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Mathematics, Administrative and Economic Activities in Ancient Worlds edited by Cécile Michel and Karine Chemla

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Recently, we have been fortunate to see the start of a new research project, Mathematical Sciences in the Ancient World (the SAW Project), and the publication of a new book series, Why the Sciences of the Ancient World Matter. The SAW Project was funded by an advanced research grant from the European Research Council from 2011 to 2016, which had Karine Chemla as the principal investigator, together with Christine Proust and Agathe Keller, all researchers in the history of ancient mathematics of, respectively, China, Mesopotamia, and South Asia. The book series started in 2018 and it has been one of the most important vehicles to make public the extraordinary amount of material produced by the research project.

The book under review is volume 5 of the series, but it must be said that volume 6 of the series has already been published: *Cultures of Computation and Quantification in the Ancient World* [Chemla, Keller, and Proust 2022]. This is important to mention for at least two reasons. First, in full disclosure, I am one of the contributors to volume 6 [Gonçalves 2022], although I did not participate in the conception or production of either volume. Second, because volumes 5 and 6 are, so to speak, siblings, both of them having resulted from the endeavors of phase 1 of the SAW Project, which was "focused on documents related to administrative and economic activities" [ERC/SAW Project Final Report Summary] in its approach to the general issue of the variations of mathematical cultures, not only chronologically and geographically but also in accordance with the *milieu* in which they existed [ERC/SAW Project Fact Sheet].

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As a first approximation, the book is an inquiry into the relations between mathematics as registered in strictly mathematical texts and mathematics as present in sources belonging to economic and administrative practices. Its historical scope is mainly early imperial China (2 chapters), early classical India (2 chapters), and ancient Mesopotamia (7 chapters). One additional chapter deals with medieval Europe. These chapters are preceded by a comprehensive and engaging introduction, “Mathematics, Administrative and Economic Activities in Ancient Worlds: An Introduction”, by Chemla and Michel, the editors [1–48].

The book is divided into four parts:

(1) Mathematical Writings, Regulations, Laws and Norms [chs 2–4];
(2) Quantifying Spatial Extension, Quantifying Work [chs 5–7];
(3) Quantifying Land and Surfaces [chs 8–9]; and
(4) Prices, Rates, Loans and Interests [chs 10–13].

The organization of the book, therefore, follows neither a geographical logic nor a chronological one but a logic given by the sources, their purposes, and the social *milieux* that created them. Consequently, each part has chapters dealing with more than one of the main geographical areas of interest in the book, which gives the reader a rare and rewarding opportunity for comparison.

There are also three appendices, followed by detailed bibliographic references, each containing the conventions used to represent numbers and metrological systems of cuneiform texts, texts from early imperial China, and texts in Sanskrit, as well as other Indic languages. These conventions are followed in the present review, where units of measurement are unavoidable.

One additional appendix comprises two maps of Mesopotamia: one in the Ur III period (the setting for ch. 5 and, partially, ch. 6) and the other in the old Babylonian period (the setting for chs 2 and 11). There are no maps, however, for the other periods of the Mesopotamian history touched on in chs 8–9 for the early dynastic period, ch. 10 for the old Assyrian period, or in the chapters dealing with China or South Asia.

The volume closes with a detailed index, thus providing the reader with a practical way of assessing, for instance, how recurrent certain topics such as estimation, exactness, merchant, price, rounding, and tax are treated throughout the book.

Before going into the details of the parts and chapters, it is worthwhile to take a closer look at the introduction, but before doing this I should explain three features of this review.

First, *Mathematics, Administrative and Economic Activities in Ancient Worlds* is a dense work, and, while I have tried to convey as much of it as possible, I could only provide a glimpse of what each chapter contains.

Second, in that I am a researcher in the history of cuneiform mathematics, I have tended to articulate in more depth the arguments involving the seven chapters of the book that deal with Mesopotamia. If, despite my efforts, I have left aside important points in the chapters on China and South Asia, and in the final chapter on coins and coinage in medieval Europe, this should be considered as no more than a token of my own limitations. Having said that, I should also add that I learned a lot by reading, summarizing, and commenting on these chapters and now feel much more knowledgeable of the history of ancient mathematics. This admission also encapsulates my realization of the importance of getting oneself out of the comfort zone of their specialty. This is one of the beneficial effects that the book will have on most readers.

Finally, the introduction itself draws new conclusions about the specific topics of each chapter, and it does this in part after summarizing what is presented in each chapter. My remarks on the introduction and my summaries of the chapters likewise involve some repetition, which I have tried to keep to a minimum but could not avoid altogether.

1. The introduction

The book's introduction, besides presenting each of its parts and chapters, also discusses problems, objects, and approaches in the research on the history of mathematics. This is done in several ways.

First, the introduction reflects on the nature of the sources. The sources for the research reported in the chapters are mainly textual. Exceptions are standards of measure, such as Shang Yang's cuboidal *sheng* measure; Wang Mang's cylindrical *hu* measure; and the weights, balances, and possible rods and ropes for surveyors' work. Finally, in Martin Sauvage's contribution to the volume [ch. 6], in addition to texts, bricks and construction works serve as "documents" for research.

Second, Michel and Chemla also draw the readers' attention to the different ways in which these sources came down to us: the sources from Mesopotamia were obtained by excavation, either illicit of legal; the sources from India were primarily handed down through a written tradition; and the sources from China have come down to us in both ways, through archaeological excavation and written tradition [6]. This is further refined by considering among the sources which came from practice, which from official texts, and which from a scholarly environment. Table 1 [p. 6 below] summarizes the discussion.

As already indicated, the book also has a chapter on the mathematics related to the coins and coinage of 15th-century France [ch. 13]. It draws on John of Murs' treatise on coins, *De monetis*, as well as on manuscripts used by moneyers, money changers, mint warders, and mint masters.

Third, the introduction develops what could be called the "methodological guidelines" used in the research represented in the chapters. More precisely, to facilitate study of the relationship between more strictly mathematical practices and the mathematics of administrative and economic *milieux*, the editors develop a heuristic framework:

> We will see that some of our more strictly mathematical sources adhere to administrative and economic activities to such an extent that they help us interpret some key practices in these contexts. In another respect, in cases where we are able to identify contexts in which mathematical knowledge and practices show differences, how can we approach whether and how knowledge circulated across different *milieux*? These questions define a general program to which this book offers a contribution. We hope that these forays will encourage other colleagues to look further into these issues. [4]

Finally, Michel and Chemla deal with various types of tasks, the actors, and the contexts in which the sources drawn on in the various chapters originated [15–16]. They begin by establishing a large-scale bifurcation of actors and associated tasks into state administration and the activity of merchants. State administration itself is further split into the computation of payments, taxes, salaries and wages, planning of work, and measurement of areas. The activities of merchants include computing prices, lending, and, in the case of medieval Europe, coinage and management of coins. Table 2 [p. 9 below] summarizes this. Also, in the administration of state, regulations played an important role, so they appear frequently throughout the volume [16].

The central goal of each chapter, as Michel and Chemla propose, is to analyze its chosen sources from the region of the world on which it concentrates in order to characterize how historical actors approached quantification and computation as a body of practices constituting a "culture of computation". Within the same historiographical unit, it will become clear that in some instances, administrative and economic texts will evidence a culture of computation more closely related to that of the more strictly mathematical texts; whereas in other instances, there will be discontinuities.

One could also describe the main aim of the volume as a quest to understand different cultures of computation and quantification, that is, different mathematical cultures. This preoccupation with mathematical cultures is one that can be traced in the several publications by Chemla. As acknowledged on the first page of the introduction, albeit in a footnote, this volume, as well as the next in its series [Chemla, Keller, and Proust 2022], was born from a "three-month workshop on 'cultures of computation and quantification' and its final conference" [SAW 2023]. The footnote on the first page of the introduction also explains that the "definition and founding principles of this collective research" are set out in previous publications by Chemla [2007, 2009, and 2010] as well as in the introduction to Chemla, Keller, and Proust 2022. One could also cite the article in which Chemla explains that "by the expression 'mathematical cultures', I refer to ways of doing mathematics that are collectively shared" [Chemla 2016, 1], but by no means does this suggest that a mathematical culture represents or corresponds to a "people" or "nation". Instead, the "collectives to which these shared practices of mathematics testify need to be established on the basis of evidence, and not assumed a priori" [Chemla 2016, 2].

In *Mathematics, Administrative and Economic Activities in Ancient Worlds*, the readership will often be confronted with the different ways of doing mathematics that gained life in the past. It is for this reason that comparing calculation and quantification in administrative and economic activities with more strictly mathematical texts was chosen as a starting point. This is a situation that potentially brings evidence of difference between cultures of computation and calculation, as well as similarities.

Whatever the relation between these cultures of computation and quantification, the analytical processes of the sources allow us to learn much more about each of them and "especially [to] bring their diversity to light" [4]. In order to do that, or perhaps because of doing that, the chapters, as the editors state, "provide a better understanding of the role [that] mathematical knowledge and practices have played in allowing various types of practitioners to carry out managerial and economic activities in the ancient world" [4].

In my view, the chapters comprising this collection offer a highly original and valuable insight into questions that have become central to the history of ancient mathematics.

