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Résumé de l'article

Le fait de souscrire des montants élevés dans une nouvelle entreprise et reliée à une branche d'assurance spécialisée (c'est-à-dire une branche dans laquelle on retrouve des titulaires de polices à haut risque) constitue un danger majeur pouvant affecter la solvabilité des assureurs en assurance de dommages. Une telle souscription est doublement problématique, car la tarification peut être inappropriée et les conditions de participation au risque ne sont pas toujours précises. Mentionnons à cet égard, à titre d'exemple concret, la faillite de nombreuses sociétés captives dans les années 80, dû au fait que ces sociétés se fiaient trop fortement à des gestionnaires indépendants, principalement motivés par l'augmentation du volume de primes. Nous utilisons dans cet article le modèle Cournot de marché pour étudier les conséquences financières de gestionnaires d'assureurs (c'est-à-dire des gestionnaires non-affiliées qui détiennent des informations privées sur les caractéristiques de risques des assurés) sur les opérations de ces assureurs spécialisés évoluant dans des branches à haut risque. Dans un marché où évoluent un assureur neutre au risque et des assurés démontrant une riscophobie absolue constante, nous constatons qu'un tel assureur spécialisé ferait mieux que le gestionnaire indépendant en garantissant directement le risque. Toutefois, il peut en être autrement si cet assureur utilise une optimisation moyenne-variance. Une juste association compensation-souscription et répartition de capital peut conduire à des meilleurs résultats que la souscription directe.

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Can Independent Underwriters Benefit Insurers in High-Risk Lines? A Cournot Market-Game Analysis

by Jiang Cheng and Michael R. Powers

ABSTRACT

One of the greatest dangers to the solvency of property-liability insurers is writing large amounts of new business in a high-risk line (i.e., a line of insurance in which a substantial portion of buyers consists of high-risk insureds). This practice is problematic because of both potentially inadequate pricing and potentially lax underwriting. A prominent example of the latter phenomenon was the collapse of many captive insurers in the early to mid-1980s, in which the insurers relied too heavily on independent underwriters motivated solely by increasing premium volume. In this article, we employ a Cournot market-game model to study the financial impact of informed independent underwriters (i.e., unaffiliated underwriters with private information regarding the risk characteristics of insureds) on insurers in high-risk property-liability lines. In a market with a risk-neutral insurer and CARA insureds, we find that the insurer will always do worse by using a risk-neutral underwriter than by operating on a direct-writing basis. However, for an insurer employing mean-variance optimization, the proper combination of underwriter-compensation and capital allocation may lead to better outcomes than direct writing.

Keywords: Independent underwriters, high-risk lines, Cournot market games.

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Résumé

Le fait de souscrire des montants élevés dans une nouvelle entreprise et reliée à une branche d'assurance spécialisée (c'est-à-dire une branche dans laquelle on retrouve des titulaires de polices à haut risque) constitue un danger majeur pouvant affecter la solvabilité des assureurs en assurance de dommages. Une telle souscription est doublement problématique, car la tarification peut être inappropriée et les conditions de participation au risque ne sont pas toujours précises. Mentionnons à cet égard, à titre d'exemple concret, la faillite de nombreuses sociétés captives dans les années 80, dû au fait que ces sociétés se fiaient trop fortement à des gestionnaires indépendants, principalement motivés par l'augmentation du volume de primes. Nous utilisons dans cet article le modèle Cournot de marché pour étudier les conséquences financières de gestionnaires d'assureurs (c'est-à-dire des gestionnaires non-affiliées qui détiennent des informations privées sur les caractéristiques de risques des assurés) sur les opérations de ces assureurs spécialisés évoluant dans des branches à haut risque. Dans un marché où évoluent un assureur neutre au risque et des assurés démontrant une riscophobie absolue constante, nous constatons qu'un tel assureur spécialisé ferait mieux que le gestionnaire indépendant en garantissant directement le risque. Toutefois, il peut en être autrement si cet assureur utilise une optimisation moyenne-variance. Une juste association compensation-souscription et répartition de capital peut conduire à des meilleurs résultats que la souscription directe.

Mots clés: Gestionnaires d'assurance indépendants, branche d'assurance à haut risque, modèle Cournot de marché.

1. INTRODUCTION

One of the greatest dangers to the solvency of property-liability insurers is writing large amounts of new business in a high-risk line (i.e., a line of insurance in which a substantial portion of buyers consists of high-risk insureds). This practice is problematic because of both potentially inadequate pricing and potentially lax underwriting. A prominent example of the latter phenomenon was the collapse of many captive insurers in the early to mid-1980s, in which the insurers outsourced the underwriting function, relying too heavily on independent underwriters that were motivated solely by increasing premium volume (see Porat and Powers, 1993).

We speak of "independent underwriters" rather than "independent agents" to avoid one specific connotation of the latter term – that a market with independent agents generally contains multiple insurers among which the agents are free to place business. This distinction is necessary because our study addresses high-risk lines of business in which the number of admitted and/or alternative-market insurers is typically very low. However, we recognize that both types of intermediaries occupy the same level in the insurance-transaction hierarchy, and that there is no essential difference between the systems for compensating "independent underwriters" and "independent agents." Consequently, scholarly research on independent agents is relevant to our problem to the extent that it allows for small numbers of insurers.

The independent-agency system was the earliest method of distributing property-liability insurance in the United States, and remains one of the most important systems in commercial lines today, accounting for the majority of the industry's direct written premiums. The most important distinction between the independent-agency system and other insurance distribution systems is that independent agents own their policy "expirations" or customer list. Under the independent-agency system, the insurer cannot contact the customer for policy renewal or for the sale of additional products, but must go through the agent (see Regan and Tennyson, 2000).

A number of studies have found that the independent-agency system is not as efficient as direct-writing alternatives, such as those using mass marketing, employee sales representatives, and exclusive agents (see Joskow, 1973; Cummins and VanDerhei, 1979; Barrese and Nelson, 1992; Berger et al., 1997; and Regan, 1999). These and other studies have proposed a number of theories to explain the coexistence of the "high-cost" independent-agency system and other "low-cost" distribution systems in the property-liability insurance industry: (1) the slow adjustment hypothesis (see Joskow, 1973; Grabowski et al., 1989; Gron, 1995; Regan and Tennyson, 1996; Schlesinger et al., 1991; and Suponcic and Tennyson, 1998); (2) the incentive conflicts hypothesis (see Marvel,1982; Grossman and Hart, 1986; Sass and Gisser, 1989; and Kim et al., 1996); (3) the transaction costs hypothesis (see Regan, 1997); and (4) the search costs hypothesis (see Posey and Yavas, 1995; and Posey and Tennyson, 1998). However, it is difficult, if not impossible, to determine empirically if the type of service provided by independent agents is the source of their high costs. It is also difficult to obtain empirical support for one specific hypothesis to the exclusion of others.

To assess the economic value of independent underwriters theoretically, we employ the Cournot market-game model of Powers et al. (1998) and Powers and Shubik (1998, 2001) to determine whether or not an insurer can give an underwriter appropriate incentives to provide information about insureds in a high-risk property-liability insurance line. To simplify, we focus on the case of a single monoline insurer operating in a commercial insurance market. We assume that an independent underwriter possesses private information about its customers' risk profiles, and that the underwriter may or may not disclose its information to the insurer depending on the underwriter's own self-interest. We then study how the insurer will adjust both its underwriter-compensation system and its allocation of capital to minimize the negative effects of high-risk insureds in the context of asymmetric information.

Game-theoretic models are fairly well established in the insurance literature (see Rothschild and Stiglitz, 1976; Kihlstrom and Roth, 1982; Schlesinger, 1984; and Kunreuther and Pauly, 1985). The seminal article of Rothschild and Stiglitz (1976) is closely related to the present work because the authors considered competitive insurance markets with incomplete information and a buyer self-selection mechanism. Assuming that insurers offer a menu of price-quantity pairs, and that insureds select among these pairs to maximize expected utility, Rothschild and Stiglitz (1976) found that: (1) in some cases equilibrium does not exist; and (2) when equilibrium does exist, the existence of high-risk insureds creates a negative externality in that low-risk insureds are worse off in the presence of high-risk insureds, but the latter are no better off in the presence of the former. Recently, Ligon and Thistle (2005) applied Rothschild and Stiglitz' (1976) framework to study how mutual insurers address adverse-selection problems by separating high- and low-risk insureds into different risk pools.

In other related work, Boyer (2004) developed a claiming game to study "agent *ex post* moral hazard," in which an agent has an incentive to misreport the true state of a claim to extract rents from the insurer. To the best of our knowledge, the present article is the first to apply game theory to the study of "agent *ex ante* moral hazard," in which an underwriter may have incentive to misreport the true risk profile of a customer at the time of the insurance purchase. Given the information-asymmetry among the insureds, underwriter, and insurer, it is not clear that market forces alone will achieve an equilibrium that maximizes social welfare. Thus, it is of particular interest to regulators to understand both the nature of market equilibrium and how the market reaches it.

In the next section, we develop a model of a single-period Cournot market game under a direct-writing scenario, and study how the relative proportions of high- and low-risk insureds affect equilibrium price and quantity. In the subsequent section, we extend this model to permit the seller to use an independent underwriter. We consider the effect of a risk-neutral underwriter on markets with constant absolute risk-averse (CARA) insureds for two different cases: a risk-neutral insurer and a mean-variance-optimizing (MVO) insurer.

In the market with a risk-neutral insurer, we find that the insurer will always do worse by using a risk-neutral underwriter than by operating on a direct-writing basis. However, for an MVO insurer, the proper combination of underwriter-compensation and capital allocation may lead to better outcomes than direct writing.

Naturally, using an underwriter to select insureds immediately raises the question of how the insurer can best allocate its capital to support coverage for different risk classifications. Consequently, our analysis also offers some limited insight into the research literature on capital allocation in the insurance industry (see Phillips et al., 1998; Myers and Read, 2001; and Sherris, 2006).

2. MARKET WITH A DIRECT WRITER

Consider a commercial insurance market with a single monoline insurer (the "seller") with initial net worth of W_s, operating on a direct-writing basis, and N_D insureds (the "buyers"), each of which is either of type H (high-risk) or type L (low-risk). Assume that each buyer's type is known only to itself,[1] but that the overall proportion of high-risk buyers, ρ_H, is common knowledge. Denote the set of $N_D\rho_H$ high-risk buyers by $\mathrm{B}_{H,D}$, and the set of $N_D(1-\rho_H)$ low-risk buyers by $\mathrm{B}_{L,D}$.

We describe the economics of this market using a special case of the Cournot market-game model of Powers et al. (1998) and Powers and Shubik (1998). Initially, each buyer $i \in \mathbf{B}_{j,D}$ possesses an endowment of $V + W_B$, consisting of one unit of property with replacement value V and $W_B (\geq V)$ dollars in cash. During a given policy period, each high-risk buyer's property is subject to a random loss with probability π_H, each low-risk buyer's property is subject to a random loss with probability $\pi_L < \pi_H$, and all losses are total. For each buyer $i \in \mathbf{B}_{j,D}$, let $\delta^{(i)}_{j,D}$ be a random variable that equals 1 if the buyer suffers a property loss, and equals 0 otherwise, where the $\delta^{(i)}_{j,D} \sim$ i.i.d. Bernoulli (π_j).

Prior to the policy period – at a point in time we will call t = 0 – the seller announces a quantity offer, $y_D \in [0, W_s]$, representing the total dollar amount of potential loss, *aggregated over all buyers*, that the seller is willing to assume. Subsequently, at the beginning of the policy period – at a point in time we will call $t = 1$ – each buyer $i \in \mathbf{B}_{j,D}$ simultaneously announces a strategic price bid, $x^{(i)}_{j,D} \in \left[0, V\right]$ representing the amount that is willing to pay for insurance.

We assume that the seller is required to accept all nonzero price bids as one lot (i.e., it cannot "pick and choose" among them), as long as the seller's final expected utility is greater than its initial expected utility (but that the seller may "walk away" from the market otherwise). Since the seller is not able to determine the risk profile of any individual buyer, it must offer a single price (per unit) of insurance. The market price, expressed as a ratio comparable to the "rate-on-line" of reinsurance markets, is therefore given by

$$P_D = P_D\left(\mathrm{x}_{H,D}, \mathrm{x}_{L,D}, y_D\right) = \frac{1}{y_D}\left(\sum_{h=1}^{N_D\rho_H} x_{H,D}^{(h)} + \sum_{h=1}^{N_D(1-\rho_H)} x_{L,D}^{(h)}\right).$$

Finally, the allocation of insurance coverage admits of two interpretations: a *global formulation*, which is identical to the approaches of Powers et al. (1998) and Powers and Shubik (1998, 2001); and a *local formulation*, which requires only that the assumptions underlying the global interpretation apply in a small neighborhood of the market-equilibrium solution.

In the global formulation, we define how insurance coverage is allocated to buyers for any particular strategy vector $[\mathbf{x}_{H,D}, \mathbf{x}_{L,D}, y_D]$. Specifically, we assume that if buyer $i \in \mathbf{B}_{j,D}$ suffers a loss during the policy period, then he or she will receive a loss payment in the amount $y_D\left[x_{j,D}^{(i)} \Big/ \left(\sum_{h=1}^{N_D\rho_H} x_{H,D}^{(h)} + \sum_{h=1}^{N_D(1-\rho_H)} x_{L,D}^{(h)}\right)\right]$, which is proportional not only to the buyer's premium bid, $x_{j,D}^{(i)}$, but also to the seller's exposure offer, . In all analytical work, this loss payment will be bounded above by to reduce problems of moral hazard.

Unlike the global formulation, the local formulation does not entail a specific functional form for the loss payments. Rather, we simply define what happens to the insurance-coverage allocations *on the margin* as the strategy vector $[\mathbf{x}_{H,D}, \mathbf{x}_{L,D}, y_D]$ undergoes changes in a small neighborhood of equilibrium. Specifically, we require that if buyer $i \in \mathbf{B}_{j,D}$ increases its premium bid $x_{j,D}^{(i)}$ by a small amount, then *i*'s share of its indemnity payment also increases proportionately. Obviously, this condition is implied by the general formulation, but it also may result from less restrictive assumptions. For example, if buyer *i* increases its premium bid, then *i* quite reasonably should be able to purchase more coverage.

