History of Science in South Asia

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Dr Sita Sundar Ram, Bījapallava of Kṛṣṇa Daivajña: Algebra in Sixteenth Century India. A Critical Study

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Volume 2, 2014

URI : https://id.erudit.org/iderudit/1116387ar
DOI : https://doi.org/10.18732/H2059Q

Aller au sommaire du numéro

Éditeur(s)

University of Alberta Library

ISSN

2369-775X (numérique)

Découvrir la revue

Citer ce compte rendu

Montelle, C. (2014). Compte rendu de [Dr Sita Sundar Ram, Bījapallava of Kṛṣṇa Daivajña: Algebra in Sixteenth Century India. A Critical Study]. *History of Science in South Asia*, *2*, 1–6. https://doi.org/10.18732/H2059Q

History of Science in South Asia

A journal for the history of all forms of scientific thought and action, ancient and modern, in all regions of South Asia

BOOK REVIEW

MLA style citation form: Clemency Montelle, Book Review. *History of Science in South Asia* 2 (2014): R1–R6.
Online version available at: http://hssa.sayahna.org/.

HISTORY OF SCIENCE IN SOUTH ASIA
A journal for the history of all forms of scientific thought and action, ancient and modern, in all regions of South Asia, published online at http://hssa.sayahna.org

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BOOK REVIEW

Dr Sita Sundar Ram, *Bījapallava of Kṛṣṇa Daivajña: Algebra in Sixteenth Century India. A Critical Study* (Chennai: Kuppuswami Sastri Research Institute, 2012) 287 pp. Rs 400/–. ISBNs 9788185170503, 8185170509.

The *Bījapallava* of Kṛṣṇa Daivajña is a substantial commentary on Bhāskara II's (fl. 1114) *Bījagaṇita*, composed some time in the late sixteenth century. Kṛṣṇa Daivajña came from an impressive lineage of famous *jyotiṣīs* and was an active scholar at the Mughal court in Agra. The author of an original work on eclipse reckoning, the *Chādakanirṇaya*, and several other commentaries, including Bhāskara's *Līlāvatī* and Śrīpati's *Jātakapaddhati*, Kṛṣṇa Daivajña was also part of a group of scholars at the court who were responsible for translating Ulugh Beg's astronomical tables into Sanskrit. He is therefore a key figure in second-millennium astral sciences in India. Dr Sita Sundar Ram's expository study on the *Bījapallava* is a welcome addition to the field as it makes more accessible the writings of this important scholar.

Commentaries are valuable for a number of reasons. Most directly, they are crucial for helping the reader understand the text they are analysing. In many instances, they can clear up confusion caused by often tersely expressed rules or concepts in the base text by parsing the grammar, paraphrasing expressions, giving synonyms to technical terms, providing a worked example, or sometimes even deriving the parameters invoked by the original author. But more broadly, commentaries give modern scholars a sense of the status of the text in the time contemporary with the commentator. They can also offer vital insight into deeper issues relating to the ways in which the technical content was generated, justified, and understood by those practitioners who used the texts. Modern scholars have deemed this latter point to be particularly apt when it comes to the *Bījapallava*. Kṛṣṇa Daivajña provided a commentary to Bhāskara's text which included demonstrations or derivations of the mathematical rules and relations (the so-called *upapatti*), mock debates between himself and an imagined interlocutor, analyses of the mathematical concepts that underpin the algorithms, such as zero, unknown quantities, negative numbers, explorations

on issues regarding determinability and conditions for solvability, and the like. This work is therefore a rich and nuanced source for our understanding of the practice of mathematics in the Indian subcontinent.

Kṛṣṇa Daivajña's *Bījapallava* has been edited several times, with editions by Āpaṭe et al. (1930), by Radhakrishna Sastri (1958), and by Vasistha (1982). Dr Sita Sundar Ram bases her study on these three editions and presents excerpts of key passages which she translates and analyses with detailed mathematical worked examples. She also includes a variety of cross-references to related mathematical discoveries throughout her analysis where appropriate. These are drawn from other works of Bhāskara, from related authors in the same tradition, and also from a variety of Eurasian cultures of inquiry. She also offers modern insightful reflection on the scope and context of the mathematical features that appear throughout the work.