	Excavations			Written Tradition
	Sources from Practice	*Official or Display Texts*	*Mathematical Sources*	
Ch. 2	Administrative texts	Royal inscriptions and collections of laws	Catalog texts	
3				*Arthaśāstra of Kauṭilya*
4	The Juyan administrative documents	Bamboo slips from Qin tomb 11 at Shuihudi, sealed in *ca* 217 BC; Han tomb 247 at Zhangjiashan, sealed in *ca* 186 BC	• *Writings on Mathematical Procedures* (*Suanshu shu*), found in Han tomb 247 in official excavations • *Mathematics (Shu)* (*terminus ante quem* in 212 BC) from the antiquities market	• *The Nine Chapters on Mathematical Procedures* (*Jiu zhang shuanshu*), first century AD • Monographs in *The History of the [former] Han [dynasty]* (*Han shu*) and commentaries on them handed down with them
5	Administrative documents from irrigation projects			
6	Administrative texts containing work norms		Catalog texts dealing with bricks and work norms	
7	see ch. 4	see ch. 4	see ch. 4	• see ch. 4 • monographs in the *History of the Sui (Dynasty)* (*Sui shu*)
8	Survey texts			
9			Lists of area computation in tabular or linear format	
10	Merchants' letters		School texts	

Table 1. Summary of how sources have come down to us

Administrative texts mentioned in this table also include those concerning economic matters

	Excavations			Written Tradition
	Sources from Practice	*Official or Display Texts*	*Mathematical Sources*	
Ch. 11	Administrative official texts		Numerical texts, metrological tables	
12				• *Arthaśāstra of Kauṭilya* • Legal sources (uncertain date) from the beginning of the Common Era and the sixth century • Models of administrative and business letters compiled in the *Lekhapaddhati* (which documents the court of the Caulukya kings) from the middle of the 10th century to the early 14th • Mathematical writings: *Āryabhaṭīya* (AD 499), the *Brāhmasphuṭasiddhānta* (628) and the *Mahāsiddhānta* (second half of the 10th century); whereas others writings, such as the *Gaṇitasārasaṃgraha*, the *Pāṭīgaṇita* (ninth century), the *Līlāvatī* (12th century), and the *Gaṇitakaumudī* (1356), are devoted to mathematics

Table 1. Summary of how sources have come down to us

Administrative texts mentioned in this table also include those concerning economic matters

First is the “disunity” [21, 22, 44] of the mathematics (seen as either theory or practice or as a blurred mix of the two) in each of the historiographical units investigated: China, India, and Mesopotamia. This disunity, or heterogeneity, of mathematics is an idea of deep impact on how one thinks of mathematical knowledge and practice. It comes, as the book widely exemplifies, from the variability of approaches in communities of practice within each selected historical unit. As Michel and Chemla suggest, each *milieu* corresponding to a community of practice must have had its own kind of relationship to the more strictly mathematical texts usually associated with teaching environments or the environments in which mathematics was practiced for its own sake [3]. A practical consequence for the book was the need to focus on sources that can bring out these differences clearly [4]. All chapters taken together, the book clarifies how mathematical contents are shaped in each specific *milieu* and then circulate, as they are reshaped and reorganized according to the needs and values of each *milieu* in which they become a practice [44]. Finally, the case studies in the book allow one to see the “disunity of mathematics” and “invite…us to rethink key operations like quantification and computation” [44].

Second, although less emphasized by Michel and Chemla, the research published in this book demonstrates how the sources that have come down to us can be seen to evidence not only the relations between mathematics and the different spheres of life in a society—as if these were actually separable entities—but also the very constitutive role of mathematical knowledge and practices in a society. This seems to be in line with the statement by Michel and Chemla that “mathematical knowledge and practice played an important role for the ruling houses” [3]. Also, this seems to accord with Peng’s view that “measurement values and numbers defined and brought together the social and administrative hierarchy” [125]. The phenomenon resurfaces in other passages of the book, for instance, in Mark McClish’s analysis of how fines for erroneous weights and measures were determined, where a major role was attributed more to certain “relations between denominated measuring instruments than [to] computation” [112], thus implying that these relations silently shaped and became integral to the final determination procedures.

Both the disunity of mathematics and its constitutive role in society have been in the scope of historians of mathematics for a while, as I pointed out in Gonçalves 2020, but the book under review represents a new and invaluable large-scale, collective, and systematic endeavor in these directions.

State Administration

Computation of payments, taxes, salaries, and wages

Ch. 3	General management of the state, noting that the *Arthaśāstra* was not connected to any specific context; overlap with merchants, especially in moneylending
4	Computation of wages of officials and taxes on land
7	Measurement of grain for computation of salaries of officials and state taxes
	Planning of work and related tasks
5	Computation of volume (as spatial extension) and workload in irrigation projects
6	Computation of workloads involving brick in construction works
	Measurement of areas
8	Land surveying and land recording
9	More scholarly scribal tradition

Merchants

	Computing prices
10	Assyrian merchants, computing prices through conversion of values
11	Focus on an overseer of merchants computing prices; overlap with state administration
	Lending
12	Moneylenders
	Management of coins
13	Money changers, mint wardens, minters, management of the production of coins

Table 2. Summary of *milieux* treated

The remainder of the introduction comments on the four sections of the book, with special attention to the meta-historiographical question on the relationship of administrative and economic sources with the more strictly mathematical ones. More precisely, when do they and when do they not coincide? If they seem to indicate similar cultures of computation and quantification, can one determine whether practitioners had scholarly training? Or was it the contrary, with the scholarly texts reflecting, imitating, and being shaped by managerial and economic practices. Yet, to do that the introduction often builds on the then unpublished chapters of the now published volume 6 of the series.

1.1 An overview of the volume

Part 1 "confirms, if it were needed, that there was no uniformity of mathematical practice across a society" [19], but the cases of China and Mesopotamia show some continuity between the mathematics of practice and the mathematics of strictly mathematical texts. Chapter 2, by Michel, Robert Middeke-Conlin, and Proust shows that the use of numerical values in school texts was similar to the use in administrative texts but different from what is found in official and the so-called display texts, that is to say, inscriptions that were made for the public eye. Chapter 3, by McClish, by comparing the *Arthaśāstra* and the extant mathematical writings in Sanskrit, demonstrates that there was discrepancy in the "terms referring to computation and quantification" and how fractions were used in the expression of measurements [21]. Chapter 4, by Peng Hao, shows that mathematical manuscripts from excavations exhibit notable consistency with legal texts in respect to the mathematics necessary to implement the prescribed regulations. *The Nine Chapters on Mathematical Procedures*, a text handed down in the written tradition, is also shown to echo the mathematics involved in putting regulations into practice, although less intensely than the manuscripts, which together with Chemla 2016 (2018) may testify "to the emergence of a more common and less situated mathematics" than the one practiced by state officials [25].

Part 2, which is motivated by part 1, is an effort to detail, in the cases of China and Mesopotamia, practices by which spatial extension and work were quantified. The procedure adopted involves paying attention to the "various ways of quantifying spatial extension and their relationship with one another" [26]. In assessing spatial extension, mentions of vessels are frequent in all kinds of sources. In some cases, real vessels are also known from archaeology. The first takeaway of part 2 that is underlined in the introduction is an enlightened understanding of the reason why two different ways of measuring spatial extension, namely, capacity and volume,[1]

[1] Here capacity is a spatial extension value in a system where some or all of the units

are attested in the ancient sources from China, Mesopotamia, and South Asia, a topic especially worked out by Chemla and Ma in ch. 7. Capacity could be measured using standard vessels. Volume, on the other hand, was calculated and thus required mathematical knowledge. In measuring the capacity of large amounts of a commodity like grain, volume intervened, and the result could be converted into capacity. Furthermore, the units of measurement used to express spatial extension were also an indication of the method used to obtain the value measured [27].

A second takeaway is related to a similar enmeshment between the various forms of assessing spatial extension, but in the context of the cuneiform sources. Volumes in cuneiform sources are indicated with the same units as areas. An important textual genre in cuneiform mathematics, the metrological tables, is subdivided into capacities, weights, lengths, heights, and surfaces. There were no exclusive metrological tables for volumes; those for area were used for volumes too. This is the first entanglement between units of measurement, namely, spatial extension and surfaces. Besides, in the cuneiform sources, measuring the spatial extension of a pile of bricks also involved some specifics. More precisely, this was "the use of specific coefficients (*nalbanum*), expressed using the sexagesimal place value system and depending on the size of the brick", and the result was sometimes understood "as a volume expressed in a number of bricks" [29]. In order to understand why practitioners used different types of volumes, it is necessary to look at their practice. So, Stephanie Rost [ch. 5] unscrambles the administration of canals in the Ur III period, of which both the planning and the execution required computation of volumes and conversions involving workloads, capacities, volumes, and wages. In the same line, Martin Sauvage [ch. 6] decodes several aspects of building projects that involved managerial operations with (sometimes extraordinarily) high numbers of bricks. In relation to volumes of bricks expressed as a number of bricks, Michel and Chemla argue that it "may constitute a mathematical reflection of a type of unit that has a specific usefulness for administrative quantifications of a given kind" [31]. Giving more precision to the statement, they say:

> Brick units were units of volume used for specific purposes. In these contexts, and in these contexts only, they may be considered as being standards of measurement that could be used to measure a volume directly. [31]]

are dependent on measuring vessels. Volume is a spatial extension value in a system where some or all of the units are related to units of surface and length.