From the preceding description it follows that, within some neighborhood of equilibrium: the final wealth of buyer $i \in \mathbf{B}_{j,D}$ is

$$B_{j,D}^{(i)} = \left(1-\delta_{j,D}^{(i)}\right)\left(W_B + V - x_{j,D}^{(i)}\right)$$
$$+\delta_{j,D}^{(i)}\left\{W_B - x_{j,D}^{(i)} + y_D\left[x_{j,D}^{(i)} \Big/ \left(\sum_{h=1}^{N_D\rho_H} x_{H,k}^{(h)} + \sum_{h=1}^{N_D(1-\rho_H)} x_{L,k}^{(h)}\right)\right]\right\}; \quad (1)$$

whereas the seller's final wealth is

$$S = W_S + y_D P_D - \frac{y_D\left(\sum_{h=1}^{N_D\rho_H} \delta_{H,D}^{(h)} x_{H,D}^{(h)} + \sum_{h=1}^{N_D(1-\rho_H)} \delta_{L,D}^{(h)} x_{L,D}^{(h)}\right)}{\left(\sum_{h=1}^{N_D\rho_H} x_{H,D}^{(h)} + \sum_{h=1}^{N_D(1-\rho_H)} x_{L,D}^{(h)}\right)} \quad (2)$$

Now let $\varphi_{B_j}(\cdot):\Re\rightarrow\Re$ denote the utility function of buyer $i\in\mathbf{B}_{j,D}$, and let $\varphi_S(\cdot):\Re\rightarrow\Re$ denote the utility function of the seller, where all utility functions are increasing and weakly concave downward (so that the case of risk neutrality is permitted). It then follows that the expected utilities corresponding to the wealth amounts (1) and (2) are, respectively:

$$E\left[\varphi_{B_j}\left(B_{j,D}^{(i)}\right)\right] = \left(1-\pi_j\right)\varphi_{B_j}\left(W_B + V - x_{j,D}^{(i)}\right)$$
$$+\pi_j\,\varphi_{B_j}\left(W_B - x_{j,D}^{(i)} + y_D\left[x_{j,D}^{(i)} \Big/ \left(\sum_{h=1}^{N_D\rho_H} x_{H,D}^{(h)} + \sum_{h=1}^{N_D(1-\rho_H)} x_{L,D}^{(h)}\right)\right]\right) \quad (3)$$

and

$$E\left[\varphi_S(S)\right]$$
$$= \sum_{r_{H,D}=0}^{N_D\rho_H} \sum_{\substack{R(r_{H,D})\\ \in \mathrm{B}_{H,D}}} \sum_{r_{L,D}=0}^{N_D(1-\rho_H)} \sum_{\substack{R(r_{L,D})\\ \in \mathrm{B}_{L,D}}} \left[\left(\pi_H\right)^{r_{H,D}}\left(1-\pi_H\right)^{N_D\rho_H - r_{H,D}}\left(\pi_L\right)^{r_{L,D}}\left(1-\pi_L\right)^{N_D(1-\rho_H)-r_{L,D}}\right]$$

$$\times\varphi_S\left(W_S + y_D P_D - \frac{y_D\left(\sum_{h=1}^{N_D\rho_H} \delta_{H,D}^{(h)} x_{H,D}^{(h)} + \sum_{h=1}^{N_D(1-\rho_H)} \delta_{L,D}^{(h)} x_{L,D}^{(h)}\right)}{\left(\sum_{h=1}^{N_D\rho_H} x_{H,D}^{(h)} + \sum_{h=1}^{N_D(1-\rho_H)} x_{L,D}^{(h)}\right)}\right) \quad (4)$$

where $\boldsymbol{r}_{j,D}$ denotes the total number of loss claims from buyers in the set $\mathbf{B}_{j,D}$, and $R(\boldsymbol{r}_{j,D})$ denotes the set of all distinct subsets of $\mathbf{B}_{j,D}$ of $\boldsymbol{r}_{j,D}$ size .

Ideally, we seek an interior Nash-equilibrium solution to the direct-writing market game. If such a solution exists, then it is given by the vector $\mathbf{v}^* = [\mathbf{x}^*_{H,D},\ \mathbf{x}^*_{L,D},\ y_D\ ]$ satisfying the system of first-order conditions:

$$\left.\frac{\partial E\left[\varphi_{B_j}\left(B^{(i)}_{j,D}(t)\right)\right]}{\partial x^{(i)}_{j,D}}\right|_{v^*} = 0 \tag{5}$$

for $j = H$ and L; and

$$\left.\frac{\partial E\left[\varphi_S(S)\right]}{\partial y_D}\right|_{v^*} = 0\,. \tag{6}$$

Now consider the system (5) - (6) for a market with CARA buyers with utility functions $\varphi_{Bj}(w) = (1\text{-}e^{-\beta_j w})/\beta_j$, with $\beta_L > \beta_H \geq 0$,[2] and a risk-neutral seller with $\varphi_S(w) = \sigma w$, for some $\sigma > 0$.

It is mathematically straightforward but technically lengthy to show that the first-order conditions (5) may be written as follows:[3]

$$\begin{aligned}&\left(1-\pi_H\right)\exp\left(-\beta_H V\right)+\pi_H \exp\left(-\beta_H \frac{y_D^* x_{H,D}^*}{\left[N_D\rho_H x_{H,D}^* + N_D\left(1-\rho_H\right)x_{L,D}^*\right]}\right)\\ &\times\left\{1-\frac{y_D^*\left[\left(N_D\rho_H-1\right)x_{H,D}^*+N_D\left(1-\rho_H\right)x_{L,D}^*\right]}{\left[N_D\rho_H x_{H,D}^*+N_D\left(1-\rho_H\right)x_{L,D}^*\right]^2}\right\}=0\end{aligned} \tag{5'a}$$

and

$$\begin{aligned}&\left(1-\pi_L\right)\exp\left(-\beta_L V\right)+\pi_L \exp\left(-\beta_L \frac{y_D^* x_{L,D}^*}{\left[N_D\rho_H x_{H,D}^* + N_D\left(1-\rho_H\right)x_{L,D}^*\right]}\right)\\ &\times\left\{1-\frac{y_D^*\left\{N_D\rho_H x_{H,D}^*+\left[N_D\left(1-\rho_H\right)-1\right]x_{L,D}^*\right\}}{\left[N_D\rho_H x_{H,D}^*+N_D\left(1-\rho_H\right)x_{L,D}^*\right]^2}\right\}=0\,.\end{aligned} \tag{5'b}$$

Examining the derivative on the left-hand side of (6), however, we quickly see that there is no interior solution to the game because

$$\left.\frac{\partial E\left[\varphi_S(S)\right]}{\partial y_D}\right|_{v^*} = \sigma\left[P_D^* - \frac{\rho_H \pi_H x_{H,D}^* + \left(1-\rho_H\right)\pi_L x_{L,D}^*}{\rho_H x_{H,D}^* + \left(1-\rho_H\right) x_{L,D}^*}\right] \quad (7)$$

is constant over all $y_D \in [0, W_S]$.

To illustrate this outcome, consider a market characterized by the following parameter values: $N_D = 125$, $V = 10{,}000$, $W_B = 11{,}000$, $W_S = 90{,}000$, $\pi_H = 0{,}5$, $\pi_L = 0{,}025$, $\beta_H = 0{,}00025$, $\beta_L = 0{,}0005$, and $y^*_D = W_S = 90{,}000$.[4] For feasible values of ρ_H (i.e., values for which equilibrium is found to exist), Figure 1 presents the equilibrium price (per unit),

$$P_D^* = \frac{1}{y_D^*}\left[N_D \rho_H x_{H,D}^* + N_D\left(1-\rho_H\right)x_{L,D}^*\right],$$

Figure 2 presents the equilibrium quantities (per buyer),

$$Q_{j,D}^* = \frac{y_D^* x_{j,D}^*}{N_D \rho_H x_{H,D}^* + N_D\left(1-\rho_H\right)x_{L,D}^*},$$

for $j = H$ and L, and Figure 3 presents the expected utilities of buyers of both types.

FIGURE 1
PRICE VS. ρ_H FOR H AND L BUYERS

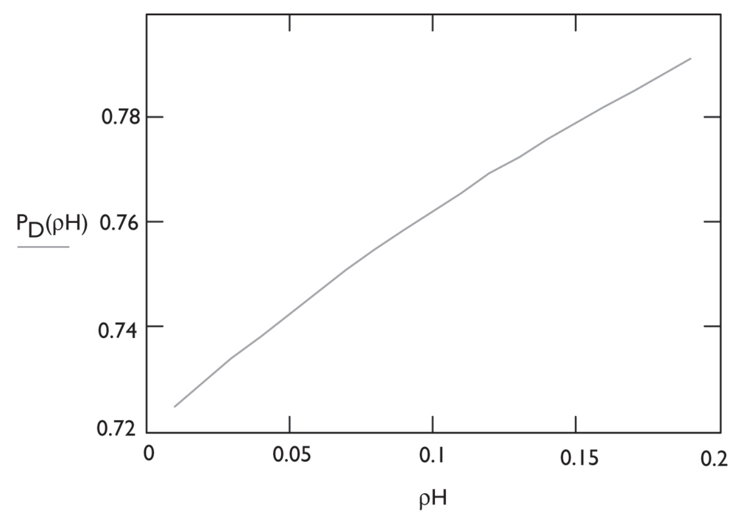


FIGURE 2A
QUANTITY VS. ρ_H FOR H BUYERS

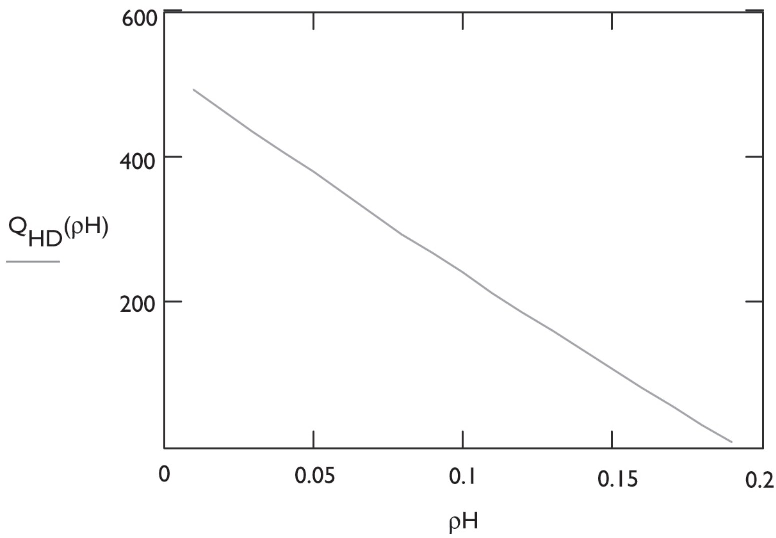


FIGURE 2B
QUANTITY VS. ρ_H FOR L BUYERS

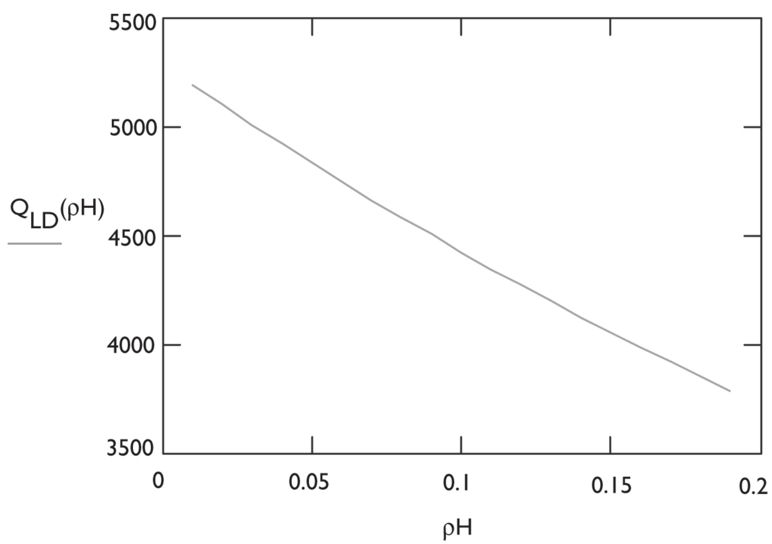


FIGURE 3A
EXPECTED UTILITY VS. ρ_H FOR H BUYERS

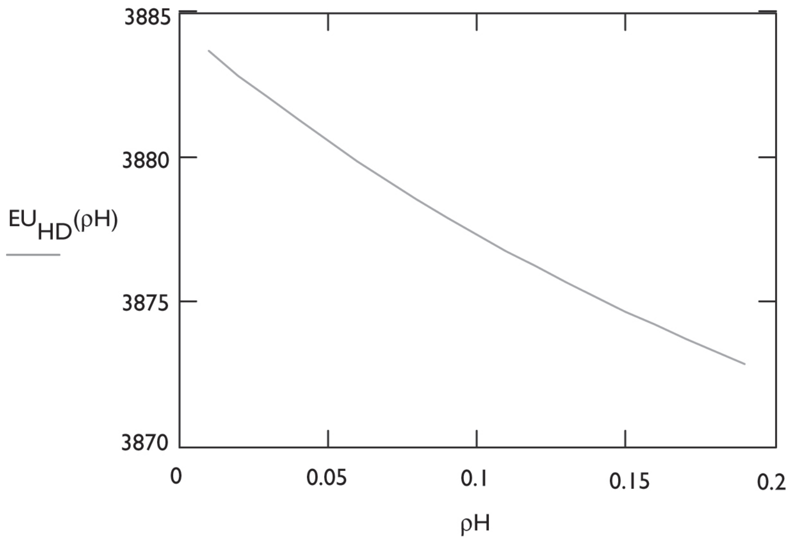


FIGURE 3B
EXPECTED UTILITY VS. ρ_H FOR L BUYERS

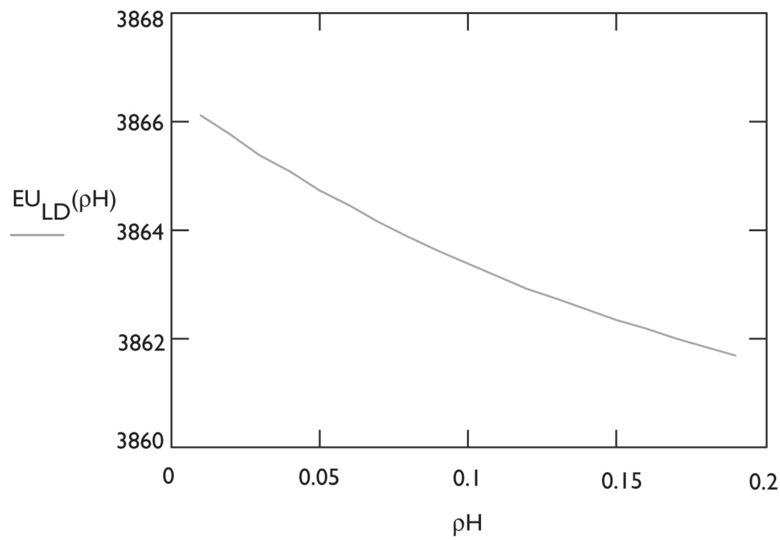


Figure 1 shows that price increases over ρ_H, whereas Figure 2(a, b) shows that quantity decreases over ρ_H for both high- and low-risk buyers. These two observations, taken together, imply that expected utilities should decrease over ρ_H for both types of buyers (as shown in Figure 3(a, b)). Given that the total number of buyers remains the same as ρ_H changes, the reason for both the increase in price and the decreases in quantity is the increased demand for insurance caused by replacing low-risk buyers by high-risk buyers. Although high-risk buyers are less risk-averse than low-risk buyers, the former group's greater loss probabilities override this effect. If we were to choose π_H sufficiently close π_L to , then demand would decrease over ρ_H, leading to lower prices and greater quantities.

We note that our results in this case differ markedly from those of Rothschild and Stiglitz (1976) in that the existence of high-risk buyers does not created a negative externality. Although low-risk buyers are worse off in the presence of high-risk buyers, the latter are actually better off in the presence of the former. This difference in results is not attributable to the assumption that $ß_H < ß_L$ (since changing the direction of this inequality would cause an even greater increase in demand over ρ_H). Rather, the difference arises primarily from the fact that Rothschild and Stiglitz (1976) assumed that there are no capacity constraints on the amount of insurance offered by sellers in the market, whereas we posit a maximum possible insurance offer of W_S.

3. MARKET WITH AN INDEPENDENT UNDERWRITER

We now assume that the seller described in the previous section can retain an independent underwriter to evaluate all potential buyers, and set up two subsidiary pools, one for high-risk buyers and the other for low-risk buyers. For expositional clarity, we will treat the two pools as separately incorporated insurance companies, so that the seller's liability is limited to its invested capital if either subsidiary becomes insolvent. Under U.S. corporate law, the owners of a group of companies may decide to bail out a failing subsidiary (e.g., to protect the group's reputation or the subsidiary's franchise value), but they are under no legal obligation to do so. Claimants against the insolvent subsidiary have no right to access the assets of the group parent or other affiliates unless they are able to "pierce the corporate veil," a heavy legal burden for the plaintiff to meet (see Easterbrook and Fischel, 1985; and Porat and Powers, 1993).

Under the independent-underwriter system, the responsibility for assigning buyers to the two different subsidiaries falls to the underwriter. We assume that the underwriter possesses each buyer's risk profile, and the only unresolved issue in the underwriting process is whether or not the underwriter is truthful in its assignments.

The most direct way to align an independent underwriter's interests with those of the seller is for the seller to pay commissions in direct proportion to its underwriting profit. However, because of timing difficulties involved in accounting for both losses and expenses when a policy is written, underwriter compensation is typically based primarily upon the underwriter's volume of direct written premiums, with some further modifications based upon the overall loss ratio of the underwriter's book of business. Given an independent underwriter's opportunities to diversify earnings by representing insurers in different lines of business, we assume that the underwriter is risk-neutral. This assumption is supported (against the alternative of risk aversion) by observing that independent underwriters are willing to produce business without the assurance of any fixed salary or guaranteed fee-for-service compensation.