The book opens with a brief survey of Indian mathematics, followed by five chapters each dedicated to a section from the *Bījapallava*. These are: six mathematical operations, the pulverizer (*kuṭṭaka*), indeterminate equations of the second degree, equations with one unknown, and lastly equations with many unknowns. The work is then wrapped up with an appraisal of Kṛṣṇa Daivajña's expertise in other areas, including philosophy, grammar, prosody. An account of his reception by later authors considers his scholarly legacy and several appendices contain useful reference information including the 219 verses of Bhāskara's *Bījagaṇita* and an ample glossary of technical terms.

The *Bījapallava* is a demanding work and many of the mathematical problems discussed therein are complex. Mathematical highlights that Dr Sita Sundar Ram brings to the reader's attention include: Kṛṣṇa Daivajña's characterisation of moving east or west on a number line (p. 19) to conceptualise the effects of operating with positive and negative numbers; the discussion on the 'symbols' for representing unknowns and their implication (p. 30ff); analysis of the pulverizer (*kuṭṭaka* method, p. 39ff); the classification of equations with one or more unknowns (p. 119–122); commentarial confusion over the lack of integer solutions for a particular indeterminate equation of the second degree (p. 105–107), to name a few.

Dr Sita Sundar Ram notes on several occasions that a distinctive feature of Kṛṣṇa Daivajña's commentary is his inclusion of *upapatti*s, often translated as "proof" or "demonstration" or "derivation". She notes (p. xiv) *upapatti* can have two senses: it is used "in the sense of both proof and method". This particular topic is becoming increasingly important in modern scholarship when considering the salient features of mathematical practice in India as it gives historians a window into some of the ways in which mathematical

propositions were verified and how and why the original authors believed their results to be justified, an area which has long been under-studied. Examining key textual passages amongst the original authors on this topic helps us understand the role of demonstration in the astral sciences.[1] *Upapatti*s in commentaries are also critical for better comprehending the many ways in which commentators responded to the base texts and the strategies the deemed useful and relevant for the interpretation of mathematical material (including *udāharaṇa*, *artham*, *vāsanā*, *utpatti*, and the like) . The critical nature of these types of commentarial passages and what they involve ultimately helps us understand the role and function of the commentary and sheds light on some of the deeper epistemological issues relating to the status of mathematical knowledge in this tradition.

As more critical scholarly publications of commentaries become available, scholars are in a better position to reflect on the genre. Several recent studies have explored the broader role of the commentary and its constituent parts as an integral part of scientific industry. Bronkhorst (2006), for instance, considers the ways in which commentaries support the texts they comment on. He observes that while the primary function of the commentary is to clarify the fundamental text, that often in the process these technical exegeses go beyond this aim, and develop and advance the content in ways that the original authors might never have anticipated. Dr Sita Sundar Ram provides many examples which support these more general reflections. For instance, one of the examples she presents (pp. 212–215) is a worked solution to a *bhāvita* problem (expressions dealing with the product of two unknown variables). The problem amounts to solving $4x + 3y + 2 = xy$ for integer x and y.[2]

She notes that Bhāskara stated that such equations can be solved both 'algebraically' and 'diagrammatically', although he didn't provide the latter.[3] Kṛṣṇa Daivajña's commentary supplies a worked solution specifically using diagrams, where the products of the constants and unknowns are imagined to be rectangles with yet to be determined sides and various diagrammatic manoeuvres produce the unknown 'lengths'. Her careful and methodical treatment of Kṛṣṇa Daivajña's account with accompanying diagrams and identification of the vari-

[1] See, for instance, the discussion in Sarma et al. (2009, pp. 267–310), In particular, Kṛṣṇa Daivajña's *upapatti* of the *kuṭṭaka* process is translated and discussed in Appendix B.