All in all, part 2 deals with topics in which historians can benefit

> from bringing together mathematical documents, administrative regulations and documents of practice, in cases when these sources seem to have been related with each other in actual practice. [31]

Part 3 comprises only chapters 8 and 9. The geographical and thematic scopes of this section are very focused: the calculation of surfaces (both of plots of land and geometrical figures) toward the end of the early dynastic period in Mesopotamia, indicated by Assyriologists as ED IIIa and ED IIIb, which in political history correspond to the two and a half centuries that preceded the accession of Sargon I of Akkad. Chapter 8, by Camille Lecompte, analyzes a group of texts originating from field survey practices. Chapter 9, by Proust, treats a small number of texts that are, at least in part, a bit older than those analyzed by Lecompte. These are, by the way, considered the earliest surviving mathematical cuneiform texts [35, 346]. The tablets analyzed by Proust are numerical and do not give the context of the calculation of surface areas presented. In the introduction of the book, Michel and Chemla, comparing chapters 8 and 9, draw a conclusion that has deep theoretical consequences for the work of historians of mathematics:

> In any event, we can derive several conclusions, essential for us here. First, we see how a historiography that would rewrite the actual numbers and measurement values used by the actors may obscure the actual problems they were facing. Moreover, here, in relation to the specific features of the measurement units, what we may consider to be a single problem—the computation of an area—, solved by a single operation—multiplication—, appears clearly to have constituted different problems, depending on the size of the surfaces considered. [37]

So, they conclude, part 3 "has allowed us to highlight differences precisely in the mathematical knowledge practices brought into play to address the same problem" [38]. These differences make the comparison meaningful to us, but perhaps actors would understand it differently. This brings us to the final part of the book.

Part 4 is more specific. It examines

> more systematically, and more broadly, how documents of practice and mathematical texts compare with each other with regard to [the] mathematical knowledge and practices to which they attest, in cases when they can be meaningfully compared. [38]

That is, the four chapters comprising part 4 of the book treat prices, rates, loans, and interest in the contexts of both cuneiform and Sanskrit texts, plus the European Middle Ages. According to Michel and Chemla, the chapters in part 4 can be classified under three headings in diverse ways.

There is a case in which values of a different mathematical nature are used differently. This is examined in chapter 11 by Middeke-Conlin, who brings strong evidence that a certain official in the kingdom of Larsa received mathematical training in the style found in the more strictly mathematical sources, attesting thus to a continuity between the mathematical cultures of the school and the practice [38–40]. But there are also cases in which different cultures of computation can be identified in the ways in which quantities are handled, ways which are examined in chapter 10 by Michel and chapter 12 by Sreeamula Rajeswara Sarma and Takanori Kusuba.

In chapter 10, Michel shows that Assyrian merchants learned to calculate and made calculations of prices and interests directly using values of measurement but not the sexagesimal place value notation; thus, their mathematical culture of computation was entirely different from what is found in the Babylonian region during the same period in the same context of converting prices and computing interest [40–41]. Sarma and Kusuba offer a comparison of interest values in mathematical texts with those found in administrative and legal treatises. While such treatises demonstrate a systematic preoccupation with the notion of fairness in setting a limit on the value of the interest rate, mathematical texts are mostly concerned with the mathematical aspects of the topic and sometimes they mention interest rates that would be considered too high if put into practice [41–42].

There is, finally, a case in which different cultures of computation are identified in the way knowledge is organized, a topic also examined in chapter 12 as well as in chapter 13 by Marc Bompaire and Matthieu Husson. In chapter 12, Sarma and Kusuba argue convincingly that the way in which knowledge about the calculation of interest is organized is seen to differ when the mathematical chapters in the Sanskrit treatises on astral sciences are contrasted with other mathematical texts. While the latter seem to reflect a state of things closer to actual practice, the former contain problems on interest calculation that, because of their visual way of handling data and the introduction of a new conceptual framework, seem to indicate their desire “to address new types of mathematical issues” [42].

Finally, according to Michel and Chemla [43], chapter 13 provides us with a “broader view of the contrast between the two mathematical cultures” reflected by John of Murs’ treatise on coins, the *De monetis*, and by manuscripts used by professionals who managed coins. John Murs’ treatise values generality in situations where the professionals’ manuscripts prefer instead to use shortcuts both to simplify calculations and to make operations visible [43–44]. Michel and Chemla conclude:

> As for most of the cases analyzed in this book, the cultures of quantification and computation thus appear to be very strongly influenced by the values meaningful to the professional *milieu* considered. [44]

2. Summaries of the chapters

2.1 Mathematical writings, regulations, laws, and norms (Part 1)

Cécile Michel, Robert Middeke-Conlin, and Christine Proust, in "A Comparative Study of Prices and Wages in Royal Inscriptions, Administrative Texts and Mathematical Texts in the Old Babylonian Kingdom of Larsa" [ch. 2], concentrate on wages and prices in the kingdom of Larsa during the Old Babylonian period (20th and 19th centuries BC). The goal of the chapter is to assess the extent to which prices and wages in royal inscriptions and mathematical texts "match those of administrative texts" [57]. The rationale for such a comparison is that, although administrative texts did have to conform to reality because "they document economic and everyday activities" [57], both royal inscriptions and mathematical texts did not necessarily do that. Royal inscriptions present "the king's point of view", so they "provide a biased perspective and cannot be used uncritically" [54]. Mathematical texts offer a view of the situation in two ways: "Scribes were aware of possible fluctuations in price linked to the availability of goods" [56], but in several "mathematical texts, problems deal with abstract situations", thus having less connection with real administrative and economic activities [57].

The inscriptions of the kings of Larsa show stability of wages and prices. The authors describe wages paid to workers employed in the construction of several monuments: under Nūr-Adad, the great wall of Larsa; under Sîn-iddinam, the digging of the Tigris and the building of a shrine for the Sun god (the Ebabbar temple); under Warad-Sîn, the wall of Ur. The authors also compile prices that were set under these kings according to their inscriptions [58–59]. Apart from the wages said to be paid to workers under Sîn-iddinam, which were much higher than the other wages, all these inscriptions indicate "stability and uniformity" [60]. Evidence from Uruk, under the contemporaneous reign of Sîn-kašsid, shows the same average values [59]. Finally, since tablets containing passages from the Code of Hammurabi were also found in Larsa, the authors describe the sections that correspond to wages and prices, the latter being more specifically for hires [60–62], completing the treatment of royal inscriptions.

As there is no mathematical text with assured provenance from Larsa, mathematical texts are only quickly examined and conclusions are kept at generalities. The authors restrict themselves to but four "catalog texts", which is how texts containing statements of problems without solutions are referenced. Two of these catalogs concern canals (YBC 4666, YBC 7164), and the other two bear upon excavation works (YBC 4657, YBC 5037). These texts are described and their numerical contents are displayed in useful tables [63–65]. Wages are daily payments to workers for different kinds of work (excavations of canals and maintenance, for instance). The materials in use varied too: earth, mud, water, or silt, plus what the ancient vocabulary refers to as *šilutum* and *dusu*, terms for which the exact meaning is said by the authors to be unknown [63], and, finally, a term meaning "a mixture of plant-matter and earth" [63].

The administrative texts described in this chapter fall into two categories: there are those that detail transactions such as "deliveries or receipts of goods, sale contracts, lists of expenditure, loans,...animals, textiles, wool, reeds, bricks, etc." [65]; there are also others that list "wages paid to workers" [69]. One text of each kind is quoted and described in detail. Seventeen texts of the first kind have their contents summarized in a table [67–69]. They cover a century, "from year six of the reign of Sîn-iddinam until the seventh regnal year of Samsu-iluna" [66]. The data from this material permit the authors to conclude that during this period the value of silver in relation to gold increased from 1:9 and 1:7 in year 2 of the reign of Rim-Sîn to 1:4 and 1:3 in year 27 of the reign of Rim-Sîn. This phenomenon is similar to what Farber [1978] described concerning northern Babylonia. The second kind of text, listing wages paid to workers, covers the period from year 10 of Warad-Sîn until year 38 of Rim-Sîn, making 40 years. From these data, the authors draw our attention to the fact that the "wages range from a few *silas* to 1 *bariga* per day when officials are concerned" [72].

The data concerning wages and prices in the three genres of text are then organized in summary tables, depicting values equivalent to 1 *gin* of silver [Table 2.8], wages per day of workers in the construction of buildings [Table 2.9], and wages of various types of work in grain or silver, per year, month, or day [Table 2.10]. The authors note that, as the texts analyzed were produced in different environments and for different purposes, comparison has limits [73]. Having said that, a few insights are elaborated [77]. First, a comparison of prices and wages in royal inscriptions and administrative texts makes clear the difference in purpose of those royal inscriptions describing a king's building projects and those depicting the king as a bearer of justice (as in the Code of Hammurabi and the Laws of Ešnunna). More precisely, the prices in the administrative texts are systematically higher than those in royal inscriptions, but only similar or lower in comparison to the prices stipulated in the Laws of Ešnunna. Besides, while in building project inscriptions the kings wanted to boast of how well paid their workforce was, in the law inscriptions the kings intended to create an image of protector of common workers from exploitation by fixating on what were probably minimum values for wages.

Second, topics in mathematical problems are reminiscent of the building concerns of kings and may be a "heritage of the Ur III period from which tens of thousands of texts" relate to these topics [77]. However, the numerical values, as apparent in the comparison with administrative texts, are real. This means that the "authors of these problems" might "have been inspired by the organization of the work of large construction projects" in the turn of the third to the second millennia but used values as practiced in their own time [77].

Mark McClish's "Computation in the *Arthaśāstra*" [ch. 3] is a philological study of computation practices in the *Arthaśāstra of Kauṭilya*. According to McClish, the *Arthaśāstra* was first composed in the first century BC and was extensively rewritten around the third century AD [81–82]. It circulated in "royal courts and offices" as well as in elite educational circles [82]. Despite not being relatable to any specific context, the *Arthaśāstra* gives detailed instructions about several activities of interest to state administration, from which "something can be gleaned regarding practices of computation" [82]. The text has many limitations as a historic source because "its purpose was never to describe the practices of any given place or time" [83]. However, as it is "clearly informed by observations and knowledge practices" [83], it is nevertheless informative of at least the "framework within which educated administrators, kings and others thought about the activities of the state" [83].