We now extend the model of the previous section to include one independent underwriter serving as an intermediary between the seller and N_U buyers. As before, the overall proportion of high-risk buyers, ρ_H, is known to all players. Denote the set of $N_U \rho_H$ high-risk buyers by $\mathbf{B}_{H,U}$, and the set of $N_U(1-\rho_H)$ low-risk buyers by $\mathbf{B}_{L,U}$.

Given that the underwriter knows each buyer's risk category (high-risk or low-risk), and may report it either truthfully or untruthfully to the seller, the underwriter's strategy consists of the private probabilities, γ_H and γ_L, that it will report (respectively) a high-risk or low-risk buyer truthfully. Thus, the set $\mathbf{B}_{H,U}$ is subdivided into $\mathbf{B}_{H,UH}$ and $\mathbf{B}_{H,UL}$, the former containing $N_{H,UH} = N_U\rho_H \gamma_H$ correctly identified high-risk buyers, and the latter containing $N_{H,UL} = N_U\rho_H (1-\gamma_H)$ incorrectly identified high-risk buyers. Similarly, the set $\mathbf{B}_{L,U}$ is subdivided into $\mathbf{B}_{L,UH}$ and $\mathbf{B}_{L,UL}$, the former containing $N_{L,UL} = N_U (1-\rho_H) \gamma_L$ correctly identified low-risk buyers, and the latter containing $N_{L,UH} = N_U(1-\rho_H)(1-\gamma_L)$ incorrectly identified low-risk buyers. We will assume that the underwriter selects its private probabilities (γ_H and γ_L) and announces the buyers' risk categories at some point in time between $t = 0$ (when the seller announces its strategic quantity offer) and $t = 1$ (when the buyers announce their strategic price bids); for simplicity, we will call this point $t = 1/2$.

Note that the seller now makes two strategic quantity offers: y_{UH}, to the pool $\mathbf{UH} = \mathbf{B}_{H,UH} \cup \mathbf{B}_{L,UH}$, and y_{UL} to the pool $\mathbf{UL} = \mathbf{B}_{H,UL} \cup \mathbf{B}_{L,UL}$, subject to the constraint $y_U = y_{UH} + y_{UL} \in [0, W_S]$; however, for conceptual consistency with the model of the previous section, we will work with the strategy pair $[y_U, y_{UH}]$, where y_U is comparable to y_D. A further strategic aspect of the seller is the choice of a compensation scheme designed to guide the underwriter's behavior in a way that is favorable to the seller. For simplicity, we will assume that this scheme consists of two components, a commission payment given by κ_1 x (Total Premiums Written) and a loss penalty given by -κ_2 x (Expected Total Losses), for positive parameters κ_1 and κ_2. Since the underwriter-commission loading, κ_1, is often set by historical and/or institutional market conditions, we will take it as exogenous in this work. However, the selection of the penalty parameter κ_2 will be modeled as an explicit strategy of the seller that is announced simultaneously with the seller's quantity offer (i.e., at $t = 0$).

At the beginning of the policy period (i.e., at $t = 1$) each buyer $i \in \mathbf{B}_{j,k}$ simultaneously announces a strategic price bid, $x^{(i)}_{j,k} \in [0, V]$ As in the previous section, we assume that the seller is required to accept all nonzero price bids as one lot, as long as the seller's final expected utility is greater than its initial expected utility. Likewise, we assume that the underwriter is required to participate as long as its final expected utility is greater than its initial expected utility (but that the underwriter may "walk away" from the market otherwise).

Since the seller is not able to determine the risk profile of any individual buyer, it must offer a single price (per unit) of insurance for each of the two pools, **UH** and **UL**. These market prices are respectively:

$$P_{UH} = P_{UH}\left(\mathrm{x}_{H,UH}, \mathrm{x}_{L,UH}, y_{UH}\right) = \frac{1}{y_{UH}} \left(\sum_{h=1}^{N_U \rho_H \gamma_H} x_{H,UH}^{(h)} + \sum_{h=1}^{N_U(1-\rho_H)(1-\gamma_L)} x_{L,UH}^{(h)} \right)$$

and

$$P_{UL} = P_{UL}\left(\mathrm{x}_{H,UL}, \mathrm{x}_{L,UL}, y_{UH}\right) = \frac{1}{\left(y_U - y_{UH}\right)} \left(\sum_{h=1}^{N_U \rho_H (1-\gamma_H)} x_{H,UL}^{(h)} + \sum_{h=1}^{N_U(1-\rho_H)\gamma_L} x_{L,UL}^{(h)} \right).$$

Finally, the allocation of insurance coverage again admits of both a global formulation and a local formulation. Under the former, if buyer $i \in \mathbf{B}_{j,k}$ suffers a loss during the policy period, then he or she will receive a loss payment in the amount of

$$y_{UH}\left[x^{(i)}_{j,UH}\Big/\left(\sum_{h=1}^{N_U\rho_H\gamma_H} x^{(h)}_{H,UH}+\sum_{h=1}^{N_U(1-\rho_H)(1-\gamma_L)} x^{(h)}_{L,UH}\right)\right] \text{ for } k=UH \text{ and}$$

$$\left(y_U-y_{UH}\right)\left[x^{(i)}_{j,UL}\Big/\left(\sum_{h=1}^{N_U\rho_H(1-\gamma_H)} x^{(h)}_{H,UL}+\sum_{h=1}^{N_U(1-\rho_H)\gamma_L} x^{(h)}_{L,UL}\right)\right] \quad \text{for} \quad k=UL.$$

Under the latter, we simply require that if buyer $i\in \mathbf{B}_{j,k}$ increases its premium bid $x^{(i)}_{j,k}$. by a small amount in a neighborhood of equilibrium, then 's share of its indemnity payment also increases proportionately.

Within some neighborhood of equilibrium: the final wealth of buyer $i\in \mathbf{B}_{j,k}$ is thus

$$\begin{aligned}B^{(i)}_{j,UH}&=\left(1-\delta^{(i)}_{j,UH}\right)\left(W_B+V-x^{(i)}_{j,UH}\right)\\&+\delta^{(i)}_{j,UH}\left\{W_B-x^{(i)}_{j,UH}+y_{UH}\left[x^{(i)}_{j,UH}\Big/\left(\sum_{h=1}^{N_U\rho_H\gamma_H} x^{(h)}_{H,UH}+\sum_{h=1}^{N_U(1-\rho_H)(1-\gamma_L)} x^{(h)}_{L,UH}\right)\right]\right\}\end{aligned}$$

for $k = UH$, and

$$\begin{aligned}B^{(i)}_{j,UL}&=\left(1-\delta^{(i)}_{j,UL}\right)\left(W_B+V-x^{(i)}_{j,UL}\right)\\&+\delta^{(i)}_{j,UL}\left\{W_B-x^{(i)}_{j,UL}+\left(y_U-y_{UH}\right)\left[x^{(i)}_{j,UL}\Big/\left(\sum_{h=1}^{N_U\rho_H(1-\gamma_H)} x^{(h)}_{H,UL}+\sum_{h=1}^{N_U(1-\rho_H)\gamma_L} x^{(h)}_{L,UL}\right)\right]\right\}\end{aligned}$$

for $k = UL$; the seller's final wealth is

$$\begin{aligned}S&=W_S+\left(1-\kappa_1\right)\left[y_{UH}P_{UH}+\left(y_U-y_{UH}\right)P_{UL}\right]\\&-\left(1-\kappa_2\right)\left[\frac{y_{UH}\left(\sum_{h=1}^{N_U\rho_H\gamma_H}\delta^{(h)}_{H,UH}x^{(h)}_{H,UH}+\sum_{h=1}^{N_U(1-\rho_H)(1-\gamma_L)}\delta^{(h)}_{L,UH}x^{(h)}_{L,UH}\right)}{\left(\sum_{h=1}^{N_U\rho_H\gamma_H} x^{(h)}_{H,UH}+\sum_{h=1}^{N_U(1-\rho_H)(1-\gamma_L)} x^{(h)}_{L,UH}\right)}\right.\\&\left.+\frac{\left(y_U-y_{UH}\right)\left(\sum_{h=1}^{N_U\rho_H(1-\gamma_H)}\delta^{(h)}_{H,UL}x^{(h)}_{H,UL}+\sum_{h=1}^{N_U(1-\rho_H)\gamma_L}\delta^{(h)}_{L,UL}x^{(h)}_{L,UL}\right)}{\left(\sum_{h=1}^{N_U\rho_H(1-\gamma_H)} x^{(h)}_{H,UL}+\sum_{h=1}^{N_U(1-\rho_H)\gamma_L} x^{(h)}_{L,UL}\right)}\right];\end{aligned}$$

and the underwriter's final wealth is

$$U = W_U + \kappa_1\left[y_{UH}P_{UH} + \left(y_U - y_{UH}\right)P_{UL}\right]$$

$$-\kappa_2\left[\frac{y_{UH}\left(\sum_{h=1}^{N_U\rho_H\gamma_H}\delta_{H,UH}^{(h)}x_{H,UH}^{(h)} + \sum_{h=1}^{N_U(1-\rho_H)(1-\gamma_L)}\delta_{L,UH}^{(h)}x_{L,UH}^{(h)}\right)}{\left(\sum_{h=1}^{N_U\rho_H\gamma_H}x_{H,UH}^{(h)} + \sum_{h=1}^{N_U(1-\rho_H)(1-\gamma_L)}x_{L,UH}^{(h)}\right)}\right.$$

$$\left.+\frac{\left(y_U - y_{UH}\right)\left(\sum_{h=1}^{N_U\rho_H(1-\gamma_H)}\delta_{H,UL}^{(h)}x_{H,UL}^{(h)} + \sum_{h=1}^{N_U(1-\rho_H)\gamma_L}\delta_{L,UL}^{(h)}x_{L,UL}^{(h)}\right)}{\left(\sum_{h=1}^{N_U\rho_H(1-\gamma_H)}x_{H,UL}^{(h)} + \sum_{h=1}^{N_U(1-\rho_H)\gamma_L}x_{L,UL}^{(h)}\right)}\right],$$

where W_U is its initial endowment. Letting $\varphi_{Bj}(\cdot):\Re\rightarrow\Re$ denote the utility function of buyer $\in\mathbf{B}_{j,k}$, $\varphi_S(\cdot):\Re\rightarrow\Re$ denote the utility function of the seller, and $\varphi_U(\cdot):\Re\rightarrow\Re$ denote the utility function of the underwriter (where all utility functions are increasing and weakly concave downward), it then follows that the payoffs to the various players are given by the following expressions:

$$E\left[\varphi_{B_j}\left(B_{j,UH}^{(i)}\right)\right] = \left(1-\pi_j\right)\varphi_{B_j}\left(W_B + V - x_{j,UH}^{(i)}\right)$$

$$+\pi_j\varphi_{B_j}\left(W_B - x_{j,UH}^{(i)} + y_{UH}\left[x_{j,UH}^{(i)}\Big/\left(\sum_{h=1}^{N_U\rho_H\gamma_H}x_{H,UH}^{(h)} + \sum_{h=1}^{N_U(1-\rho_H)(1-\gamma_L)}x_{L,UH}^{(h)}\right)\right]\right)$$

and

$$E\left[\varphi_{B_j}\left(B_{j,UL}^{(i)}\right)\right] = \left(1-\pi_j\right)\varphi_{B_j}\left(W_B + V - x_{j,UL}^{(i)}\right)$$

$$+\pi_j\varphi_{B_j}\left(W_B - x_{j,UL}^{(i)} + \left(y_U - y_{UH}\right)\left[x_{j,UL}^{(i)}\Big/\left(\sum_{h=1}^{N_U\rho_H(1-\gamma_H)}x_{H,UL}^{(h)} + \sum_{h=1}^{N_U(1-\rho_H)\gamma_L}x_{L,UL}^{(h)}\right)\right]\right);$$

$$
\begin{aligned}
&E\left[\varphi_S(S)\right] \\
&= \sum_{r_{H,UH}=0}^{N_U\rho_H\gamma_H} \sum_{\substack{R(r_{H,UH})\\ \in \mathrm{B}_{H,UH}}} \sum_{r_{L,UH}=0}^{N_U(1-\rho_H)(1-\gamma_L)} \sum_{\substack{R(r_{L,UH})\\ \in \mathrm{B}_{L,UH}}} \sum_{r_{H,UL}=0}^{N_U\rho_H(1-\gamma_H)} \sum_{\substack{R(r_{H,UL})\\ \in \mathrm{B}_{H,UL}}} \sum_{r_{L,UL}=0}^{N_U(1-\rho_H)\gamma_L} \\
&\quad \times \sum_{\substack{R(r_{L,UL})\\ \in \mathrm{B}_{L,UL}}} \left[(\pi_H)^{r_{H,UH}+r_{H,UL}} (1-\pi_H)^{N_U\rho_H - r_{H,UH} - r_{H,UL}} \times (\pi_L)^{r_{L,UH}+r_{L,UL}} (1-\pi_L)^{N_U(1-\rho_H) - r_{L,UH} - r_{L,UL}} \right] \\
&\quad \times \varphi_S\left(W_S + (1-\kappa_1)\left[y_{UH}P_{UH} + (y_U - y_{UH})P_{UL} \right] - (1-\kappa_2) \right. \\
&\quad \left. \left[\frac{y_{UH}\left(\sum_{h\in R(r_{H,UH})} x_{H,UH}^{(h)} + \sum_{h\in R(r_{L,UH})} x_{L,UH}^{(h)} \right)}{\left(\sum_{h=1}^{N_U\rho_H\gamma_H} x_{H,UH}^{(h)} + \sum_{h=1}^{N_U(1-\rho_H)(1-\gamma_L)} x_{L,UH}^{(h)} \right)} + \frac{(y_U - y_{UH})\left(\sum_{h\in R(r_{H,UL})} x_{H,UL}^{(h)} + \sum_{h\in R(r_{L,UL})} x_{L,UL}^{(h)} \right)}{\left(\sum_{h=1}^{N_U\rho_H(1-\gamma_H)} x_{H,UL}^{(h)} + \sum_{h=1}^{N_U(1-\rho_H)\gamma_L} x_{L,UL}^{(h)} \right)} \right] \right),
\end{aligned}
$$

where $r_{j,k}$ denotes the total number of loss claims from buyers in the set $\mathbf{B}_{j,k}$, and $R(r_{j,k})$ denotes the set of all distinct subsets of $\mathbf{B}_{j,k}$ of size $r_{j,k}$; and

$$
\begin{aligned}
&E\left[\varphi_U(U)\right] \\
&= \sum_{r_{H,UH}=0}^{N_U\rho_H\gamma_H} \sum_{\substack{R(r_{H,UH})\\ \in \mathrm{B}_{H,UH}}} \sum_{r_{L,UH}=0}^{N_U(1-\rho_H)(1-\gamma_L)} \sum_{\substack{R(r_{L,UH})\\ \in \mathrm{B}_{L,UH}}} \sum_{r_{H,UL}=0}^{N_U\rho_H(1-\gamma_H)} \sum_{\substack{R(r_{H,UL})\\ \in \mathrm{B}_{H,UL}}} \sum_{r_{L,UL}=0}^{N_U(1-\rho_H)\gamma_L} \\
&\quad \times \sum_{\substack{R(r_{L,UL})\\ \in \mathrm{B}_{L,UL}}} \left[(\pi_H)^{r_{H,UH}+r_{H,UL}} (1-\pi_H)^{N_U\rho_H - r_{H,UH} - r_{H,UL}} \times (\pi_L)^{r_{L,UH}+r_{L,UL}} (1-\pi_L)^{N_U(1-\rho_H) - r_{L,UH} - r_{L,UL}} \right] \\
&\quad \times \varphi_U\left(W_U + \kappa_1\left[y_{UH}P_{UH} + (y_U - y_{UH})P_{UL} \right] - \kappa_2 \right. \\
&\quad \left. \left[\frac{y_{UH}\left(\sum_{h\in R(r_{H,UH})} x_{H,UH}^{(h)} + \sum_{h\in R(r_{L,UH})} x_{L,UH}^{(h)} \right)}{\left(\sum_{h=1}^{N_U\rho_H\gamma_H} x_{H,UH}^{(h)} + \sum_{h=1}^{N_U(1-\rho_H)(1-\gamma_L)} x_{L,UH}^{(h)} \right)} + \frac{(y_U - y_{UH})\left(\sum_{h\in R(r_{H,UL})} x_{H,UL}^{(h)} + \sum_{h\in R(r_{L,UL})} x_{L,UL}^{(h)} \right)}{\left(\sum_{h=1}^{N_U\rho_H(1-\gamma_H)} x_{H,UL}^{(h)} + \sum_{h=1}^{N_U(1-\rho_H)\gamma_L} x_{L,UL}^{(h)} \right)} \right] \right).
\end{aligned}
$$

If an interior Nash-equilibrium solution exists, then it is given by the vector $\mathbf{v}^* = [\mathbf{x}^*_{H,UH}, \mathbf{x}^*_{L,UH}, \mathbf{x}^*_{H,UL}, \mathbf{x}^*_{L,UL}, y^*_{UH}, \kappa^*_2, \gamma^*_H, \gamma^*_L]$ satisfying the system of first-order conditions:

$$\left.\frac{\partial E\left[\varphi_{B_j}\left(B^{(i)}_{j,k}\right)\right]}{\partial x^{(i)}_{j,k}}\right|_{\mathrm{v}^*} = 0 \tag{8}$$

for $j = H$ and L and $k = UH$ and UL;

$$\left.\frac{\partial E\left[\varphi_S(S)\right]}{\partial y_U}\right|_{\mathrm{v}^*} = 0\,, \tag{9}$$

and

$$\left.\frac{\partial E\left[\varphi_S(S)\right]}{\partial y_{UH}}\right|_{\mathrm{v}^*} = 0\,, \tag{10}$$

$$\left.\frac{\partial E\left[\varphi_S(S)\right]}{\partial \kappa_2}\right|_{\mathrm{v}^*} = 0\,; \tag{11}$$

and

$$\left.\frac{\partial E\left[\varphi_U(U)\right]}{\partial \gamma_j}\right|_{\mathrm{v}^*} = 0\,; \tag{12}$$

for $j = H$ and L.

