios in which a scribe might use leaves of different dates to write out a text.

We express regret that the Bodleian Library kept their carbon-dating findings embargoed for many months, and then chose a newspaper press-release and YouTube as media for a first communication of these technical and historical matters. The Library thus bypassed standard academic channels that would have permitted serious collegial discussion and peer review prior to public announcements. While the excitement inspired by intriguing discoveries benefits our field and scholarly research in general, the confusion generated by broadcasting over-eager and carelessly inferred conclusions, with their inevitable aftermath of caveats and disputes,[47] does not.

Without wishing to dampen the laudable ardor shown in this project for scientifically investigating the material characteristics of ancient documents, we urge the investigators to consider the importance of reconciling their findings with historical knowledge and inferences obtained by other means. It should not be hastily assumed that the apparent implications of results from physical tests must be valid even if the conclusions they suggest appear historically absurd.

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