The sources used by the *Arthaśāstra* were probably composed after Emperor Aśoka's rule (268–231 BC). They belonged to a stream of "expert traditions devoted specifically to the topic pertaining to governance" [84]. The *Arthaśāstra*, however, does not offer details of computation, but the state that it envisions is one that "required specialized expertise, often including specific computational practices" [84]. Thus, what can be learned about computational details of that state depends on "close study of the language used to express and work with numbers and measurement values" [85].

McClish informs us that the *Arthaśāstra* expresses numbers by using the conventional systems of Sanskrit words, and that they are used together only with measurement units or when items are counted [85]. According to the *Arthaśāstra*, there are four ways a value can be specified: "weighing, measuring, counting and estimating" [85]. The author then presents a detailed account of the words and expressions used for measuring weight, capacity, space, and time [86–95]. This is followed by descriptions of how the *Arthaśāstra* treats counting [96] and appraisal (*tarka*), the latter understood as the "process for assigning values to objects alongside the acts of weighing, measuring and counting" and underlining "human agency to a greater extent than a simple measurement" [97].

When studying the operations and procedures in the *Arthaśāstra*, it is important to understand the previous processes of assigning value because "the *Arthaśāstra* does not engage in any computation with decontextualized numbers" [97]. Even so, it is difficult to identify in the text any piece of evidence for computation, both because the *Arthaśāstra* is silent in relation to operations and procedures and because it is not always clear if processes depended on measurement rather than on computation [97]. In any case, in the *Arthaśāstra*, operations or procedures can be divided into increase and decrease. Increase includes addition and iterative addition [98–102] as well as multiplication [102–103]; decrease, subtraction and iterative subtraction [103–106] as well as fractional division [106–107].

One example is found in the inspections of changes of volume of grain that were carried out under the responsibility of the superintendent of the storehouse. The text of the *Arthaśāstra* presents the variation of volume both when the grain is worked in several ways and for several types of grain. McClish draws our attention to the ways these variations are described: as a fractional ratio decrease or increase, as the subtraction or addition of a fraction, and as a multiple of the amount of reference [108]. Besides, the *Arthaśāstra* presents cases when fractions correspond to successive operations, which may be "some indication of a two-step procedure" [109]. The author explains, "Grain kernels are not conceived as a fraction of the unhusked volume per se, but as a fraction of *half* of the unhusked volume" [109]. Although it is not possible to know exactly the operations to which such representations corresponded or if they are witness to a preference for computing with fractions, it is certain that "the Superintendent's activities require computation" [109]. Besides, it opens a possible avenue of research, "comparing the use of fractions here with other quantitative practices in the text" [110].

The second operation examined concerns the application of fines "for the use of erroneous measuring instruments by merchants" [110]. By analyzing a passage of the *Arthaśāstra* that specifies a series of increasing fines for a series of increasing errors [111–112], McClish arrives at a very interesting conclusion. The calculation of fines is less a matter of effectively carrying out the calculation with fractions of the units of measurement and more a consequence of the way the system of weight units is structured: “Deviations were identified through measuring practices rather than computation” and “the application of these rules in the field need only have relied on knowing the measurement intervals at which fines were assessed” [113]. So, the implied computation in the analyzed passage would only involve “carrying out the iterative addition of standardized fines” [113].

After analyzing other topics of the *Arthaśāstra* that imply computation practices, namely, interest for coinage fees [113–114] and the excavation and construction of defenses [114–116], McClish summarizes the concern of the state with computation by noticing that it must have been most important in areas such as “state finance; commerce; fines; stores; regulation; manufacturing; and the planning of construction” [116].

The term «gaṇita» is the “primary Sanskrit term for the tradition of mathematics” [117], but in the *Arthaśāstra* its two occurrences mean counting only [117]. The closest the *Arthaśāstra* comes to mentioning mathematics or computing is by means of the term «saṃkhyāna» [117], a term which in the pair «lipi» and «saṃkhyāna» “captures the fundamental distinction between literacy and numeracy” [117]. To McClish, the uses of the verbal form «sam + khyā», the elements composing «saṃkhyāna», “refer specifically to processes that result in quantification, whether the processes themselves are computational or not” [118]. The term «anumāna» occurs mainly in referring “to processes or results of processes that use some kind of inferential framework to determine a quantity” [119], as “an inference” that is “made as to the quantity of food consumed by an animal in one day” [119] or “an inference of the loss in value suffered by different kinds of objects because of damage” [119]. The final term mentioned is «tarka» (appraisal), as seen above.

In the last section of his chapter, McClish explains how this analysis of the *Arthaśāstra* fits in a wider research program on “South Asian intellectual history” [121]. For example, the operations and procedures common in the *Arthaśāstra* are afterward borrowed by the legal tradition. A comparison between astronomical contents of the *Arthaśāstra* and the *Jyotiṣavedānga* (an astronomical treatise dated to 400 BC) also reveals similarities [121]. The chapter finishes by presenting exciting avenues of research in which the analysis of the *Arthaśāstra* constitutes a possible framework connecting it to “other cultures of computation in classical South Asia” [122].

"Official Salaries and State Taxes as Seen in Qin-Han Manuscripts, with a Focus on Mathematical Texts" [ch. 4] by Hao Peng/彭浩 is a study of how mathematical texts from the Qin (221–207 BC) and Han (206 BC – AD 220) periods deal with salaries. The sources used for this study are, for the Qin period, *Mathematics* (*Shu*) and, for the Han period, the *Statutes and Ordinances of the Second Year (of the Empress Lü)* (*Er nian lü ling)*, drafted in 186 BC, and *Writings on Mathematical Procedures*, dating to no later than 187–180 BC. In addition, Peng takes into account the Juyan administrative documents, covering the interval from 103 BC to AD 293 but dating mostly to eastern Han (AD 23–220). Finally, Peng also considers *The Nine Chapters on Mathematical Procedures*, a text believed to preserve contents from the Han period [126–127].

Salaries of official employees (*zhi*) reflected their ranks (also associated with the meaning of «zhi» as "order" [127]) both in the Qin and the Han sources. The former mentions officers of 50 *dan* and 100 *dan* as well as prefects of 600, 700, 800, and 1000 *dan*. The "capital clerk and commandery governor are 2000-*dan* offices" [128]. The latter mentions officers of 2000, 1000, 600, 500, 400, 300, 200, 160, and 120 *dan* plus lower ranked officers described as *dou* eaters (the *dou* was 1⁄10 of a *dan*). Salaries corresponded to amounts of grain, but mixed payments could occur, as attested in the Juyan administrative documents, which mention "cash" and "salt" payments [129–130]. Also, for nongraded officials, payments had to be deducted for periods of leave and meals taken on the road that were paid by the state [130].

The mathematics of the payroll depended also on conversions. First, it was necessary to convert harvested amounts of grain to unhulled grain (*su*) because the latter was the standard for all official calculations [132]. It was also necessary to use conversion rates between coarse and refined grains [133] as well as between hulled and unhulled grain [134], a conversion useful in the case of the deduction of the on the road meals once these were given in hulled grain [134]. Finally, the Qin and Han manuscripts also deal with the daily rations of salt and condiments [136]. Peng ends this part of the chapter with two remarks that accentuate the importance of metrology: fines were instituted for deviations greater than the one legally stipulated [137] and physical standards were produced, like the Shang Yang bronze *sheng* and the Gaonu [Prefecture] harvest plant *dan* bronze weights [138].

The next section of the chapter deals with the issue of state tax collection. Peng introduces the reader to a very diversified universe. Field tax, levied in grain, and hay-straw field tax, levied in fodder, are the first of the primary forms of taxation [139]. The taxed lands continued to belong to the throne, but the lessees paid the taxes and had the right to manage it. An interesting aspect is that lessees were entitled to "indemnities for crops eaten by another man's domestic animal" [139]. Another interesting aspect is the amount of land granted, which was closely related to the rank of the lease [139].

Another form of primary taxation appears in pass and market taxes. A pass was an inspection point, either in the frontiers or in strategic commercial places [140]. The details about how pass taxes were collected appear only in the mathematical works. As regards market taxes, this "refers to periodic country markets and permanent commercial zones possessed of specialized stalls for the exchange of urban and rural commodities" [141]. Both the Shuihudi and the Yuelu Academy manuscript corpora contain details about its management. One interesting aspect is the money box: cash markets had to be set by market administrators, who personally deposited the cash in a money box and certified the deposit in triplicate [142]. Peng then describes mining and salt taxes [142–143], poll taxes and mouth taxes [143], and government service [144].

The last section of Peng's chapter is devoted to detailing the collection and management of field taxes, which depended mainly on the maintenance of "complete land records and statistics on cultivated land" [145]. There are national statistics such as the Book of Han registers, but also statistics in lower administrative levels, commandery, and prefecture. The documentation mentions delimited fields, allotted fields, and borrowed fields. Peng discusses details of these different ways of describing a field [145–148]. The determination of how much to pay on tax fields depended on the field norm, which was the surface "required to achieve a production value in a given unit (be it capacity, volume or weight)" [148–149]. The norm was the "unit for calculating tax", and tax fields had to be registered in duplicate tickets [149]. Mistakes in ticketing appear as a problem in the mathematical texts, where they are corrected by "reducing the area of the tax field in question" [150]. Related legal statutes contained provisions of punishment for "officers for any mistakes they might make in the calculation, auditing or collection of field tax" [152].