We now consider the system (8) - (12) for a market with a risk-neutral underwriter with utility function $\varphi_U(w) = \upsilon w$, for some $\upsilon > 0$, and CARA buyers with utility functions $\varphi_{Bj}(w) = (1 - e^{-\beta_j w})/\beta_j$, with $\beta_L > \beta_H \geq 0$. The first subsection below addresses the case of a risk-neutral seller (i.e., $\varphi_S(w) = \sigma w$, for some $\sigma > 0$), whereas the second addresses the case of an MVO seller.

3.1 Market with a Risk-Neutral Seller

As with the first-order conditions presented in Section 2, it is mathematically straightforward but technically lengthy to show that the first-order conditions (8) and (12) may be written as follows:[5]

$$
\begin{aligned}
&\left(1-\pi_H\right)\exp\left(-\beta_H V\right)+\pi_H \exp\left(-\beta_H \frac{y_{UH}^* x_{H,UH}^*}{\left[N_U\rho_H\gamma_H^* x_{H,UH}^* + N_U\left(1-\rho_H\right)\left(1-\gamma_L^*\right)x_{L,UH}^*\right]}\right) \\
&\times\left\{1-\frac{y_{UH}^*\left[\left(N_U\rho_H\gamma_H^*-1\right)x_{H,UH}^* + N_U\left(1-\rho_H\right)\left(1-\gamma_L^*\right)x_{L,UH}^*\right]}{\left[N_U\rho_H\gamma_H^* x_{H,UH}^* + N_U\left(1-\rho_H\right)\left(1-\gamma_L^*\right)x_{L,UH}^*\right]^2}\right\}=0\,,
\end{aligned}
\tag{8'a}
$$

$$
\begin{aligned}
&\left(1-\pi_H\right)\exp\left(-\beta_H V\right)+\pi_H \exp\left(-\beta_H \frac{\left(y_U^*-y_{UH}^*\right)x_{H,UL}^*}{\left[N_U\rho_H\left(1-\gamma_H^*\right)x_{H,UL}^* + N_U\left(1-\rho_H\right)\gamma_L x_{L,UL}^*\right]}\right) \\
&\times\left\{1-\frac{\left(y_U^*-y_{UH}^*\right)\left\{\left[N_U\rho_H\left(1-\gamma_H^*\right)-1\right]x_{H,UL}^* + N_U\left(1-\rho_H\right)\gamma_L^* x_{L,UL}^*\right\}}{\left[N_U\rho_H\left(1-\gamma_H^*\right)x_{H,UL}^* + N_U\left(1-\rho_H\right)\gamma_L^* x_{L,UL}^*\right]^2}\right\}=0\,,
\end{aligned}
\tag{8'b}
$$

$$
\begin{aligned}
&\left(1-\pi_L\right)\exp\left(-\beta_L V\right)+\pi_L \exp\left(-\beta_L \frac{y_{UH}^* x_{L,UH}^*}{\left[N_U\rho_H\gamma_H^* x_{H,UH}^* + N_U\left(1-\rho_H\right)\left(1-\gamma_L^*\right)x_{L,UH}^*\right]}\right) \\
&\times\left\{1-\frac{y_{UH}^*\left\{N_U\rho_H\gamma_H^* x_{H,UH}^* + \left[N_U\left(1-\rho_H\right)\left(1-\gamma_L^*\right)-1\right]x_{L,UH}^*\right\}}{\left[N_U\rho_H\gamma_H^* x_{H,UH}^* + N_U\left(1-\rho_H\right)\left(1-\gamma_L^*\right)x_{L,UH}^*\right]^2}\right\}=0\,,
\end{aligned}
\tag{8'c}
$$

$$
\begin{aligned}
&\left(1-\pi_L\right)\exp\left(-\beta_L V\right)+\pi_L \exp\left(-\beta_L \frac{\left(y_U^*-y_{UH}^*\right)x_{L,UL}^*}{\left[N_U\rho_H\left(1-\gamma_H^*\right)x_{H,UL}^* + N_U\left(1-\rho_H\right)\gamma_L^* x_{L,UL}^*\right]}\right) \\
&\times\left\{1-\frac{\left(y_U^*-y_{UH}^*\right)\left\{N_U\rho_H\left(1-\gamma_H^*\right)x_{H,UL}^* + \left[N_U\left(1-\rho_H\right)\gamma_L^*-1\right]x_{L,UL}^*\right\}}{\left[N_U\rho_H\left(1-\gamma_H^*\right)x_{H,UL}^* + N_U\left(1-\rho_H\right)\gamma_L^* x_{L,UL}^*\right]^2}\right\}=0\,;
\end{aligned}
\tag{8'd}
$$

and

$$
\begin{aligned}
&\left(1-\gamma_H^*\right)\pi_H^{N_U\left(1-\gamma_H^*\right)\rho_H}\Big\{\kappa_1\left[\rho_H\gamma_H^* x_{H,UH}^* + \left(1-\rho_H\right)\left(1-\gamma_L^*\right)x_{L,UH}^* + \rho_H\left(1-\gamma_H^*\right)x_{H,UL}^* + \left(1-\rho_H\right)\gamma_L^* x_{L,UL}^*\right] \\
&-\kappa_2^*\Bigg\{\frac{y_{UH}^*\left[\rho_H\gamma_H^*\pi_H\left(1-\pi_H\right)x_{H,UH}^* + \left(1-\rho_H\right)\left(1-\gamma_L^*\right)\pi_L\left(1-\pi_L\right)x_{L,UH}^*\right]}{N_U\rho_H\gamma_H^* x_{H,UH}^* + N_U\left(1-\rho_H\right)\left(1-\gamma_L^*\right)x_{L,UH}^*} \\
&+\frac{\left(y_U^*-y_{UH}^*\right)\left[\rho_H\left(1-\gamma_H^*\right)x_{H,UL}^* + \left(1-\rho_H\right)\gamma_L^*\pi_L\left(1-\pi_L\right)x_{L,UL}^*\right]}{N_U\rho_H\left(1-\gamma_H^*\right)x_{H,UL}^* + N_U\left(1-\rho_H\right)\gamma_L^* x_{L,UL}^*}\Bigg\}\Big\} \\
&=\gamma_H^*\pi_H^{N_U\gamma_H^*\rho_H}\Big\{\kappa_1\left[\rho_H\gamma_H^* x_{H,UH}^* + \left(1-\rho_H\right)\left(1-\gamma_L^*\right)x_{L,UH}^* + \rho_H\left(1-\gamma_H^*\right)x_{H,UL}^* + \left(1-\rho_H\right)\gamma_L^* x_{L,UL}^*\right] \\
&-\kappa_2^*\Bigg\{\frac{y_{UH}^*\left[\rho_H\gamma_H^* x_{H,UH}^* + \left(1-\rho_H\right)\left(1-\gamma_L^*\right)\pi_L\left(1-\pi_L\right)x_{L,UH}^*\right]}{N_U\rho_H\gamma_H^* x_{H,UH}^* + N_U\left(1-\rho_H\right)\left(1-\gamma_L^*\right)x_{L,UH}^*} \\
&+\frac{\left(y_U^*-y_{UH}^*\right)\left[\rho_H\left(1-\gamma_H^*\right)\pi_H\left(1-\pi_H\right)x_{H,UL}^* + \left(1-\rho_H\right)\gamma_L^*\pi_L\left(1-\pi_L\right)x_{L,UL}^*\right]}{N_U\rho_H\left(1-\gamma_H^*\right)x_{H,UL}^* + N_U\left(1-\rho_H\right)\gamma_L^* x_{L,UL}^*}\Bigg\}\Big\},
\end{aligned}
\tag{12'a}
$$

$$
\begin{aligned}
&\left(1-\gamma_L^*\right)\pi_L^{N_U\left(1-\gamma_L^*\right)\left(1-\rho_H\right)}\left\{\kappa_1\left[\rho_H\gamma_H^*x_{H,UH}^*+\left(1-\rho_H\right)\left(1-\gamma_L^*\right)x_{L,UH}^*+\rho_H\left(1-\gamma_H^*\right)x_{H,UL}^*+\left(1-\rho_H\right)\gamma_L^*x_{L,UL}^*\right]\right.\\
&-\kappa_2^*\left\{\frac{y_{UH}^*\left[\rho_H\gamma_H^*\pi_H\left(1-\pi_H\right)x_{H,UH}^*+\left(1-\rho_H\right)\left(1-\gamma_L^*\right)x_{L,UH}^*\right]}{N_U\rho_H\gamma_H^*x_{H,UH}^*+N_U\left(1-\rho_H\right)\left(1-\gamma_L^*\right)x_{L,UH}^*}\right.\\
&\left.\left.+\frac{\left(y_U^*-y_{UH}^*\right)\left[\rho_H\left(1-\gamma_H^*\right)\pi_H\left(1-\pi_H\right)x_{H,UL}^*+\left(1-\rho_H\right)\gamma_L^*\pi_L\left(1-\pi_L\right)x_{L,UL}^*\right]}{N_U\rho_H\left(1-\gamma_H^*\right)x_{H,UL}^*+N_U\left(1-\rho_H\right)\gamma_L^*x_{L,UL}^*}\right\}\right\}\\
&=\gamma_L^*\pi_L^{N_U\gamma_L^*\left(1-\rho_H\right)}\left\{\kappa_1\left[\rho_H\gamma_H^*x_{H,UH}^*+\left(1-\rho_H\right)\left(1-\gamma_L^*\right)x_{L,UH}^*+\rho_H\left(1-\gamma_H^*\right)x_{H,UL}^*+\left(1-\rho_H\right)\gamma_L^*x_{L,UL}^*\right]\right.\\
&-\kappa_2^*\left\{\frac{y_{UH}^*\left[\rho_H\gamma_H^*\pi_H\left(1-\pi_H\right)x_{H,UH}^*+\left(1-\rho_H\right)\left(1-\gamma_L^*\right)\pi_L\left(1-\pi_L\right)x_{L,UH}^*\right]}{N_U\rho_H\gamma_H^*x_{H,UH}^*+N_U\left(1-\rho_H\right)\left(1-\gamma_L^*\right)x_{L,UH}^*}\right.\\
&\left.\left.+\frac{\left(y_U^*-y_{UH}^*\right)\left[\rho_H\left(1-\gamma_H^*\right)\pi_H\left(1-\pi_H\right)x_{H,UL}^*+\left(1-\rho_H\right)\gamma_L^*x_{L,UL}^*\right]}{N_U\rho_H\left(1-\gamma_H^*\right)x_{H,UL}^*+N_U\left(1-\rho_H\right)\gamma_L^*x_{L,UL}^*}\right\}\right\},
\end{aligned}
\tag{12'b}
$$

However, studying the derivatives on the left-hand sides of (9) - (11), we find that there is no interior solution to the game because

$$
\left.\frac{\partial E\left[\varphi_S\left(S\right)\right]}{\partial y_U}\right|_{v^*}=\sigma\left\{\left(1-\kappa_1\right)P_{UL}^*-\left(1-\kappa_2\right)\left[\frac{\rho_H\left(1-\gamma_H^*\right)\pi_Hx_{H,UL}^*+\left(1-\rho_H\right)\gamma_L^*\pi_Lx_{L,UL}^*}{\rho_H\left(1-\gamma_H^*\right)x_{H,UL}^*+\left(1-\rho_H\right)\gamma_L^*x_{L,UL}^*}\right]\right\}
$$

is constant over all $y_U\in\left[0,W_S\right]$,

$$
\begin{aligned}
\left.\frac{\partial E\left[\varphi_S\left(S\right)\right]}{\partial y_{UH}}\right|_{v^*}&=\sigma\left(1-\kappa_1\right)\left(P_{UH}^*-P_{UL}^*\right)\\
&-\sigma\left(1-\kappa_2^*\right)\left[\frac{\rho_H\gamma_H^*\pi_Hx_{H,UH}^*+\left(1-\rho_H\right)\left(1-\gamma_L^*\right)\pi_Lx_{L,UH}^*}{\rho_H\gamma_H^*x_{H,UH}^*+\left(1-\rho_H\right)\left(1-\gamma_L^*\right)x_{L,UH}^*}\right.\\
&\left.-\frac{\rho_H\left(1-\gamma_H^*\right)\pi_Hx_{H,UL}^*+\left(1-\rho_H\right)\gamma_L^*\pi_Lx_{L,UL}^*}{\rho_H\left(1-\gamma_H^*\right)x_{H,UL}^*+\left(1-\rho_H\right)\gamma_L^*x_{L,UL}^*}\right]
\end{aligned}
$$

is constant over all $y_{UH}\in\left[0,y_U\right]$ and

$$
\begin{aligned}
\left.\frac{\partial E\left[\varphi_S\left(S\right)\right]}{\partial \kappa_2}\right|_{v^*}&=\sigma\left[\frac{y_{UH}^*\left[\rho_H\gamma_H^*\pi_Hx_{H,UH}^*+\left(1-\rho_H\right)\left(1-\gamma_L^*\right)\pi_Lx_{L,UH}^*\right]}{\rho_H\gamma_H^*x_{H,UH}^*+\left(1-\rho_H\right)\left(1-\gamma_L^*\right)x_{L,UH}^*}\right.\\
&\left.-\frac{\left(y_U^*-y_{UH}^*\right)\left[\rho_H\left(1-\gamma_H^*\right)\pi_Hx_{H,UL}^*+\left(1-\rho_H\right)\gamma_L^*\pi_Lx_{L,UL}^*\right]}{\rho_H\left(1-\gamma_H^*\right)x_{H,UL}^*+\left(1-\rho_H\right)\gamma_L^*x_{L,UL}^*}\right]
\end{aligned}
$$

is constant over all $\kappa_2 \geq 0$ such that equations (12'a) and (12'b) are satisfied.