[2] Bhāskara *Bījagaṇita* verse 204 (Sita Sundar Ram 2012: 267):

> *catustriguṇayo raśyoḥ saṃyutir dviyutā tayoḥ* |
> *rāśighātena tulyā syāt tau rāśī vetsi ced vada* ||

[3] Here, Dr Sita Sundar Ram translates *kṣetra* (lit. a figure) as 'geometrically', but what this amounts to is using diagrams.

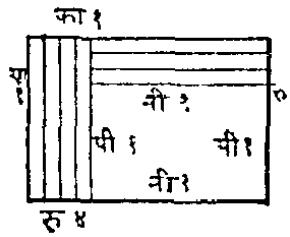
ous steps of working with the resulting rectangles, gives a sound appreciation of the original mathematical steps of working. A reproduction of a page from one of the editions can be seen in figure 1. Of course, this edition may be quite different from the way in which the scribes presented the text in their manuscripts, however it gives something of an impression of the layout and aspects invoked when solving this problem.

Dr Sita Sundar Ram includes many *upapatti*s throughout the work, sometimes supplementing *upapatti*s from other authors for completeness. Her decision to paraphrase Kṛṣṇa Daivajña's *upapatti*s using modern mathematical terminology has the benefit of making the mathematics tractable to the modern reader, however her symbolic summaries which are expressed using modern algebraic forms of reasoning can obscure the original phrasing and processes which are key to gaining insight into the original thought processes and practices of the Kṛṣṇa Daivajña. Readers are left with many questions. How precisely were these original 'equations' expressed in prose? And how were they manipulated, reduced, and simplified and how were various symmetries and similarities spotted without symbolic styles of reasoning? What is the language and grammar of these *upapatti*s and how do they contrast with other technical exegetical passages? All of these features are vital to understanding more fully the contrasting and rich ways of operating in the specific circumstances that commentators such as Kṛṣṇa Daivajña were working in. There is great potential here to expand on this topic.

For example, a theme of considerable interest is Bhāskara's treatment of interdeterminate equations of the second degree (invoked by the Sanskrit compound *vargaprakṛti*) which can be captured in modern symbolic notation as $Nx^2 + 1 = y^2$. Before turning to Bhāskara's approach to solving expressions such as these, Kṛṣṇa Daivajña presents the details of earlier mathematician Brahmagupta's approach, known as the *bhavana* method. Kṛṣṇa Daivajña gives three *upapatti*s to illustrate this method. Dr Sita Sundar Ram begins her depiction of Kṛṣṇa Daivajña's commentary by setting up auxiliary equations $Nx_1^2 + k_1 = y_1^2$ and $Nx_2^2 + k_2 = y_2^2$, multiplying both sides by y_2^2 and so on, working through entirely algebraic steps of reasoning to derive the final expressions which generate the required integer solutions. However, Kṛṣṇa Daivajña's passage is entirely in prose. It employs specific inflections of grammar and technical vocabulary to express relations and operations between unknowns, which are themselves expressed by lexical units *jye* for *jyeṣṭha*; *kṣe* for *kṣepa* and *ka* for *kaniṣṭa*. Kṛṣṇa Daivajña's 'auxiliary equations' are thus expressed: *āka 1 ājye 1 ākṣe 1 dvika 1 dvijye 1 dvikṣe 1*. Algebraic substitutions and simplifications, grouping of like terms, indexical relations, and the like do not 'appear' quite so obvious in prose, so the process of reworking equations to new expressions is achieved quite differently.