Peng concludes his "overview of the application of mathematical knowledge in the state administration of salaries and taxation in the Qin-Han period" [152] by underlining the "variety of policies, economic regimes and laws on the macro-level" and feasible "methods for their implementation" [152]. This attests, the author concludes, to the "fairly high level of mathematical knowledge" [152] that officials of higher ranks involved in the state administration had to have. This is consistent with the fact that such officials were evaluated, among other criteria, for their writing and accounting skills [152–153].

2.2 Quantifying spatial extension, quantifying work (Part 2)

Stephanie Rost's "Insights into the Administration of Ancient Irrigation Systems in Third Millennium BC Mesopotamia" [ch. 5] is a study of the mathematics found in irrigation-related texts of the Ur III dynasty.

Her chapter begins with a survey of Ur III society, focusing on the sociopolitical background and the management of the agricultural land. It details the division of the kingdom into provinces, with special attention to the provinces of Umma and Girsu/Lagaš [162–163], the inner administrative division of the province of Umma into four districts, and the description of the main families that held important official posts and the governorship of the province [163–164]. Agricultural work is described by specifying the hierarchical relations between ox drivers, cultivators, inspectors of plow oxen, and scribes of plow oxen [164–166]. Connected to agricultural work was the issue of how land was separated into institutional and royal sectors [162–164]. Finally, there was the further issue of how a system of taxation, the *bala* system, connected all the provinces to the central administration [164]. Rost explains that

> in order to administer these complex labor systems and keep track of the fulfillment of a province's *bala* tax obligations, a bureaucracy of previously unknown dimensions was employed to keep records of nearly all aspects of life, including irrigation management. [170]

One of the most important innovations in this context was the introduction of standardized work norms that enabled officials in charge to convert total workloads into individual workloads and workdays [171]. The majority of the documents from the Ur III period are reports of expenditures of labor and goods, including the documents that are the object of Rost's inquiry, texts related to irrigation as exemplified by text 1 (A 02644), in which tasks were specified together with the number of workers and the duration (although all of them are intended to be done in just one day).

As Rost explains, "There are also texts that represent the planning stage" of irrigation projects [171], which is very important for the whole reasoning of the chapter. In this way, the bulk of the chapter [171–195] is dedicated to presenting a few representative texts, each one instantiating a different phase of work in an irrigation project: initial inspection [text 2: YBC 00952]; surveying and computation of costs, such as amount of work [text 3: YBC 01821 and text 4: Um. 1594]; assignment of workload to supervisors [text

5: Um. 0757 and text 6: BM 105352]; execution of work [text 7: Um. 0993, a receipt for the execution of the assigned work]; and payment of personnel [text 8: VAT 07384].

In presenting and translating the texts, Rost covers a large amount of contextual information and offers hypotheses for the interpretation of the less well-documented (or undocumented) aspects of the administration of an irrigation-related work. In all, the author delineates a comprehensive view of how the different phases of the process might have been connected among themselves and to the other spheres of the Ur III administration. In particular, Rost comments that the appearance of members of the governing families in irrigation-related works may be an indication that these works were of great magnitude and importance [190]. Also, by analyzing the dates of some of the documents, Rost suggests that irrigation systems might have been more stable than previously thought [182] in Assyriological studies.

In her conclusion, Rost emphasizes the importance of texts containing the planning of irrigation-related works [195]. They indicate that the administration of the province preferred to take part in all the phases of the work and that midlevel administrators had to master a skill set that was broader than has been previously assumed, suggesting also "a much higher level of literacy and numeracy in Ur III society" [195].

The chapter ends with a caveat. The texts analyzed represent the reality of the so-called institutional sector of the Ur III administration and so allow one to conclude that irrigation works in this sector were tightly controlled by the province administration. Less is known about the much larger so-called royal sector, which might have had a different type of organization [196].

Martin Sauvage's "Mathematical Computations in the Management of Public Construction Work in Mesopotamia (End of the Third and Beginning of the Second Millennium BC)" [ch. 6] is an investigation on the calculations used in constructions made with bricks. The goal of the chapter is "to see what types of calculations Mesopotamian scribes were trained in for the domain of construction and to try to check how far these calculation methods were really used in practice" [202]. The sources are both the more strictly mathematical texts from the scribal schools of the old Babylonian period and "practical administrative texts" from the Ur III and the old Babylonian period [207]. The practical texts contain "provisional estimations" of the material and workforce needed, "statement of accounts" of expenses, and "lists of works and their tasks" [207].

Sauvage offers a survey of the studies of the mathematics of bricks, touching on the Ur III and the old Babylonian periods. In mathematical texts, one finds both a typology of bricks and mathematical tools to solve problems related to building. According to the author, 14 types of brick are known [211], and the mathematical tools for working with bricks are labor norms and coefficients. The labor norms (Akkadian *iškarum*, Sumerian eš$_2$-kar$_3$) indicate how much work an average worker can perform per day. Examples are the volume of bricks that can be stacked, the volume of a wall that can be demolished, and the volumes of sun-dried or baked bricks that can be made [208]. Coefficients are used to "calculate certain magnitudes from other magnitudes" [210]. The *tupšikkum* is a weight load; the *muttallikum* describes a distance that a certain amount of material can be transported in a day by one person. For bricks specifically, Sauvage presents the *nalbanum*, "the number of bricks of a specific type in a volume unit" [216]; the *nazbalum*, "the number of bricks contained in a carrying load" [219]; and the *taddîtum*, "the number of bricks of a given type in a one *sar* area" [219]. The norms and coefficients are present in many mathematical texts dealing with brickwork. In some cases, for example, when a wall must be demolished and the material be carried over a distance [221], tasks must be serialized and the coefficients combined.

In a much shorter section, Sauvage presents and comments on the contents of a few Ur III administrative tablets, the numerical contents of which suggest that different values for labor norms and different brick coefficients were in use in that period. More precisely, the values in tablet AO 5675, from Umma, indicate the use of a work norm of 10 *sar* of area per day in a reed-related work [222]. Tablet MLC 2404, also from Umma, indicates the use of a volumetric work norm of 10 *gin* per day, a very common value for some earthworks [222]. Tablet A 2976, of unknown provenance, is a "provisional review of the materials and men that would be necessary for the construction of a storehouse" [223], indicating the use of different work norms and the *nazbalum* coefficient for brick carrying [223]. Finally, CUNES 48-07-066, from Garšana, is "a list of brick deliveries" [224] that also presupposes the *nazbalum* as applied to bricks that correspond to one of the known types.

Sauvage concludes that "mathematical exercises were created with realistic values in order to provide practical training for the scribes who would work later on building operations" [224], which means that "in this respect, training of the scribes concerning building work was really practical and not only theoretical" [225]. As a last contribution, Sauvage points out possible avenues of research, namely, searching for a better understanding of the differences in labor norms between men and women—a difference not commented on in this review but present in the sources analyzed by the author—and if and how work norms were subjected to negotiation "according to specific situations" [225].

"The Use of Volume in the Measurement of Grain in Early Imperial China" [ch. 7] by Karine Chemla/林力娜 and Ma Biao/馬彪 is an inquiry into why in early imperial China (third century BC to first century AD) two different systems of measurement, namely, volume and capacity, were used to deal with the same issue, that of assessing spatial extension, sometimes being used for the same type of material, for example, grain measured by volume and grain measured by capacity.

Chemla and Ma's argument is grounded in the way in which mathematical texts deal with spatial extension, especially that of grain. The texts selected for analysis are:

- the *Writings on Mathematical Procedures* (*Shuanshu shu*/筭數書), exhumed from a tomb sealed around 186 BC;
- *Mathematics* (*Shu*/數), dated to the Qin dynasty (221–206 BC) by its editors; and
- *The Nine Chapters on Mathematical Procedures* (*Jiu zhang suanshu*/九章算術), probably compiled during the first century AD.

All of them, for different reasons, reflect the actual practices of the administration of early imperial China, and in all of them grain is a relevant topic. In addition to *The Nine Chapters*, Chemla and Ma pay attention to commentaries on them, especially those written by Liu Hui in AD 263 and by Li Chunfeng and presented to the throne in AD 656.

To prepare the argument, the authors explain that in the mathematical texts there were indeed two ways of expressing spatial extension. On the one hand are capacities, which were expressed with a specific system of units, the *dan* (later *hu*) of 10 *dou* and the *dou* of 10 *sheng* [244]. This system is well attested in the mathematical texts. On the other hand are volumes, expressed not with specific units of volume but by a specific way of using units of length. Thus, a volume of *n chi* corresponded to a "cuboid" (a square right prism) of square base with sides of 1 *chi* (here the *chi* is a unit of length) and height *n chi* (here *chi* is a linear unit of measure again). The *chi* in the result was referred to as the "*chi* of the number-product, *ji*" [252]. Chemla and Ma emphasize that this clearly shows that there were two ways of expressing spatial extension and that these were also attested in the documents of practice.

One interesting consequence of the distinction is that capacities could be measured by standard vessels (like Shang Yang's cuboidal *sheng* and Wang Mang's cylindrical *hu*), while volumes could not. Instead, volumes had to be calculated. This is exemplified with a problem from *The Nine Chapters* on a pile of millet supposed to have the form of a cone of known dimensions, for which one is asked to find the number product *ji*, the volume [253]. Chemla and Ma's insightful interpretation asserts that, in this way, volumes are a connection between geometrical forms and numbers [254]. That volumes are calculated also comes from the manuscripts because they contain tables that can be consistently understood as aids to calculating with measures of length in a way that produces volumes [257]. All this leads to the main question of the chapter: "Why is it that in some cases, the spatial extension of amounts of grain was expressed as a volume, and not as a capacity?" [259].