To illustrate this outcome, consider a market with the following parameter values: $N_U = 125$, $\rho_H = 0{,}1$, $V = 10{,}000$, $W_B = 11{,}000$, $W_S = 90{,}000$, $\pi_H = 0{,}5$, $\pi_L = 0{,}025$, $ß_H = 0{,}00025$, $ß_L = 0{,}0005$, $y^*_U = W_s = 90{,}000$ [6] and $\kappa_1 = 0{,}2$. In this case, the upper bound of κ^*_2 is a value slightly greater than 0,206, and the lower bound of y^*_{UH} is of course 0. To study the behavior of the solution as these bounds are approached, we fix $y^*_{UH} = 0{,}10 y^*_U = 9{,}000$, and let κ^*_2 range over the interval [0,0.206].

Figure 4 presents the equilibrium prices (per unit),

$$P^*_{UH} = \frac{1}{y^*_{UH}} \left[N_U \rho_H \gamma^*_H x^*_{H,UH} + N_U \left(1 - \rho_H\right)\left(1 - \gamma^*_L\right) x^*_{L,UH} \right]$$

and

$$P^*_{UL} = \frac{1}{\left(y^*_U - y^*_{UH}\right)} \left[N_U \rho_H \left(1 - \gamma^*_H\right) x^*_{H,UL} + N_U \left(1 - \rho_H\right) \gamma^*_L x^*_{L,UL} \right],$$

whereas Figure 5 presents the equilibrium quantities (per buyer),

$$Q^*_{j,UH} = \frac{y^*_{UH} x^*_{j,UH}}{N_U \rho_H \gamma^*_H x^*_{H,UH} + N_U \left(1 - \rho_H\right)\left(1 - \gamma^*_L\right) x^*_{L,UH}}$$

and

$$Q^*_{j,UL} = \frac{\left(y^*_U - y^*_{UH}\right) x^*_{j,UL}}{N_U \rho_H \left(1 - \gamma^*_H\right) x^*_{H,UL} + N_U \left(1 - \rho_H\right) \gamma^*_L x^*_{L,UL}},$$

for $j = H$ and L.

FIGURE 4
PRICE VS. κ^*_2 FOR UH AND UL BUYERS WHEN $y^*_{UH} = 0.10 y^*_U$

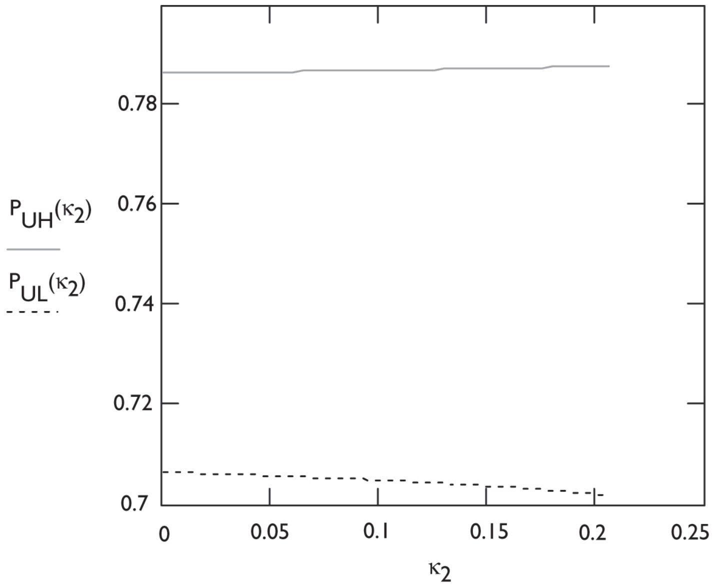


FIGURE 5A
QUANTITY VS. κ^*_2 FOR H,UH BUYERS WHEN $y^*_{UH} = 0.10 y^*_U$

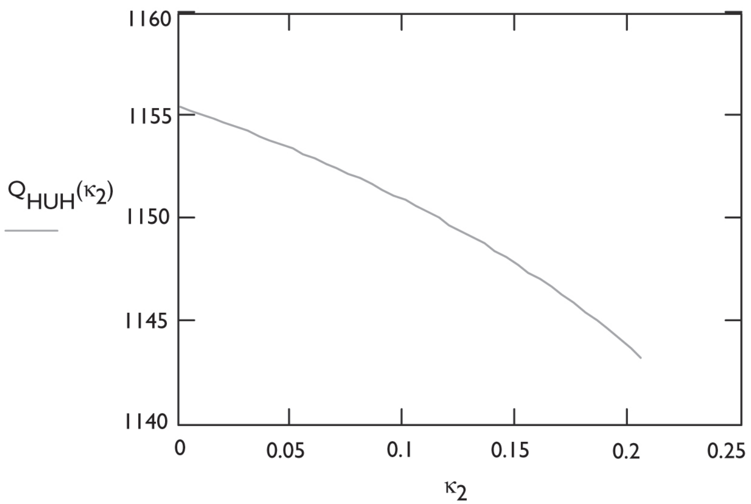


FIGURE 5B
QUANTITY VS. κ^*_2 FOR L,UH BUYERS WHEN $y^*_{UH} = 0.10y^*_U$

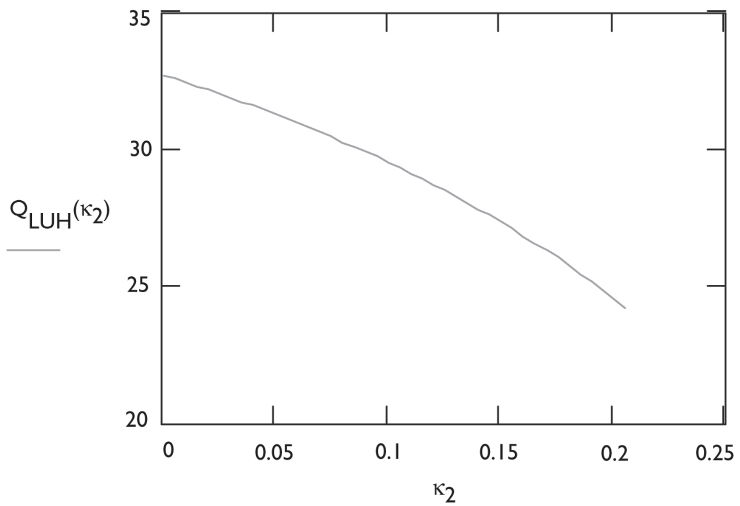


FIGURE 5C
QUANTITY VS. κ^*_2 FOR H,UL BUYERS WHEN $y^*_{UH} = 0.10y^*_U$

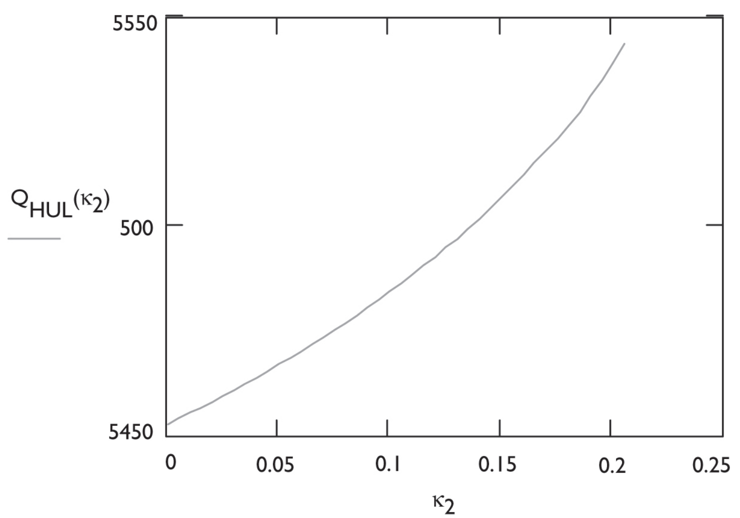
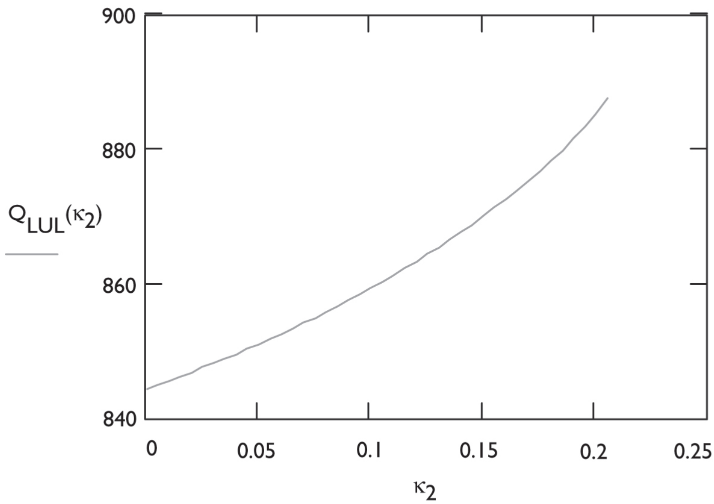


FIGURE 5D
QUANTITY VS. κ^*_2 FOR L,UL BUYERS WHEN $y^*_{UH} = 0.10y^*_U$

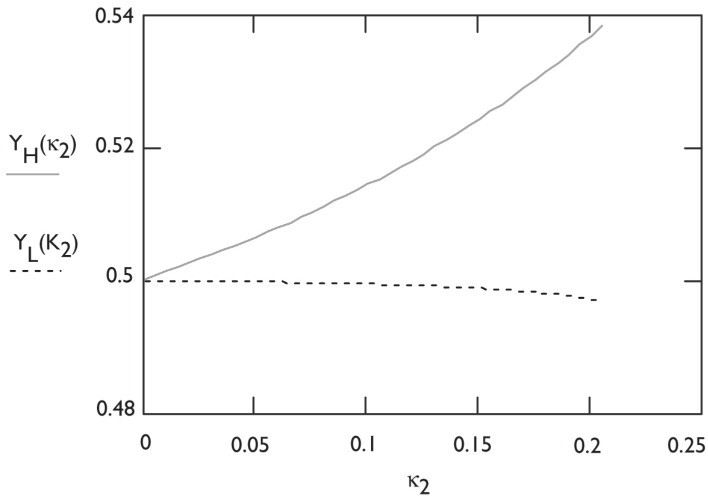


Interestingly, the reason that $P^*_{UH} > P^*_{UL}$ in Figure 4 is not that pool **UH** contains substantially more high-risk buyers than pool **UL**. (In fact, as we will see in Figure 6, for small values of κ^*_2, both pools contain about 50 percent high-risk buyers and 50 percent low-risk buyers.) Rather, the reason for the greater price in **UH** is simply that we are studying the case in which y^*_{UH} (the supply of insurance coverage in **UH**) is only one-ninth as big as y^*_U - y^*_{UH} (the supply of insurance coverage in **UL**). Reducing supply naturally causes the equilibrium price to rise.

Figure 6 shows clearly that, for all values of κ^*_2 considered, the underwriter is typically far from truthful in its identification of high- and low-risk buyers. In fact, because of the algebraic nature of equations (12'a) and (12'b), the parameters γ^*_H and γ^*_L both equal 0.5 when κ^*_2=0, with $\frac{\partial \gamma^*_H}{\partial \kappa^*_2} > 0$ and $\frac{\partial \gamma^*_L}{\partial \kappa^*_2} < 0$ – i.e., the underwriter becomes more truthful about high-risk buyers, but less truthful about low-risk buyers, as the loss penalty increases.

The underwriter's dissimulation is a direct result of the buyers' risk aversion. Essentially, for a fixed value of κ^*_2, the underwriter desires to maximize total written premiums, which is achieved by making both high- and low-risk buyers as uncertain as possible about the mix of high- and low-risk business in each of the two pools, **UH**

FIGURE 6
UNDERWRITER REPORTING ACCURACY VS. κ^*_2 WHEN
$y^*_{UH} = 0.10y^*_U$



and **UL**. This greater uncertainty causes the risk-averse buyers to make larger price bids to ensure themselves of more adequate coverage. Not surprisingly, as the penalty parameter κ^*_2 increases, the underwriter experiences a tradeoff between the commission income from greater total written premiums and the loss penalty associated with greater expected total losses. This encourages the underwriter to send more buyers (of both the high-risk and low-risk types) to the pool **UH**, where premiums are higher so that less coverage is purchased. (Compare Figures 5a and 5b to Figures 5c and 5d, respectively.)

Unfortunately for the seller, not even the largest value of κ^*_2 permitted under equations (12'a) and (12'b) is sufficient to encourage the underwriter to segregate high- and low-risk buyers into relatively homogeneous pools. By comparing the independent-underwriter market of our numerical example to a direct-writing market of identical size (i.e., with $N_D = N_U$ and $y^*_D = y^*_U$), it can easily be see that, after taking into account the net costs of using an underwriter (i.e., the commission payment less the loss penalty), the seller would prefer a direct-writing scheme. Figure 7 provides this comparison by plotting the ratio of expected total profits,

$$\frac{E\left[\text{Profit}_U \middle| y^*_{UH}, \kappa^*_2\right]}{E\left[\text{Profit}_D\right]}$$

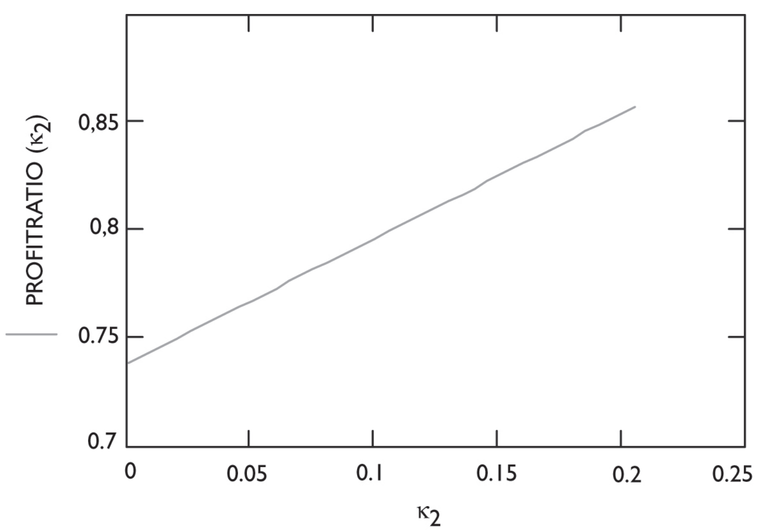
$$= \left\{ y^*_{UH} \left\{ \left(1-\kappa_1\right) P^*_{UH} - \left(1-\kappa^*_2\right) \left[\frac{\rho_H \gamma^*_H \pi_H x^*_{H,UH} + \left(1-\rho_H\right)\left(1-\gamma^*_L\right) \pi_L x^*_{L,UH}}{\rho_H \gamma^*_H x^*_{H,UH} + \left(1-\rho_H\right)\left(1-\gamma^*_L\right) x^*_{L,UH}} \right] \right\} \right.$$

$$\left. + \left(y^*_U - y^*_{UH}\right) \left\{ \left(1-\kappa_1\right) P^*_{UL} - \left(1-\kappa^*_2\right) \left[\frac{\rho_H \left(1-\gamma^*_H\right) \pi_H x^*_{H,UL} + \left(1-\rho_H\right) \gamma^*_L \pi_L x^*_{L,UL}}{\rho_H \left(1-\gamma^*_H\right) x^*_{H,UL} + \left(1-\rho_H\right) \gamma^*_L x^*_{L,UL}} \right] \right\} \right\}$$

$$\Bigg/ \left\{ y^*_D \left[P^*_D - \frac{\rho_H \pi_H x^*_{H,D} + \left(1-\rho_H\right) \pi_L x^*_{L,D}}{\rho_H x^*_{H,D} + \left(1-\rho_H\right) x^*_{L,D}} \right] \right\}$$

against κ^*_2 for various choices of y^*_{UH}, and reveals that these values are consistently less than 1.0, even in the limit as $y^*_{UH} \rightarrow 0$.

FIGURE 7
RATIO OF EXPECTED TOTAL PROFITS VS. κ^*_2 WHEN $y^*_{UH} = 0.10 y^*_U$



This observation suggests that, for the case of a risk-neutral seller, the answer to the question posed in the title of the present article is clearly “No”. In the next subsection, we study whether or not independent underwriters can benefit an MVO seller.