स्यात् । एवं कालकाङ्कतुल्यै रूपैरूनो यावत्तावद्वर्णो लघुक्षेत्रस्य द्वितीयो भुजोऽस्यतोऽसौ कालकाङ्कतुल्यै रूपैर्युतः सन्यावत्तावन्मानं स्यात् । अत्रेष्टं यदि कालकखण्डात्मकस्य भुजस्य मानं कल्प्यते तदाऽनेन क्षेत्रफले भक्ते यत्फलं तद्यावत्तावत्खण्डात्मकस्य द्वितीयभुजस्य मानं स्यात् । अत इष्ट यावत्तावदङ्कयुतं कालकमानं स्यात् । (फलं कालकाङ्कयुतं यावत्तावन्मानं स्यात् ।) यदि त्विष्टं यावत्खण्डात्मकस्य भुजस्य मानं कल्प्यते तदा फलं कालकखण्डात्मकस्य भुजस्य मानं स्यात् । अत इष्टं कालकाङ्कयुतं यावत्तावन्मानं स्यात् । फलं यावत्तावदङ्कयुतं कालकमानं स्यादिति । अत उपपन्नमिष्टफलाभ्यां स्वेच्छया संयुतौ वर्णाङ्कौ व्यत्ययाद्वर्णयोर्माने ज्ञातव्ये इति ।

अथवाऽन्यथोपपत्तिः । भावितक्षेत्रान्तर्गतक्षेत्रस्य भुजयोर्माने अ-यवर्णौ कल्पिते दर्शनं



इह नीलको यावत्तावदङ्कतुल्यै रूपैर्युतो जातं कालकमानं नी १ रू ४ । एवं पीतकाङ्कः कालकाङ्कतुल्यै रूपैर्युतो जातं यावत्तावन्मानं पी १ रू ३ । एवं क्रमेण जाते यावत्तावत्कालकमाने पी १ रू ३ । नी १ रू ४ । आभ्यां पक्षयोरनयोः या ४ का ३ रू २

याकामा १

यावत्तावत्कालकावुत्थाप्य जातमुपरिगपक्षे पी ४ रू १२ नी ३ रू १२ रू २ । द्वितीयपक्षे तु यावत्कालकयोर्वधोऽनेकगुणनार्थं न्यासः पी १ । नी १ रू ४ गुण· रू ३ । नी १ रू ४

नाज्जातो द्वितीयपक्षः पीनीभा १ पी ४ नी ३ रू १२ । एवं पक्षौ

पी ४ रू १२ नी ३ रू १२ रू २

पीनीभा १ पी ४ नी ३ रू १२

अथ नीलकयोः पीतस्योश्च तुल्यत्वान्मध्ये अनेन नाशे जातौ पक्षौ·

रू १२ रू १२ रू २

नीपीभा १ रू १२

Figure 1: A page from Āpaṭe et al.'s edition of the *Bījapallava*. Here, Kṛṣṇa Daivajña is detailing the way to solve a *bhāvita* problem diagrammatically.

While taking a dozen or so lines of modern symbolic reasoning, Kṛṣṇa Daivajña's explanation takes over three pages of printed commentary! (See, for instance pp. 93–96 of Āpaṭe et al.'s edition). All of these features are highly pertinent for understanding more fully the distinct mathematical practices in this culture of inquiry. Indeed, while modern historiographical practise in recent decades has tended to overly denigrate those approaches to original text which use the resources, symbolic and otherwise, of modern mathematics, the methodological choice to depict the mathematical content of the text with modern symbolism does have a place in historical expositions. However, it must be employed with care.

Overall, preparing a technical text such as this presents many challenges. It requires broad mathematical expertise and mastery of Sanskrit, as well as historical sympathy and reflection. Dr Sita Sundar Ram's comprehensive study of this seminal work is thus an impressive and commendable contribution and her achievement has provided a solid foundation both for reference and further investigation. The work is well organised, comprehensive, and amply referenced. Sound mathematical analysis underscores the textual passages, clarity is provided on some fairly complex and intricate mathematical passages, and her work will be accessible to a wide audience including mathematicians and historians alike. This publication will serve as a important source for contributing to our knowledge of second-millennium astral sciences in India and is an incentive to continue to explore and study critically commentaries such as these.

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