The authors present a first step leading to an answer by analyzing occurrences of the units *dan* and *hu* in certain passages of the texts *Mathematics* and *The Nine Chapters*, respectively. Remembering that "dan" is the name of a unit of capacity and of a unit of weight, Chemla and Ma note that in the selected passage from *Mathematics* it must have a third meaning. Similarly, "hu" must have a different meaning from its usual meanings as either a unit of weight or a unit of capacity. The key is eventually found in Li Chunfeng's *Monograph on Pitch Pipes and the Calendar*: the third meaning of "hu" is a unit of value [262]. Thus, the passage in which it occurs in *The Nine Chapters* relates different volumes of different grains, with each having the same value of 1 *hu*. The same is valid for the "dan" in *Mathematics*. Summing up, volumes are introduced in order to assess value.

Next, Chemla and Ma show that "every time volume occurs in relation to grain, an assessment of this kind is at stake" [264]. Noting that the passage from *Mathematics* dealing with *hu* as a value associates it with a vessel, Chemla and Ma offer a thoughtful analysis of two very well-known vessel standards of capacity: Shang Yang's cuboidal *sheng* (dated to 344 BC) and Wang Mang's (45 BC – AD 23) cylindrical *hu* [265–267]. For that, they draw on the results of their previous research where they showed that for a certain state of grains ("standard coarsely husked grain" [265]) the unit of value and the unit of capacity coincided. Then, as both vessels just mentioned have inscriptions explaining how their linear dimensions were chosen by their creators to produce volumes that could be related to "standard coarsely husked grain", these vessels end up being a bridge between the volume of a specific geometric form and its capacity [268–269].

A last piece of evidence connecting volume, capacity, and value comes from three problems in *The Nine Chapters*. These problems deal with piles of grain, first in the form of a cone, then a semicone (because the pile is against a wall), and finally a quarter of a cone (because the pile is against the corner of a wall) [270]. The solutions are computed from the corresponding volumes and, with division, the corresponding values in *hu*. It is interesting that the results are expressed as an integer part followed by a fraction, which according to Chemla and Ma, "could by no means result from a measurement" [271]. Similar problems occur in *Writings on Mathematical Procedures*, but now they are related to the *dan* instead of the *hu*.

Next, Chemla and Ma compare the use of volume and capacity in the measurement of grain [273] when value was at issue. First, a calculated volume could be divided by the volume of the unit of value. In mathematical writings, in the context of "tax payment and also in relation to issuing or receiving grain in the context of granaries" [273–274], the value of small amounts of grain was assessed by measuring capacity, and the way in which the results were written indicates that they were the result of measurement with "sequences of vessels" [274]. In mathematical texts, when it comes to large amounts of grain whose value was sought, units of volume measurement are used, possibly because this would reduce the number of operations and so give more precision to the operation, which might be in turn the reason why volume as "a theoretical kind of magnitude" was introduced, as Chemla and Ma suggest [275].

In their conclusions, the authors argue that two types of quantities (capacity and volume) were used to assess spatial extension because they could incorporate the way in which a measurement had been made before the conversion to value [276]: capacities were measured by vessels; volumes were obtained by assigning a geometrical form to a spatial extension and by calculation. Moreover, the spatial extension of concrete capacity vessels designed to express the value of a certain amount of grain in a certain state was defined using a calculated volume, thus confirming the connection between "extension and value" [276].

The chapter opens several avenues of research, such as whether there was any recognition of proportionality between the height and circumference in piles of grain [271], of a relation between the forms of standard vessels and granaries [273], or of value as a central unit in managing granaries [276].

2.3 Quantifying land and surfaces (Part 3)

Camille Lecompte's "The Measurement of Fields during the Pre-Sargonic Period" [ch. 8] has its starting point in 15 field survey texts from the early dynastic IIIb period. In survey texts in general, the fields themselves are divided into three categories: lease (*apin-la*$_2$), subsistence (*šuku*), and domain (*niĝ*$_2$*-en-na*) [285]. These texts come from Girsu and involve measurement of the sides of the fields and of their surfaces [286]. The majority of texts deal with domain fields, but some are surveys of subsistence fields [286]. All in all, they represent a kind of work that was essential to the management of fields [286]. This is illustrated by the fact that survey texts from Girsu are "written under the responsibility of high officials" [289], although there were certainly lower-rank individuals playing important roles in the administrative flow, such as the "land recorder" [287], the "land surveyor" [287–288], and the "field assessor" [288].

There are fields measured as rectangles, and their surfaces are calculated as the product of length by width [291–293]. That the fields were exactly rectangular is hardly possible, so the method for computing the area represents a "first-order approximation" [291]. In other fields, the surface is obtained by the product of the average length by the average width, a procedure known as the land surveyor's formula in the historiography of mathematics [293–297]. In these cases, the fields were irregular in shape "but still roughly correspond to quadrilaterals" [293]. The land surveyor's formula is equally applied in the survey texts from Girsu in situations where the fields have one pair of parallel sides (called trapezoids in the chapter). In this case, only three measurements are given, for instance, a length, a second length, and a width, and the area is calculated as the product of the average of the lengths by the width [293].

Lecompte then notes that neither of these two procedures matches the "surface of the fields as indicated in the tablets, which seems to be due to rounding" [298]. Because the tablets do not explicitly show how the computations were made, it is impossible to know the exact reasons for the discrepancy. However, it is possible to offer a tentative classification of the types of divergences, as Lecompte does in the conclusion [307]: the fields are not exactly the shape of the geometrical figures assumed by the calculations; scribes sometimes preferred rounded values, which were obtained by the omission of smaller units of surface; and the use of the surveyor's formula may produce a large difference in relation to the surface indicated on a tablet.

The chapter also provides important insight into what has been called the "agricultural landscape", adding detail and nuance to previous studies on this topic by Mario Liverani [1990; 1996; and 1997]. Lecompte's research leads to the conclusion that "the Sumerian agricultural landscape seems to have been more diverse than that suggested by Liverani" [305].

Christine Proust's "Early-Dynastic Tables from Southern Mesopotamia, or the Multiple Facets of the Quantification of Surfaces" [ch. 9] is a contribution to the understanding of how surfaces were quantified in the early dynastic period, thus establishing a parallel to the chapter by Lecompte.

More precisely, Proust examines five texts (VAT 12593, MS 3047, Feliu 2012, A 681, CUNES 50-08-001), two of them dated to the early dynastic IIIa (ED

IIIa = Fara period, 2600–2500 BC) and two of them dated to the ED IIIb (2500–2340 BC). Each of these texts contains entries specifying the length, width, and surface area of fields. Proust takes into account several features of the tablets, the first of which is whether they have a tabular format (three tablets) or whether they are organized as lists of clauses (two tablets). Also considered is the shape of the fields, i.e., whether they are square or rectangular. In addition, Proust explores whether they are large or small and whether the order of presentation of surfaces follows a decreasing or increasing order. Last discussed is whether the notation for areas is additive or subtractive. All this information is summarized in Table 9.2 [353] and Table 9.12 [383].

One central part of the argument comes from the analysis of tablet A 681, an ED IIIb tablet from Adab in which information is given as a list of clauses involving small squares in increasing order. The units of surface used in this text are:

1 *gan* = 100 *sar*,
1 *sar* = 60 *gin*,
1 *gin* = 3 *samana*, and
1 *samana* = 60 *še*.

Surfaces are expressed by an aggregate of multiples of these units, for instance, 1 *gin* 2 *samana*. What is interesting here is that the same surface can also be expressed using a subtractive notation, e.g., 2 *gin* minus 1 *samana*, such as is found in A 681 [367–368]. The analysis carried out by Proust leads her to conclude that A 681 was not a mere reference for surface values corresponding to length values, as the previous tablets analyzed in the chapter seemed to be [373–374]. The way surfaces are expressed in A 681 is consistent with the possibility that the scribe operated by cutting and pasting with area elements. If so, this text may have been offered as a list of mathematical problems, the aim of which was to determine surfaces in a specific way [374], where the technique used was typical of the foundation of "a long mathematical tradition" that included so-called quadratic problems from the Old Babylonian period, which, as Jens Høyrup [2002] has shown, were solved through cut-and-paste operations.

A second enlightening takeaway from Proust's chapter derives from her analysis of mathematical tablet CUNES 50-08-001. This text is divided into five sections, each one presenting the sides and the surfaces of squares of different orders of magnitude. The first section involves a very wide range of multiples of *ninda*. The following four sections deal with successively smaller fractions of the *ninda*:

1 to 10 *nikkas*, where 1 *nikkas* = ¼ *ninda*;
1 to 10 *kuš-numun*, where 1 *kuš-numun* = 1⁄12 *ninda*;
1 to 10 *giš-bad*, where 1 *giš-bad* = 1⁄24 *ninda*; and
1 to 10 *šu-bad*, where 1 *šu-bad* = 1⁄48 *ninda*.

According to Proust, each section can be generated from the areas of squares of sides 1 *nikkas*, 1 *kuš-numun*, 1 *giš-bad*, and 1 *šu-bad*. As she points out, the areas of these squares are fractions of the area of the square of sides 1 *ninda*, that is to say, fractions of 1 *sar*. Thus, CUNES 50-08-001 is ultimately based on knowing how to express several fractions of 1 *sar*, exactly the type of relationship that would be embodied by the tables of reciprocals in the Ur III and the Old Babylonian periods. To Proust, this is an indication that CUNES 50-08-001 is "a systematic exploration of sexagesimal computation" [382], and that the several unexpected units representing very small fractions of the *ninda* might "have been created for the exploration of the newly discovered methods of sexagesimal calculation" [382].