3.2 Market with an MVO Seller

We now consider the possibility that an independent underwriter may offer value to a seller concerned about the stability, as well as the expected magnitude, of its financial results. To this end, we posit a seller employing mean-variance optimization, and consider the ratio of total profit variances,

$$
\begin{aligned}
&\frac{Var\left[\text{Profit}_U \middle| y^*_{UH}, \kappa^*_2\right]}{Var\left[\text{Profit}_D\right]} \\
&= \left\{ \left(y^*_{UH}\right)^2 \left(1-\kappa^*_2\right)^2 \left\{ \frac{\rho_H \gamma^*_H \pi_H \left(1-\pi_H\right)\left(x^*_{H,UH}\right)^2 + \left(1-\rho_H\right)\left(1-\gamma^*_L\right)\pi_L\left(1-\pi_L\right)\left(x^*_{L,UH}\right)^2}{N_A\left[\rho_H \gamma^*_H x^*_{H,UH} + \left(1-\rho_H\right)\left(1-\gamma^*_L\right)x^*_{L,UH}\right]^2} \right\} \right. \\
&\left. + \left(y^*_U - y^*_{UH}\right)^2 \left(1-\kappa^*_2\right)^2 \left\{ \frac{\rho_H\left(1-\gamma^*_H\right)\pi_H\left(1-\pi_H\right)\left(x^*_{H,UL}\right)^2 + \left(1-\rho_H\right)\gamma^*_L \pi_L \left(1-\pi_L\right)\left(x^*_{L,UL}\right)^2}{N_U\left[\rho_H\left(1-\gamma^*_H\right)x^*_{H,UL} + \left(1-\rho_H\right)\gamma^*_L x^*_{L,UL}\right]^2} \right\} \right\} \\
&\Bigg/ \left\{ \left(y^*_D\right)^2 \left\{ \frac{\rho_H \pi_H \left(1-\pi_H\right)\left(x^*_{H,D}\right)^2 + \left(1-\rho_H\right)\pi_L\left(1-\pi_L\right)\left(x^*_{L,D}\right)^2}{N_D\left[\rho_H x^*_{H,D} + \left(1-\rho_H\right)x^*_{L,D}\right]^2} \right\} \right\},
\end{aligned}
$$

in conjunction with the previously considered $\frac{E\left[\text{Profit}_U \middle| y^*_{UH}, \kappa^*_2\right]}{E\left[\text{Profit}_D\right]}$.

FIGURE 8
RATIO OF TOTAL PROFIT VARIANCES VS. κ^*_2 WHEN $y^*_{UH} = 0.10y^*_U$

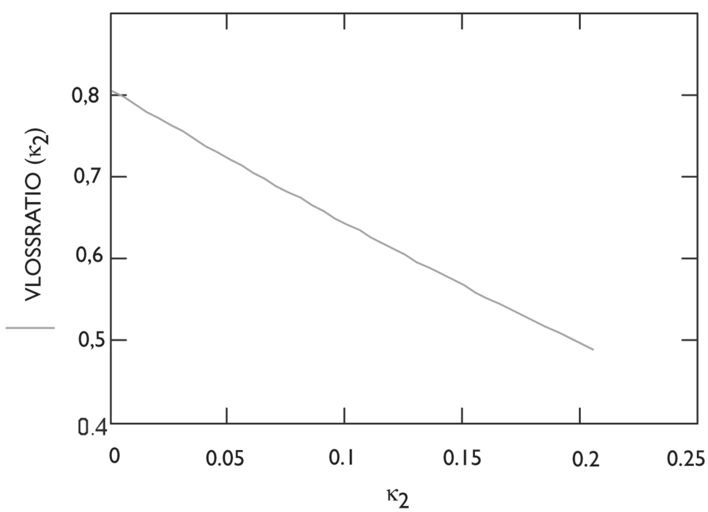
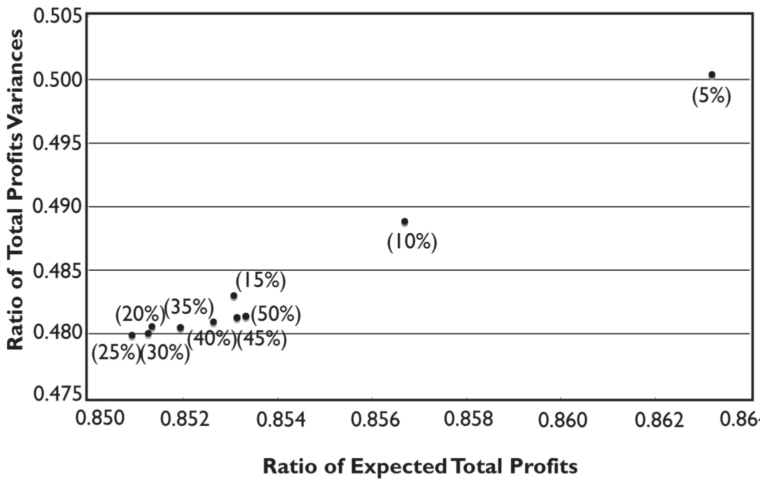


Figure 8 plots the ratio of total profit variances against κ^*_2 for various choices of y^*_{UH}, and reveals that these values, like those of the ratio of expected total profits in Figure 7, are consistently less than 1.0. This means that, by introducing an underwriter, an MVO seller is able to gain back in variance-reduction some of what it loses in expected profit.

To make the mean-variance tradeoff clear, we first note that for any fixed value of y^*_{UH}, $\frac{E\left[\text{Profit}_U \middle| y^*_{UH}, \kappa^*_2\right]}{E\left[\text{Profit}_D\right]}$ is increasing over κ^*_2, whereas $\frac{Var\left[\text{Profit}_U \middle| y^*_{UH}, \kappa^*_2\right]}{Var\left[\text{Profit}_D\right]}$ is decreasing. This means that, from an MVO perspective, the maximum feasible value of κ^*_2 under equations (12'a) and (12'b) – which we will denote by κ^{**}_2 – dominates all other choices of κ^*_2 for any value of y^*_{UH}. Therefore, the seller need consider only the locus of all possible pairs

$$\left[\frac{E\left[\text{Profit}_U \middle| y^*_{UH}, \kappa^{**}_2\right]}{E\left[\text{Profit}_D\right]}, \frac{Var\left[\text{Profit}_U \middle| y^*_{UH}, \kappa^{**}_2\right]}{Var\left[\text{Profit}_D\right]}\right],$$ as presented in Figure 9.

FIGURE9
MEAN-VARIANCE TRADEOFF FOR VARIOUS VALUES OF y^*_{UH}



In this figure, the plotted points are associated with values of y^*_{UH} ranging from $0.50y^*_U$ ("5%") to $0.50y^*_U$ ("50%"). Values of y^*_{UH} above $0.50y^*_U$ are not shown because they correspond to values of y^*_{UL} below $0.50y^*_U$, and therefore yield the same points as those plotted above (by symmetry). One can imagine the additional point [1.0, 1.0], well beyond the upper-right corner, denoting the case of direct writing. Cleary, direct writing dominates the independent-underwriter points in terms of expected total profit, but is dominated by all such points in terms of the total profit variance. Presumably, an MVO seller possesses a set of increasing and concave-downward indifference curves, and the seller's optimal choice of y^*_{UH} may be found by identifying the first point on the locus in Figure 9 to be crossed by an indifference curve as one moves from the lower right to the upper left.

4. CONCLUSIONS

In this article, we have used a game-theoretic model to study the interactive behavior of an insurer, a risk-neutral independent underwriter, and an arbitrary number of CARA insureds in a high-risk property-liability line. For the case of a risk-neutral insurer, we have found that: (1) the insurer does best by penalizing the underwriter for expected losses to the greatest extent possible; (2) the underwriter is almost completely uninformative (untruthful) in assigning insureds to high- vs. low-risk subsidiary pools; and (3) the insurer's payoff under direct writing dominates its payoff under the independent-underwriter system.

For the case of an MVO insurer, we have found that while (1) and (2) still hold, (3) is no longer necessarily true. Rather, because of the reduction in the variance of its total profit, the MVO insurer may prefer a particular allocation of capital between the subsidiary pools **UH** and **UL**. Our result is thus consistent with prior studies arguing for the coexistence of independent-agency and direct-writing distribution systems. Clearly, the market can reward an insurer for using an independent underwriter.

With respect to the capital-allocation problem, our result is consistent with the findings of Myers and Read (2001), who argued that capital allocations among an insurer's lines of business depend on both the relative degree of variability among the several lines, as well as the correlations among them. Our work provides a simplified model of an insurer with two lines to which it can allocate capital.

Through our analysis, we confirm Myers and Read's (2001) position that capital allocations are not arbitrary.

To gain additional insight into the dynamics of insurance markets with financial intermediaries, our model could be generalized in two principal ways. First, the number of insurers and underwriters could be relaxed to permit multiple players in both categories, and brokers (whose compensation is paid by the insureds rather than the insurers) could be substituted for underwriters. Second, the model could be extended from a one-period game to a multi-period model in which the insureds are free to move from insurer to insurer, or from one distribution system to another, over time. (For example, a low-risk insured assigned by an underwriter to the pool **UH** might decide to opt for direct writing.) Although these generalizations would complicate the mathematics greatly, numerical solutions would still be possible, and likely afford additional useful insights into the nature of insurance markets.

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Notes

1. This assumption, which is consistent with Rothschild and Stiglitz (1976), is reasonable for high-risk commercial property-liability lines. However, it would not be reasonable for lines such as personal automobile, in which the insurer possesses a large historical database revealing the strong connection between an insured's demographic variables and accident risk (see Powers, 2001).

2. We assume that high-risk buyers are less risk-averse than low-risk buyers because we view risk aversion as an endogenous characteristic of the insured that causes the insured to invest more resources into reducing risk.

3. The formal derivations are provided in Appendix 1.

4. To be consistent with recent aggregate U.S. property-liability industry statistics, we assume that the insurer will allocate an amount of capital that is approximately equal to 80 percent (= 1/1.25) of written premiums (see Powers, 2006). Assuming further that expected losses constitute roughly 80 percent of written premiums, we use an expected loss-to-capital ratio of approximately 1.0; i.e., $y^*_D \approx [N_D\rho_H\pi_H + N_D(1\text{-}\rho_H)\pi_L]V \approx 90{,}000$. Given that $y^*_D = W_S$ in the boundary solution implied by a positive first partial derivative in equation (7) (which is necessary for the seller to remain in the market), we then back out the assumption that W_S = 90,000.

5. The formal derivations are provided in Appendixes 2 and 3.

6. The selection is made for reasons analogous to those in Footnote 4.

Appendix 1: Derivations of Equations (5'a) and (5'b)

Substituting $\varphi_{BJ}(w) = (1-e^{-\beta jw})/\beta_j$ (for $j = L, H$ and $\beta_L > \beta_H \geq 0$) into equation (3) gives:

$$E\left[\varphi_{B_j}\left(B_{j,D}^{(i)}\right)\right]=\left(1-\pi_j\right)\frac{1-e^{-\beta_j\left(W_B+V-x_{j,D}^{(i)}\right)}}{\beta_j}+$$

$$\pi_j\frac{1-e^{-\beta_j\left\{W_B-x_{j,D}^{(i)}+y_D\left[x_{j,D}^{(i)}\Big/\left(\sum_{h=1}^{N_D\rho_H}x_{H,D}^{(h)}+\sum_{h=1}^{N_D(1-\rho_H)}x_{L,D}^{(h)}\right)\right]\right\}}}{\beta_j}.$$

We then take first partial derivatives as in equation (5):

$$\frac{\partial E\left[\varphi_{B_j}\left(B_{j,D}^{(i)}\right)\right]}{\partial x_{j,D}^{(i)}}=-\left(1-\pi_j\right)\exp\left(-\beta_j\left(W_B+V-x_{j,D}^{(i)}\right)\right)$$

$$-\pi_j\exp\left(-\beta_j\left\{W_B-x_{j,D}^{(i)}+y_D\left[x_{j,D}^{(i)}\Big/\left(\sum_{h=1}^{N_D\rho_H}x_{H,D}^{(h)}+\sum_{h=1}^{N_D(1-\rho_H)}x_{L,D}^{(h)}\right)\right]\right\}\right)$$

$$\times\left\{1-\frac{y_D\left[\left(\sum_{h=1}^{N_D\rho_H}x_{H,D}^{(h)}+\sum_{h=1}^{N_D(1-\rho_H)}x_{L,D}^{(h)}\right)-x_{j,D}^{(i)}\right]}{\left(\sum_{h=1}^{N_D\rho_H}x_{H,D}^{(h)}+\sum_{h=1}^{N_D(1-\rho_H)}x_{L,D}^{(h)}\right)^2}\right\}.$$

Setting the above derivative equal to 0 and letting $x^{(i)}_{j,D} = x^*_{j,D}$ for all i (i.e., assuming that all buyers within a given category are homogeneous) and $y_D = y^*_D$ yields:

$$\left(1-\pi_j\right)\exp\left(-\beta_jV\right)+\pi_j\exp\left(-\beta_j\frac{y_D^*x_{j,D}^*}{\left[N_D\rho_Hx_{H,D}^*+N_D\left(1-\rho_H\right)x_{L,D}^*\right]}\right)$$

$$\times\left\{1-\frac{y_D^*\left[\left(N_D\rho_H-1\right)x_{H,D}^*+N_D\left(1-\rho_H\right)x_{L,D}^*\right]}{\left[N_D\rho_Hx_{H,D}^*+N_D\left(1-\rho_H\right)x_{L,D}^*\right]^2}\right\}=0.$$

Appendix 2: Derivations of Equations (8'a), (8'b), (8'c), and (8'd)

The desired derivations are obtained from those in Appendix 1 simply by replacing the subscript D by the subscripts UH and UL, in turn.

Appendix 3: Derivations of Equations (12'a) and (12'b)

Note that:

$$
\begin{aligned}
&E\left[\varphi_U(U)\right]\\
&=\sum_{r_{H,UH}=0}^{N_U\rho_H\gamma_H}\ \sum_{\substack{R(r_{H,UH})\\ \in B_{H,UH}}}\ \sum_{r_{L,UH}=0}^{N_U(1-\rho_H)(1-\gamma_L)}\ \sum_{\substack{R(r_{L,UH})\\ \in B_{L,UH}}}\ \sum_{r_{H,UL}=0}^{N_U\rho_H(1-\gamma_H)}\ \sum_{\substack{R(r_{H,UL})\\ \in B_{H,UL}}}\ \sum_{r_{L,UL}=0}^{N_U(1-\rho_H)\gamma_L}\ \sum_{\substack{R(r_{L,UL})\\ \in B_{L,UL}}} \Big[\left(\pi_H\right)^{r_{H,UH}+r_{H,UL}}\left(1-\pi_H\right)^{N_U\rho_H-r_{H,UH}-r_{H,UL}}\\
&\times\left(\pi_L\right)^{r_{L,UH}+r_{L,UL}}\left(1-\pi_L\right)^{N_U(1-\rho_H)-r_{L,UH}-r_{L,UL}}\Big]\\
&\times\varphi_U\Bigg(W_U+\kappa_1\left[y_{UH}P_{UH}+\left(y_U-y_{UH}\right)P_{UL}\right]\\
&-\kappa_2\left[\frac{y_{UH}\left(\sum\limits_{h\in R(r_{H,UH})}x_{H,UH}^{(h)}+\sum\limits_{h\in R(r_{L,UH})}x_{L,UH}^{(h)}\right)}{\left(\sum\limits_{h=1}^{N_U\rho_H\gamma_H}x_{H,UH}^{(h)}+\sum\limits_{h=1}^{N_U(1-\rho_H)(1-\gamma_L)}x_{L,UH}^{(h)}\right)}+\frac{\left(y_U-y_{UH}\right)\left(\sum\limits_{h\in R(r_{H,UL})}x_{H,UL}^{(h)}+\sum\limits_{h\in R(r_{L,UL})}x_{L,UL}^{(h)}\right)}{\left(\sum\limits_{h=1}^{N_U\rho_H(1-\gamma_H)}x_{H,UL}^{(h)}+\sum\limits_{h=1}^{N_U(1-\rho_H)\gamma_L}x_{L,UL}^{(h)}\right)}\right]\Bigg)\Bigg].
\end{aligned}
$$

To solve for equilibrium, we substitute $\varphi_U(w) = \upsilon w$ (for some $\upsilon > 0$) and then "differentiate" the underwriter's expected utility with respect to γ_H and γ_L, in turn. We write "differentiate" in quotation marks because $E[\varphi_U(U)]$ is formed by multiple summations (rather than multiple integrals) that depend on the continuous parameters γ_H and γ_L; consequently, the optimization process must involve both first finite differences as well as first partial derivatives.