2.4 Prices, rates, loans, and interests (Part 4)

Cécile Michel's "Computation Practices of the Assyrian Merchants during the Nineteenth Century BC" [ch. 10] analyzes conversion computations in school texts and in letter extracts from Assyrian merchants and their family members [400–402]. The chapter begins with a study of six school tablets, of which four are from Aššur and two from Kaneš in Anatolia, where the merchants had business. Although there are other still unpublished school texts from these localities, the six presented by the author show quite well that converting a quantity of gold into a quantity of silver of the same value was a topic studied in the Old Assyrian learning environment from which these texts come [404–409].

That this topic was in fact useful in practice is attested by two letter extracts that Michel adduces in which the same type of computation from the school texts is found [409–411]. Next, in order to give the reader a sense of the context, Michel presents and studies a complete letter, called by Larsen [1967] a "caravan account" [411–413]. In it, the senders explain that a quantity of gold that they were given by the recipient was converted into silver. They then list all the purchases that they made with the silver. The total amount of expenses is almost equal to the original amount of silver. Differences in calculation like this are the author's focus in the rest of the chapter.

In order to carry out a systematic analysis of how Assyrian merchants made these conversion calculations, Michel provides the reader with two distinct groups of letter extracts. In the first, the conversions are from gold to silver [415–418]; in the second, from tin to silver [418–425].

In gold to silver conversions, the conversion rate is the number of *gin* of silver that correspond to 1 *gin* of gold. This means that the amount of gold has to be multiplied by this conversion rate in order to determine the amount of silver that is equivalent in value.

In tin to silver conversions, as the tin is the less expensive metal, the conversion rate is given as the number of *gin* of tin that correspond to 1 *gin* of silver. Consequently, the given amount of tin has to be divided by the conversion rate in order to determine the amount of silver equivalent in value. As is known, division is not an operation performed directly in cuneiform mathematics at the beginning of the second millennium. However, as the Assyrian merchants did not leave any trace of how they did their calculations, Michel simply presents the calculations, even the divisions, in modern notation, trying to make them as consistent as possible with the way in which the results are expressed in the ancient documents.

The documents at issue are nine letter extracts containing gold to silver conversions and 22 letter extracts containing tin to silver conversions. The author analyzes explicitly a selection of extracts of both groups because they result in either an incorrect result or a rounded one.

From this detailed analysis, Michel draws several fascinating conclusions [429]: merchants sometimes committed mistakes in calculation and sometimes used rounding (up or down) to express the result of a conversion, where these rounding operations were used, for instance, when the numerically correct result would demand using a fraction that was not available in their arithmetic (e.g., 1⁄7). In the case of the tin to silver conversion, the rationale could have been very different: the author suggests that the amount of tin to be converted into silver was determined beforehand from the conversion rate and the availability of both tin and silver. If this was true, then the calculations that the merchants performed might actually have been the multiplication of the amount of silver by the conversion rate, and the resulting amount of tin was rounded and finally set as the object of their negotiation. Another interesting result concerns how the Assyrian merchants used fractions: they consistently "preferred to use fractions of the higher [metrological] unit instead of integers of the lower unit" [429].

Finally, it should be noticed that the Assyrian merchants' procedures in the cases studied did not involve sexagesimal place value notation (SPVN). In this respect, these procedures are quite different from what is found in the administrative practices of the Babylonians, in which SPVN was the constant tool for multiplication. The chapter finishes by pointing out a possible avenue of research dealing with how Assyrian merchants might have made their calculations. As there are at least two old Assyrian texts mentioning writing boards coated with wax, one of which mentions furthermore some sort of computing tool [429–430], one should not exclude the possibility that Assyrian merchants employed some of these devices for the intermediary steps that inevitably intervened in their making conversions.

Robert Middeke-Conlin's "Connecting a Disconnect: Can Evidence for Scribal Education Be Found in a Professional Setting during the Old Babylonian Period?" [ch. 11] is an inquiry into the reasons for a discrepancy in calculation found in the administrative text YBC 7473, written by the overseer of merchants Itti-Sîn-milki in a city of the Old Babylonian kingdom of Larsa. In the first part of the tablet [441–444], the scribe lists several goods, relating their in-kind values (capacities of sesame, a certain number of rams, a weight of wool that have the value of 1 *gin* of silver), their conversion rates to silver, and the equivalent value in silver. In all these situations, the conversion rate declares the amount of the good that corresponds to one *gin* of silver. Consequently, the equivalent value in silver is obtainable by multiplying the original amount of the good by the reciprocal of its conversion rate.

In one of Itti-Sîn-milki's conversions of sesame to silver, however, Middeke-Conlin draws our attention to a two-fold discrepancy. First, the value in silver is not what we would call a numerically correct value [445–446]. Second, the given conversion rate cannot produce a reciprocal easily usable in the SPVN system in use during the Old Babylonian Period [453–454] because it does not correspond to a regular number in this notation.[2]

Middeke-Conlin brings two pieces of evidence to explain this twofold discrepancy. First, YBC 4698 is a mathematical tablet, probably from Sippar or Kiš in northern Babylonia, that shows that conversion rates were also a subject in the mathematical school curriculum [450–451]. This is crucial for the argument, for, as shown by Christine Proust [2007; 2008; and 2013], in the Old Babylonian mathematical curriculum it was common practice to take the given measurement values, convert them to SPVN, make calculations in

[2] A number in the SPVN system is said to be regular if and only if its reciprocal has finite sexagesimal representation. For instance, 3 is regular, since its reciprocal is 20 (3 times 20 gives 60); 7, however, is not regular, since its reciprocal would produce the infinite repetition 8,34,17,8,34,17,8,34,17....

SPVN, and finally convert the results back to measurement values. Middeke-Conlin explores the possibility that Itti-Sîn-milki was doing exactly this. As Middeke-Conlin states at the end of the chapter, the overseer of merchants should have been able to do all the calculations in his business himself and, therefore, may have had some mathematics education [460].

In fact, all the calculations in YBC 7473 can be easily interpreted in this way, except the one where the discrepancy popped up. The central point is that the conversion rate, 1 *barig* 5 *ban* of sesame to 1 *gin* of silver, corresponds to the number 1;50 (equal in our notation to 110 or 10 times 11), which does not have an exact reciprocal value in the sexagesimal system (because 11 is not a regular number). Itti-Sîn-milki could have used an approximate value for the reciprocal of 110 [454–455]. As the reciprocal of 110 should be obtained by the multiplication of the reciprocal of 10 by the reciprocal of 11, the real problem here was the reciprocal of 11.

This is where Middeke-Conlin adduces the mathematical tablet M10, an unprovenanced text published by Abraham Sachs in 1952. M10 contains approximations for reciprocals of a few numbers that do not have exact reciprocals in base 60, among them an approximate reciprocal of 11 [455]. What is interesting here is that the author of M10 indicated erroneously that the approximation for the reciprocal of 11 presented in M10 was one in deficit, that is to say, the author thought that the approximation multiplied by 11 would produce a result, for practical purposes, less than 1. However, the approximation for the reciprocal of 11 present in M10 is actually one in excess, but its author was apparently unaware of that.

And so was Itti-Sîn-milki. If he had used the approximation for the reciprocal of 11 found in M10 or in a similar text, he would have obtained a value larger than the correct one. Itti-Sîn-milki, however, presented an equivalent in silver even larger, which seems to be a strong indication that Itti-Sîn-milki believed the approximation he used was in deficit, justifying a rounding up [456–458].

In order to reconstruct how much Itti-Sîn-milki rounded up the result of his calculation, Middeke-Conlin analyzes the metrological tables of weights that were current in Old Babylonian schools. His conclusion is that the amount Itti-Sîn-milki added to the result that he believed was in deficit is consistent with a pattern of sexagesimal numbers present in such metrological tables [458–459].

In this way, Middeke-Conlin states that it is very probable that Itti-Sîn-milki had had some mathematical training similar to what research has found in

Old Babylonian *edubas*, thus linking the mathematics of the economic and administrative practices to the mathematics of more strictly mathematical texts and underlining the importance of the administrative and economic sources for the study of mathematics in cuneiform sources [459–460]. It remains only to add that the study of errors, rounding, and mistakes is part of a large endeavor by the author, most of the results of which are presented in Middeke-Conlin 2020.

Sreeamula Rajeswara Sarma and Takanori Kusuba's "Loans and Interest in Sanskrit Legal and Mathematical Texts" [ch. 12] focuses on loan making in ancient and medieval India as evident in Sanskrit administrative, legal, and mathematical texts.

The chapter begins with a reflection on the limitations of Sanskrit texts as historical sources. The *Arthaśāstra*, specifically, cannot reflect the variety of practices that must have existed in so vast an empire as the Maurya [464]. Plus, statistical analysis has shown that the *Arthaśāstra* is composed of several layers from different periods. Finally, the commentators are so distant in time from the period of composition that even they have difficulty in understanding its technical terms. As for legal texts, collectively referred to as *Dharmaśāstra*, it is not known if and how they were used in courts of law and whether they prevailed over local customs (*deśācāra*). In addition, it is difficult to relate these texts to specific periods and regions, although they do contain the views "of some ancient thinkers on the general conduct of life" [465].

Over time, as these legal texts were copied and commentaries made on them, they enjoyed some legitimacy. Sarma and Kusuba consider the three most influential texts among the legal sources: the *Manu-smṛti*, the *Yajñavalkya-smṛti*, and the *Nārada-smṛti*, in addition to the commentaries that formed with each one of them a "continuous intellectual discourse" [464].