To "differentiate" with respect to γ_H, first note that γ_H appears twice in the above summations – in the expressions $\sum\limits_{r_{H,UL}=0}^{N_U\rho_H(1-\gamma_H)}$ and $\sum\limits_{r_{H,UH}=0}^{N_U\rho_H\gamma_H}$, respectively. Thus, the "first derivative" of $E[\varphi_U(U)]$ will be given by the sum of two parts, one of which involves taking the finite first difference of the first summation, while keeping the second summation unchanged, whereas the other involves taking the finite first difference of the second summation, while keeping the first summation unchanged. In the former case, we replace $r_{H,UL}$ by $N_U\rho_H(1-\gamma_H)$ in the expression and multiply by $\dfrac{\partial\left(N_U\rho_H\left(1-\gamma_H\right)\right)}{\partial\gamma_H}=-N_U\rho_H$.

In the latter case, we replace $r_{H,UH}$ by $N_U \rho_H \gamma_H$ in the expression and multiply by $\frac{\partial \left(N_U \rho_H \gamma_H\right)}{\partial \gamma_H} = N_U \rho_H$. Denoting the indicated "differentiation" operator by $\frac{\delta}{\delta \gamma_H}$, we find:

$$
\begin{aligned}
\frac{\delta E\left[\varphi_U(U)\right]}{\delta \gamma_H} = & \sum_{r_{H,UH}=0}^{N_U \gamma_H \rho_H} \sum_{R(r_{H,UH}) \in B_{H,UH}} \sum_{r_{L,UH}=0}^{N_U (1-\gamma_L)(1-\rho_H)} \sum_{R(r_{L,UH}) \in B_{L,UH}} \sum_{R(r_{H,UL}) \in B_{H,UL}} \sum_{r_{L,UL}=0}^{N_U \gamma_L (1-\rho_H)} \sum_{R(r_{L,UL}) \in B_{L,UL}} \Big[\pi_H^{r_{H,UH} + N_U (1-\gamma_H) \rho_H} \\
& \times (1-\pi_H)^{N_U \rho_H - r_{H,UH} - N_U (1-\gamma_H)\rho_H} \pi_L^{r_{L,UH} + r_{L,UL}} (1-\pi_L)^{N_U (1-\rho_H) - r_{L,UH} - r_{L,UL}} \Big] \\
& \times \upsilon \left\{ \kappa_1 \left(y_{UH} P_{UH} + y_{UL} P_{UL} \right) - \kappa_2 \left[\frac{y_{UH} \left(\sum_{h \in R(r_{H,UH})} x_{H,UH}^{(h)} + \sum_{h \in R(r_{L,UH})} x_{L,UH}^{(h)} \right)}{\left(\sum_{i \in B_{H,UH}} x_{H,UH}^{(i)} + \sum_{i \in B_{L,UH}} x_{L,UH}^{(i)} \right)} + \frac{(y_U - y_{UH}) \left(\sum_{h \in R(r_{H,UL})} x_{H,UL}^{(h)} + \sum_{h \in R(r_{L,UL})} x_{L,UL}^{(h)} \right)}{\left(\sum_{i \in B_{H,UL}} x_{H,UL}^{(i)} + \sum_{i \in B_{L,UL}} x_{L,UL}^{(i)} \right)} \right] \right\} \\
& \times (-N_U \rho_H) \\
& + \sum_{R(r_{H,UH}) \in B_{H,UH}} \sum_{r_{L,UH}=0}^{N_U (1-\gamma_L)(1-\rho_H)} \sum_{R(r_{L,UH}) \in B_{L,UH}} \sum_{r_{H,UL}=0}^{N_U (1-\gamma_H) \rho_H} \sum_{R(r_{H,UL}) \in B_{H,UL}} \sum_{r_{L,UL}=0}^{N_U \gamma_L (1-\rho_H)} \sum_{R(r_{L,UL}) \in B_{L,UL}} \Big[\pi_H^{N_U \gamma_H \rho_H + r_{H,UL}} \\
& \times (1-\pi_H)^{N_U \rho_H - N_U \gamma_H \rho_H - r_{H,UL}} \pi_L^{r_{L,UH} + r_{L,UL}} (1-\pi_L)^{N_U (1-\rho_H) - r_{L,UH} - r_{L,UL}} \Big] \\
& \times \upsilon \left\{ \kappa_1 \left(y_{UH} P_{UH} + y_{UL} P_{UL} \right) - \kappa_2 \left[\frac{y_{UH} \left(\sum_{h \in R(r_{H,UH})} x_{H,UH}^{(h)} + \sum_{h \in R(r_{L,UH})} x_{L,UH}^{(h)} \right)}{\left(\sum_{i \in B_{H,UH}} x_{H,UH}^{(i)} + \sum_{i \in B_{L,UH}} x_{L,UH}^{(i)} \right)} + \frac{(y_U - y_{UH}) \left(\sum_{h \in R(r_{H,UL})} x_{H,UL}^{(h)} + \sum_{h \in R(r_{L,UL})} x_{L,UL}^{(h)} \right)}{\left(\sum_{i \in B_{H,UL}} x_{H,UL}^{(i)} + \sum_{i \in B_{L,UL}} x_{L,UL}^{(i)} \right)} \right] \right\} \\
& \times (N_U \rho_H).
\end{aligned}
$$

Now set the above "derivative" equal to 0 and let: $x^{(i)}{}_{j,k} = x^*{}_{j,k}$ for all i (i.e., assume that all buyers in each category are homogeneous); $y_U = y^*{}_U$ and $y_{UL} = y^*{}_{UL}$; and the parameters κ_2, γ_H and γ_L be given by their equilibrium values (κ_2, γ_H, and γ_L, respectively). This yields the following equation:

$$
\begin{aligned}
& \sum_{r_{H,UH}=0}^{N_U \gamma_H \rho_H} \sum_{R(r_{H,UH}) \in B_{H,UH}} \sum_{r_{L,UH}=0}^{N_U (1-\gamma_L^*)(1-\rho_H)} \sum_{R(r_{L,UH}) \in B_{L,UH}} \sum_{R(r_{H,UL}) \in B_{H,UL}} \sum_{r_{L,UL}=0}^{N_U \gamma_L^* (1-\rho_H)} \sum_{R(r_{L,UL}) \in B_{L,UL}} \Big[\pi_H^{r_{H,UH} + N_U (1-\gamma_H^*) \rho_H} \\
& \times (1-\pi_H)^{N_U \gamma_H^* \rho_H - r_{H,UH}} \pi_L^{r_{L,UH} + r_{L,UL}} (1-\pi_L)^{N_U (1-\rho_H) - r_{L,UH} - r_{L,UL}} \Big] \\
& \times \upsilon \Big\{ \kappa_1 \left[\left(N_{H,UH} x_{H,UH}^* + N_{L,UH} x_{L,UH}^* \right) + \left(N_{H,UL} x_{H,UL}^* + N_{L,UL} x_{L,UL}^* \right) \right] \\
& - \kappa_2^* \left[\frac{y_{UH}^* \left(r_{H,UH} x_{H,UH}^* + r_{L,UH} x_{L,UH}^* \right)}{\left(N_{H,UH} x_{H,UH}^* + N_{L,UH} x_{L,UH}^* \right)} + \frac{\left(y_U^* - y_{UH}^* \right) \left(r_{H,UL} x_{H,UL}^* + r_{L,UL} x_{L,UL}^* \right)}{\left(N_{H,UL} x_{H,UL}^* + N_{L,UL} x_{L,UL}^* \right)} \right] \Big\} \\
& \times (-N_U \rho_H)
\end{aligned}
$$

$$+\sum_{R(r_{H,UH})\in B_{H,UH}}\sum_{r_{L,UH}=0}^{N_U(1-\gamma_L^*)(1-\rho_H)}\sum_{R(r_{L,UH})\in B_{L,UH}}\sum_{r_{H,UL}=0}^{N_U(1-\gamma_H^*)\rho_H}\sum_{R(r_{H,UL})\in B_{H,UL}}\sum_{r_{L,UL}=0}^{N_U\gamma_L^*(1-\rho_H)}\sum_{R(r_{L,UL})\in B_{L,UL}}\Big[\pi_H^{N_U\gamma_H^*\rho_H+r_{H,UL}}$$

$$\times(1-\pi_H)^{N_U\rho_H(1-\gamma_H^*)-r_{H,UL}}\,\pi_L^{r_{L,UH}+r_{L,UL}}\,(1-\pi_L)^{N_U(1-\rho_H)-r_{L,UH}-r_{L,UL}}\Big]$$

$$\times\upsilon\Big\{\kappa_1\Big[\big(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\big)+\big(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\big)\Big]$$

$$-\kappa_2^*\left[\frac{y_{UH}^*\big(r_{H,UH}x_{H,UH}^*+r_{L,UH}x_{L,UH}^*\big)}{\big(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\big)}+\frac{\big(y_U^*-y_{UH}^*\big)\big(r_{H,UL}x_{H,UL}^*+r_{L,UL}x_{L,UL}^*\big)}{\big(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\big)}\right]\Big\}$$

$$\times(N_U\rho_H)=0.$$

After canceling out the factor from both sides, we have:

$$\sum_{r_{H,UH}=0}^{N_U\gamma_H^*\rho_H}\sum_{R(r_{H,UH})\in B_{H,UH}}\sum_{r_{L,UH}=0}^{N_U(1-\gamma_L^*)(1-\rho_H)}\sum_{R(r_{L,UH})\in B_{L,UH}}\sum_{R(r_{H,UL})\in B_{H,UL}}\sum_{r_{L,UL}=0}^{N_U\gamma_L^*(1-\rho_H)}\sum_{R(r_{L,UL})\in B_{L,UL}}\Big[\pi_H^{r_{H,UH}+N_U(1-\gamma_H^*)\rho_H}$$

$$\times(1-\pi_H)^{N_U\gamma_H^*\rho_H-r_{H,UH}}\,\pi_L^{r_{L,UH}+r_{L,UL}}\,(1-\pi_L)^{N_U(1-\rho_H)-r_{L,UH}-r_{L,UL}}\Big]$$

$$\times\Big\{\kappa_1\Big[\big(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\big)+\big(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\big)\Big]$$

$$-\kappa_2^*\left[\frac{y_{UH}^*\big(r_{H,UH}x_{H,UH}^*+r_{L,UH}x_{L,UH}^*\big)}{\big(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\big)}+\frac{\big(y_U^*-y_{UH}^*\big)\big(r_{H,UL}x_{H,UL}^*+r_{L,UL}x_{L,UL}^*\big)}{\big(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\big)}\right]\Big\}$$

$$=\sum_{R(r_{H,UH})\in B_{H,UH}}\sum_{r_{L,UH}=0}^{N_U(1-\gamma_L^*)(1-\rho_H)}\sum_{R(r_{L,UH})\in B_{L,UH}}\sum_{r_{H,UL}=0}^{N_U(1-\gamma_H^*)\rho_H}\sum_{R(r_{H,UL})\in B_{H,UL}}\sum_{r_{L,UL}=0}^{N_U\gamma_L^*(1-\rho_H)}\sum_{R(r_{L,UL})\in B_{L,UL}}\Big[\pi_H^{N_U\gamma_H^*\rho_H+r_{H,UL}}$$

$$\times(1-\pi_H)^{N_U\rho_H(1-\gamma_H^*)-r_{H,UL}}\,\pi_L^{r_{L,UH}+r_{L,UL}}\,(1-\pi_L)^{N_U(1-\rho_H)-r_{L,UH}-r_{L,UL}}\Big]$$

$$\times\Big\{\kappa_1\Big[\big(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\big)+\big(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\big)\Big]$$

$$-\kappa_2^*\left[\frac{y_{UH}^*\big(r_{H,UH}x_{H,UH}^*+r_{L,UH}x_{L,UH}^*\big)}{\big(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\big)}+\frac{\big(y_U^*-y_{UH}^*\big)\big(r_{H,UL}x_{H,UL}^*+r_{L,UL}x_{L,UL}^*\big)}{\big(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\big)}\right]\Big\}.$$

Recalling that $r_{j,k}$ denotes the total number of loss claims from buyers in the set $\mathbf{B}_{j,k}$, and $R(r_{j,k})$ denotes the set of all distinct subsets of $\mathbf{B}_{j,k}$ of size $r_{j,k}$, we observe that the number of incorrectly identified high-risk buyers, $r_{H,UL}$, is given by $N_U(1\text{-}\gamma^*_H)\rho_H$, and the number of correctly identified high-risk buyers, $r_{H,UH}$, is given by $N_U\gamma^*_H\rho_H$. Since $r_{H,UL}$ is not an index of summation on the left-hand side of the equation, and $r_{H,UH}$ is not an index on the right-hand side, we can replace $r_{H,UL}$ with $N_U(1\text{-}\gamma^*_H)\rho_H$ and $r_{H,UH}$ with $N_U\gamma^*_H\rho_H$, yielding:

$$\sum_{r_{H,UH}=0}^{N_U\gamma_H^*\rho_H}\sum_{R(r_{H,UH})\in B_{H,UH}}\sum_{r_{L,UH}=0}^{N_U(1-\gamma_L^*)(1-\rho_H)}\sum_{R(r_{L,UH})\in B_{L,UH}}\sum_{R(r_{H,UL})\in B_{H,UL}}\sum_{r_{L,UL}=0}^{N_U\gamma_L^*(1-\rho_H)}\sum_{R(r_{L,UL})\in B_{L,UL}}\left[\pi_H^{r_{H,UH}+N_U(1-\gamma_H^*)\rho_H}\right.$$
$$\left.\times(1-\pi_H)^{N_U\gamma_H^*\rho_H-r_{H,UH}}\pi_L^{r_{L,UH}+r_{L,UL}}(1-\pi_L)^{N_U(1-\rho_H)-r_{L,UH}-r_{L,UL}}\right]$$

$$\times\left\{\kappa_1\left[\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)+\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)\right]\right.$$
$$\left.-\kappa_2^*\left[\frac{y_{UH}^*\left(r_{H,UH}x_{H,UH}^*+r_{L,UH}x_{L,UH}^*\right)}{\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)}+\frac{\left(y_U^*-y_{UH}^*\right)\left(r_{H,UL}x_{H,UL}^*+r_{L,UL}x_{L,UL}^*\right)}{\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)}\right]\right\}$$

$$=\sum_{R(r_{H,UH})\in B_{H,UH}}\sum_{r_{L,UH}=0}^{N_U(1-\gamma_L^*)(1-\rho_H)}\sum_{R(r_{L,UH})\in B_{L,UH}}\sum_{r_{H,UL}=0}^{N_U(1-\gamma_H^*)\rho_H}\sum_{R(r_{H,UL})\in B_{H,UL}}\sum_{r_{L,UL}=0}^{N_U\gamma_L^*(1-\rho_H)}\sum_{R(r_{L,UL})\in B_{L,UL}}\left[\pi_H^{N_U\gamma_H^*\rho_H+r_{H,UL}}\right.$$
$$\left.\times(1-\pi_H)^{N_U\rho_H(1-\gamma_H^*)-r_{H,UL}}\pi_L^{r_{L,UH}+r_{L,UL}}(1-\pi_L)^{N_U(1-\rho_H)-r_{L,UH}-r_{L,UL}}\right]$$

$$\times\left\{\kappa_1\left[\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)+\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)\right]\right.$$
$$\left.-\kappa_2^*\left[\frac{y_{UH}^*\left(r_{H,UH}x_{H,UH}^*+r_{L,UH}x_{L,UH}^*\right)}{\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)}+\frac{\left(y_U^*-y_{UH}^*\right)\left(r_{H,UL}x_{H,UL}^*+r_{L,UL}x_{L,UL}^*\right)}{\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)}\right]\right\}.$$