The analysis starts with the explanation that the texts were not in principle aimed at the caste of merchants (*Vaiśyas*, or *Banias*), because as a caste they had no access to Sanskrit administrative, legal, and mathematical texts [466]. As for their contents, Sarma and Kusuba begin by describing the position of each of the chosen texts on interest rates. The *Arthaśāstra* prescribes different interest rates according to the risk of the capital (from ordinary loans at 1.25% per month to those at 20% per month). The *Manu-smṛti* suggests different interest rates according to the caste of the borrower (2% per month to *Brahmins*, 3% to *Kṣatriyas*, 4% to *Vaiśyas*, and 5% to *Śūdras*, the members of the lowest caste). The *Yājñavalkya-smṛti* distinguishes the 1.2% rate as secured by a pledge and the 2% as for unsecured loans. The

Nārada-smṛti "acknowledges that lending money is a legitimate means of increasing one's wealth" [471] and mentions four types of interest: corporal, that is to say, daily interest without any diminution on the capital; periodical, which is interest paid monthly; stipulated or interest agreed upon by the debtor; and cyclical, interest on interest, which, according to Sarma and Kusuba is erroneously taken by some authors as "compound interest" but "is not mentioned in any Sanskrit mathematical text, even as a hypothetical case" [472]. The section ends with a few considerations about loans of commodities found in the administrative and legal texts, prescriptions mainly of limits on the interest rates that can be applied [472–473].

The authors then analyze the compilation of the model, or sample, document *Lekhapaddhati*, composed in the period *ca* AD 950–1350, when the Caulukya kings governed Gujarat. Three documents are selected from the *Lekhapaddhati* [473–474]. These documents are interesting because they attest to the fact that the 2% interest rate suggested by the *Manu-smṛti* seems to have been in common practice in medieval India. Moreover, one of the documents gives detailed instructions for the recovery of a loan [476].

Sarma and Kusuba bring several pieces of evidence to the idea that limiting the accumulated interest to the value of the principal, the *dāmdupat* of Indian law, dates from the time of the *Manu-smṛti* and is still in practice in some legal cultures of South Asia, which underlies the importance of the *Manu-smṛti* [480].

The second part of the chapter is devoted to the same topics that appear in mathematical and astronomical texts in Sanskrit, including commentaries. The computation of interest is treated in Bhāskara's commentary on the Āryabhaṭa's presentation of the rule of three in his *Āryabhaṭīya*. Bhāskara shows how the rule of three can be applied twice, becoming the rule of five, to solve problems in which a principal, a time duration, and an interest are given as arguments, together with another principal and another time duration, given as required, in order to compute the corresponding interest [482–484]. The rule of five is also abstractly described by Brahmagupta in his *Brāhmasphuṭa-siddhānta*, while his commentator Pṛthūdaka offers examples of its application, for example, by Bhāskara [484–488]. Sarma and Kusuba note that in these mathematical texts, the interest rate is frequently 5%, and in some other texts it is sometimes even higher, so it is "purely hypothetical and has no contemporary evidence" [488]. Another concern of mathematical texts is to calculate the time necessary for the "sum of the principal and the interest" to become a given multiple of the principal [488].

The remainder of the chapter covers other types of mathematical problems involving interest rates that appear in Sanskrit mathematical treatises. One of them is represented by problems in which one is given the sum or *miśra* (mixture) of two of the involved quantities. It is mentioned by several mathematical texts [489–495].

Sarma and Kusuba close the chapter by noticing that administrative and legal texts tend to emphasize that interest rates must be fair and that a loan should not be allowed to accumulate beyond a certain limit (twice the principal), while mathematical and astronomical texts use bigger interest rates and also deal with problems in which the principal plus the interest becomes multiples of the interest. The authors conclude that mathematical texts cannot be expected to “show any coherent picture” [499] because their contents “are not based on the actual state of affairs of any period or region, but on the mathematician’s ingenuity” [499].

Marc Bompaire and Matthieu Husson’s “Computational Practices around Coins and Coinage: John of Murs’ *Quadripartitum numerorum* and French Money Changers’ Books” [ch. 13] is the only chapter of the book bearing on European medieval mathematics. The authors focus on the mathematics related to coins and coinage, and state that similar practices were common in several environments of medieval Europe, the three most important being those of:

(1) moneyers, money changers, mint wardens, and mint masters;
(2) merchants dealing with coins as goods; and
(3) scholars with an interest in mathematics who were connected
 (a) sometimes to merchants and
 (b) sometimes to faculties of arts.

Bompaire and Husson’s goal is to compare the calculation practices related to coins and coinage as found in (1) and (3b) [504].

The sources for the mathematics of coins and coinages of scholars related to the faculties of arts are represented in the section of John of Murs’ *Quadripartitum numerorum* (1343) on money, *De monetis*. The sources for the study of the calculative practices of moneyers and associated practitioners are three manuscripts:

- the literary BnF Arsenal 8315, which contains annotations on money changing dating to 14th-century southern France (Montolieu);
- BnF Fr. 5917, produced by money changers or mint wardens, dating to *ca* 1420 in northern France (possibly Normandy); and

  - BnF nouv. acq. Fr. 471, reflecting the practice of a royal mint warden and produced *ca* 1466 possibly near La Rochelle [504–505].

Bompaire and Husson start their comparison of the two *milieux* with the respective approaches to metrology. In *De monetis*, metrology is simple: the *marc* is a unit measure of weight, and it is made of 12 *deniers*, where a *denier* is made up of 24 *grains* [505–506]. On the other hand, the analysis of the manuscripts from the money changers' *milieu* reveals that, besides the units of measure for weight, there was a metrological system devised to measure the quantity of silver in an alloy, using the same term, "denier":

> 12 *deniers* of alloy corresponds to a type of money made entirely of silver. 6 *deniers* of alloy correspond to a type of money with equal amounts of silver and copper. These *deniers* were further divided into 24 *grains*. [507]

This "manifold metrology" [507] was put into action in calculations in which the language used suggests that an abacus was used; and the conversions between the different units are represented in tables, called *comptes faits*, to make the conversion easier [508–510].

In the following sections, the authors describe two techniques used by money changers to assess the alloy of a large number of coins of different metal composition: the common money [511] and the differences methods [515]. The explanation of these methods would go into detail that is not necessary here, but it must be said that in both the money changers preceded their calculations by mixing the coins and sampling them. The calculations were then performed on the sample, which was considered representative of all the money.

Bompaire and Husson also provide examples of the common money and differences methods from John of Murs' *De monetis* [513, 518]. The main distinction of the money changers' practice is that they are not applied to a sample of the money. Indeed, no sampling is made. Another important difference is that the money changers' calculations were apparently designed to be made in one's head, since they proceeded step by step with tight control of the nature of the numbers involved. Bompaire and Husson chose examples from John of Murs [510–521] that were in the context of another problem faced by money changers, that of changing the alloy of a coin or set of coins, the subject of the penultimate section of the chapter.

Money changers dealt with the problem of changing the alloy of money by adding pure metal to a mix, but this was seen as a solution of last resort because of the difficulty and cost of obtaining pure metals. The money changers' method was to calculate the alloy that, when mixed in equal quantity with the given money, would produce the desired new alloy and then to calculate *via* fractions and *comptes faits* the amount of pure metal that would produce the same effect. This is completely different from the procedure John of Murs proposes in similar situations: he works mainly with rules of three and with the introduction of a new quantity, the weight of the mix [525–526].

Bompaire and Husson close the chapter by reviewing the points of comparison and indicating that they "show two cultures of computation confronted in different ways and to greater and lesser extents by different aspects of the same reality", [529] being "shaped by different values" [529] according to the different *milieux* to which they belonged.

3. Final remarks

The book does not have a general concluding chapter. Instead, it is the introduction [ch. 1] that summarizes the main results, many of which have been discussed above, especially with regard to the disunity of mathematics and its constitutive role in society.

It remains to be said, however, that *Mathematics, Administrative and Economic Activities in Ancient Worlds* may be considered a starting point for similar research in that it presents specific points to keep in mind when comparing mathematical knowledge and practices from different *milieux* or different cultures of computation and quantification, points such as "the mathematical problems actors addressed", the terminology, the number systems, "the material environments they shaped to carry out quantification and computation", the "actors' expectations regarding the answers they wanted to obtain" and "their aspirations regarding the procedures they wanted to use" [5], as well as epistemological and professional values [43–44].

In addition, each chapter opens up uncountable avenues of research by pointing out issues that can be developed further and questions that still deserve treatment. Some of these avenues of research are indicated above in my summaries of the chapters, but there are many others.

In relation to that, as a historian of cuneiform mathematics, it would be unfair on my part not to add that the seven chapters of the book that deal specifically with cuneiform mathematical texts establish exciting new possibilities of research in at least two major directions, besides those that have already been presented. The first concerns the form and purpose of mathematics education in Mesopotamia, a subject vastly explored in the Assyriological literature [e.g., in Robson 2002 and 2004; Proust 2007 and 2008].

The second focuses on the technical aspects of cuneiform mathematics and its sociohistorical development, equally a central theme for historians working in this field [e.g., Robson 1999 and 2008; Høyrup 2002; Friberg 2000 and 2007; Friberg and al-Rawi 2016; Chambon 2011].

The book has typos and minor errors, which do not impede understanding but may deserve correction in a second edition.

In sum, the book is part of a long conversation about the diverse richness of the ancient mathematical traditions, or rather cultures of computation and quantification. On a very general historiographic level, it is a tributary of the idea that unifying narratives are, if not altogether flawed, at least problematic. On a very specific level, it is the consequence of the specialties of each contributor taking part in the collective and collaborative research initiative that was the SAW Project. It is an up-to-date window on what currently advances the historiography of ancient mathematics and, therefore, a work that I recommend without any shadow of doubt.

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