Using the notation of the binomial distribution, we simplify the above equation as follows:

$$\sum_{R(r_{H,UL})\in B_{H,UL}}\sum_{r_{H,UH}=0}^{N_U\gamma_H^*\rho_H}\sum_{R(r_{H,UH})\in B_{H,UH}}\sum_{r_{L,UH}=0}^{N_U(1-\gamma_L^*)(1-\rho_H)}\sum_{R(r_{L,UH})\in B_{L,UH}}\sum_{r_{L,UL}=0}^{N_U\gamma_L^*(1-\rho_H)}\left[\binom{N_U\gamma_L^*(1-\rho_H)}{r_{L,UL}}\pi_L^{r_{L,UL}}(1-\pi_L)^{N_U\gamma_L^*(1-\rho_H)-r_{L,UL}}\right]$$
$$\times\left[\pi_H^{r_{H,UH}+N_U(1-\gamma_H^*)\rho_H}(1-\pi_H)^{N_U\gamma_H^*\rho_H-r_{H,UH}}\pi_L^{r_{L,UH}}(1-\pi_L)^{N_U(1-\gamma_L^*)(1-\rho_H)-r_{L,UH}}\right]$$

$$\times\left\{\kappa_1\left[\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)+\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)\right]\right.$$
$$\left.-\kappa_2^*\left[\frac{y_{UH}^*\left(r_{H,UH}x_{H,UH}^*+r_{L,UH}x_{L,UH}^*\right)}{\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)}+\frac{\left(y_U^*-y_{UH}^*\right)\left[N_U\left(1-\gamma_H^*\right)\rho_Hx_{H,UL}^*+r_{L,UL}x_{L,UL}^*\right]}{\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)}\right]\right\}$$

$$=\sum_{R(r_{H,UH})\in B_{H,UH}}\sum_{r_{L,UH}=0}^{N_U(1-\gamma_L^*)(1-\rho_H)}\sum_{R(r_{L,UH})\in B_{L,UH}}\sum_{r_{H,UL}=0}^{N_U(1-\gamma_H^*)\rho_H}\sum_{R(r_{H,UL})\in B_{H,UL}}\sum_{r_{L,UL}=0}^{N_U\gamma_L^*(1-\rho_H)}\left[\binom{N_U\gamma_L^*(1-\rho_H)}{r_{L,UL}}\pi_L^{r_{L,UL}}(1-\pi_L)^{N_U\gamma_L^*(1-\rho_H)-r_{L,UL}}\right]$$
$$\times\left[\pi_H^{N_U\gamma_H^*\rho_H+r_{H,UL}}(1-\pi_H)^{N_U\rho_H(1-\gamma_H^*)-r_{H,UL}}\pi_L^{r_{L,UH}}(1-\pi_L)^{N_U(1-\gamma_L^*)(1-\rho_H)-r_{L,UH}}\right]$$

$$\times\left\{\kappa_1\left[\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)+\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)\right]\right.$$
$$\left.-\kappa_2^*\left[\frac{y_{UH}^*\left(N_U\gamma_H^*\rho_Hx_{H,UH}^*+r_{L,UH}x_{L,UH}^*\right)}{\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)}+\frac{\left(y_U^*-y_{UH}^*\right)\left(r_{H,UL}x_{H,UL}^*+r_{L,UL}x_{L,UL}^*\right)}{\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)}\right]\right\}.$$

Then, since

$$\sum_{r_{L,UL}=0}^{N_U\gamma_L^*(1-\rho_H)}\left[\binom{N_U\gamma_L^*(1-\rho_H)}{r_{L,UL}}\pi_L^{r_{L,UL}}(1-\pi_L)^{N_U\gamma_L^*(1-\rho_H)-r_{L,UL}}\right]=1$$

and

$$\sum_{r_{L,UL}=0}^{N_U\gamma_L^*(1-\rho_H)}\left[\binom{N_U\gamma_L^*(1-\rho_H)}{r_{L,UL}}\pi_L^{r_{L,UL}}(1-\pi_L)^{N_U\gamma_L^*(1-\rho_H)-r_{L,UL}}\right]r_{L,UL}=N_U\gamma_L^*(1-\rho_H)\pi_L(1-\pi_L),$$

we can simplify further:

$$\sum_{R(r_{H,UL})\in B_{H,UL}}\sum_{r_{H,UH}=0}^{N_U\gamma_H^*\rho_H}\sum_{R(r_{H,UH})\in B_{H,UH}}\sum_{r_{L,UH}=0}^{N_U(1-\gamma_L^*)(1-\rho_H)}\sum_{R(r_{L,UH})\in B_{L,UH}}\left[\pi_H^{r_{H,UH}+N_U(1-\gamma_H^*)\rho_H}(1-\pi_H)^{N_U\gamma_H^*\rho_H-r_{H,UH}}\right.$$
$$\left.\times\pi_L^{r_{L,UH}}(1-\pi_L)^{N_U(1-\gamma_L^*)(1-\rho_H)-r_{L,UH}}\right]$$

$$\times\left\{\kappa_1\left[\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)+\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)\right]\right.$$
$$\left.-\kappa_2^*\left[\frac{y_{UH}^*\left(r_{H,UH}x_{H,UH}^*+r_{L,UH}x_{L,UH}^*\right)}{\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)}+\frac{\left(y_U^*-y_{UH}^*\right)\left[N_U(1-\gamma_H^*)\rho_H x_{H,UL}^*+N_U\gamma_L^*(1-\rho_H)\pi_L(1-\pi_L)\right]x_{L,UL}^*}{\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)}\right]\right\}$$

$$=\sum_{R(r_{H,UH})\in B_{H,UH}}\sum_{r_{L,UH}=0}^{N_U(1-\gamma_L^*)(1-\rho_H)}\sum_{R(r_{L,UH})\in B_{L,UH}}\sum_{r_{H,UL}=0}^{N_U(1-\gamma_H^*)\rho_H}\sum_{R(r_{H,UL})\in B_{H,UL}}\left[\pi_H^{N_U\gamma_H^*\rho_H+r_{H,UL}}(1-\pi_H)^{N_U\rho_H(1-\gamma_H^*)-r_{H,UL}}\right.$$
$$\left.\times\pi_L^{r_{L,UH}}(1-\pi_L)^{N_U(1-\gamma_L^*)(1-\rho_H)-r_{L,UH}}\right]$$

$$\times\left\{\kappa_1\left[\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)+\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)\right]\right.$$
$$\left.-\kappa_2^*\left\{\frac{y_{UH}^*\left(N_U\gamma_H^*\rho_H x_{H,UH}^*+r_{L,UH}x_{L,UH}^*\right)}{\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)}+\frac{\left(y_U^*-y_{UH}^*\right)\left[r_{H,UL}x_{H,UL}^*+N_U\gamma_L^*(1-\rho_H)\pi_L(1-\pi_L)x_{L,UL}^*\right]}{\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)}\right\}\right\}.$$

We denote the left-hand side of the above equation by T_1, and the right-hand side by . For the left-hand side, we can use the notation of the binomial distribution to show:

$$T_1=\sum_{R(r_{H,UL})\in B_{H,UL}}\sum_{r_{H,UH}=0}^{N_U\gamma_H^*\rho_H}\sum_{R(r_{H,UH})\in B_{H,UH}}\sum_{r_{L,UH}=0}^{N_U(1-\gamma_L^*)(1-\rho_H)}\left[\binom{N_U(1-\gamma_L^*)(1-\rho_H)}{r_{L,UH}}\pi_L^{r_{L,UH}}(1-\pi_L)^{N_U(1-\gamma_L^*)(1-\rho_H)-r_{L,UH}}\right]$$
$$\times\left[\pi_H^{r_{H,UH}+N_U(1-\gamma_H^*)\rho_H}(1-\pi_H)^{N_U\gamma_H^*\rho_H-r_{H,UH}}\right]$$

$$\times\left\{\kappa_1\left[\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)+\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)\right]\right.$$
$$\left.-\kappa_2^*\left[\frac{y_{UH}^*\left(r_{H,UH}x_{H,UH}^*+r_{L,UH}x_{L,UH}^*\right)}{\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)}+\frac{\left(y_U^*-y_{UH}^*\right)\left[N_U(1-\gamma_H^*)\rho_H x_{H,UL}^*+N_U\gamma_L^*(1-\rho_H)\pi_L(1-\pi_L)\right]x_{L,UL}^*}{\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)}\right]\right\}.$$

Then, since

$$\sum_{r_{L,UH}=0}^{N_U\left(1-\gamma_L^*\right)\left(1-\rho_H\right)}\left[\binom{N_U\left(1-\gamma_L^*\right)\left(1-\rho_H\right)}{r_{L,UH}}\pi_L^{r_{L,UH}}\left(1-\pi_L\right)^{N_U\left(1-\gamma_L^*\right)\left(1-\rho_H\right)-r_{L,UH}}\right]=1$$

and

$$\sum_{r_{L,UH}=0}^{N_U\left(1-\gamma_L^*\right)\left(1-\rho_H\right)}\left[\binom{N_U\left(1-\gamma_L^*\right)\left(1-\rho_H\right)}{r_{L,UH}}\pi_L^{r_{L,UH}}\left(1-\pi_L\right)^{N_U\left(1-\gamma_L^*\right)\left(1-\rho_H\right)-r_{L,UH}}\right]r_{L,UH}=N_U\left(1-\gamma_L^*\right)\left(1-\rho_H\right)\pi_L\left(1-\pi_L\right),$$

we obtain:

$$\begin{aligned}T_1=&\sum_{R\left(r_{H,UL}\right)\in B_{H,UL}}\ \sum_{r_{H,UH}=0}^{N_U\gamma_H^*\rho_H}\ \sum_{R\left(r_{H,UH}\right)\in B_{H,UH}}\left[\pi_H^{r_{H,UH}+N_U\left(1-\gamma_H^*\right)\rho_H}\left(1-\pi_H\right)^{N_U\gamma_H^*\rho_H-r_{H,UH}}\right]\\&\times\left\{\kappa_1\left[\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)+\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)\right]\right.\\&-\kappa_2^*\left\{\frac{y_{UH}^*\left[r_{H,UH}x_{H,UH}^*+N_U\left(1-\gamma_L^*\right)\left(1-\rho_H\right)\pi_L\left(1-\pi_L\right)x_{L,UH}^*\right]}{\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)}\right.\\&\left.\left.+\frac{\left(y_U^*-y_{UH}^*\right)\left[N_U\left(1-\gamma_H^*\right)\rho_Hx_{H,UL}^*+N_U\gamma_L^*\left(1-\rho_H\right)\pi_L\left(1-\pi_L\right)x_{L,UL}^*\right]}{\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)}\right\}\right\}.\end{aligned}$$

Simplifying further yields:

$$\begin{aligned}T_1=&\sum_{R\left(r_{H,UL}\right)\in B_{H,UL}}\left[\pi_H^{N_U\left(1-\gamma_H^*\right)\rho_H}\right]\sum_{r_{H,UH}=0}^{N_U\gamma_H^*\rho_H}\left[\binom{N_U\gamma_H^*\rho_H}{r_{H,UH}}\pi_H^{r_{H,UH}}\left(1-\pi_H\right)^{N_U\gamma_H^*\rho_H-r_{H,UH}}\right]\\&\times\left\{\kappa_1\left[\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)+\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)\right]\right.\\&-\kappa_2^*\left\{\frac{y_{UH}^*\left[r_{H,UH}x_{H,UH}^*+N_U\left(1-\gamma_L^*\right)\left(1-\rho_H\right)\pi_L\left(1-\pi_L\right)x_{L,UH}^*\right]}{\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)}\right.\\&\left.\left.+\frac{\left(y_U^*-y_{UH}^*\right)\left[N_U\left(1-\gamma_H^*\right)\rho_Hx_{H,UL}^*+N_U\gamma_L^*\left(1-\rho_H\right)\pi_L\left(1-\pi_L\right)x_{L,UL}^*\right]}{\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)}\right\}\right\};\end{aligned}$$

and finally:

$$\begin{aligned}T_1=&\sum_{R\left(r_{H,UL}\right)\in B_{H,UL}}\left[\pi_H^{N_U\left(1-\gamma_H^*\right)\rho_H}\right]\\&\times\left\{\kappa_1\left[\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)+\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)\right]\right.\end{aligned}$$

$$-\kappa_2^*\Bigg\{\frac{y_{UH}^*\left[N_U\gamma_H^*\rho_H\pi_H\left(1-\pi_H\right)x_{H,UH}^*+N_U\left(1-\gamma_L^*\right)\left(1-\rho_H\right)\pi_L\left(1-\pi_L\right)x_{L,UH}^*\right]}{\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)}$$

$$+\frac{\left(y_U^*-y_{UH}^*\right)\left[N_U\left(1-\gamma_H^*\right)\rho_H x_{H,UL}^*+N_U\gamma_L^*\left(1-\rho_H\right)\pi_L\left(1-\pi_L\right)x_{L,UL}^*\right]}{\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)}\Bigg\}\Bigg\}.$$

$$=N_U\left(1-\gamma_H^*\right)\rho_H\pi_H^{N_U\left(1-\gamma_H^*\right)\rho_H}$$

$$\times\Big\{\kappa_1\left[\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)+\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)\right]$$

$$-\kappa_2^*\Bigg\{\frac{y_{UH}^*\left[N_U\gamma_H^*\rho_H\pi_H\left(1-\pi_H\right)x_{H,UH}^*+N_U\left(1-\gamma_L^*\right)\left(1-\rho_H\right)\pi_L\left(1-\pi_L\right)x_{L,UH}^*\right]}{\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)}$$

$$+\frac{\left(y_U^*-y_{UH}^*\right)\left[N_U\left(1-\gamma_H^*\right)\rho_H x_{H,UL}^*+N_U\gamma_L^*\left(1-\rho_H\right)\pi_L\left(1-\pi_L\right)x_{L,UL}^*\right]}{\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)}\Bigg\}\Bigg\}.$$

Applying analogous arguments to the right-hand side, it is straightforward to show:

$$T_2=N_U\gamma_H^*\rho_H\pi_H^{N_U\gamma_H^*\rho_H}$$

$$\times\Big\{\kappa_1\left[\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)+\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)\right]$$

$$-\kappa_2^*\Bigg\{\frac{y_{UH}^*\left(N_U\gamma_H^*\rho_H x_{H,UH}^*+r_{L,UH}x_{L,UH}^*\right)}{\left(N_{H,UH}x_{H,UH}^*+N_{L,UH}x_{L,UH}^*\right)}$$

$$+\frac{\left(y_U^*-y_{UH}^*\right)\left[N_U\left(1-\gamma_H^*\right)\rho_H\pi_H\left(1-\pi_H\right)x_{H,UL}^*+N_U\gamma_L^*\left(1-\rho_H\right)\pi_L\left(1-\pi_L\right)x_{L,UL}^*\right]}{\left(N_{H,UL}x_{H,UL}^*+N_{L,UL}x_{L,UL}^*\right)}\Bigg\}\Bigg\}.$$

Then, setting the above expressions for T_1 and T_2 equal to each other, and substituting $N_{H,UH} = N_{UH}\rho_{H/UH} = N_U\gamma^*_H\rho_H$, $N_{L,UH} = N_{UH}\rho_{L/UH} = N_U(1-\gamma^*_L)(1-\rho_H)$, $N_{H,UL} = N_{UL}\rho_{H/UL} = N_U(1-\gamma^*_H)\rho_H$, and $N_{L,UL} = N_{UL}\rho_{L/UL} = N_U\gamma^*_L(1-\rho_H)$, yields equation (12'a).

Finally, Equation (12'b) may be derived in the same manner by "differentiating" $E[\varphi_U(U)]$ with respect to γ_L, rather than γ_H.