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Article abstract

The calculation of Chezy's resistance coefficient (CRC) is typically not provided *a priori* in a design problem, and its value is often selected subjectively from the literature in most open channels and conduits for the uniform flow. The evaluation of this coefficient is crucial to channel design and for computing its normal depth. The primary purpose of this research study is to revisit the mathematical formulation for the resistance coefficient. A general explicit relation of the resistance coefficient in turbulent flow is set with different geometric profiles of conduits and channels, mainly the horseshoe-shaped tunnel using the rough model method (RMM). CRC is firmly based on the internal walls' absolute roughness of the channel, the liquid kinematic viscosity, the longitudinal slope, the discharge and the filling rate. Additionally, a simplified method is proposed to determine CRC with a restricted number of variables such as the kinematic viscosity, the absolute roughness, the slope of the conduit, and the discharge. Based on studying the variation of CRC as a function of the filling rate, another explicit expression is provided to compute this coefficient efficiently when its maximum value is reached. To demonstrate how Chezy's resistance coefficient can be calculated in a horseshoe-shaped tunnel, some examples of calculations are proposed.

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CHEZY'S RESISTANCE COEFFICIENT IN A HORSESHOE-SHAPED TUNNEL

Coefficient de résistance de Chezy dans une conduite en forme de fer à cheval

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ABSTRACT

The calculation of Chezy's resistance coefficient (CRC) is typically not provided *a priori* in a design problem, and its value is often selected subjectively from the literature in most open channels and conduits for the uniform flow. The evaluation of this coefficient is crucial to channel design and for computing its normal depth. The primary purpose of this research study is to revisit the mathematical formulation for the resistance coefficient. A general explicit relation of the resistance coefficient in turbulent flow is set with different geometric profiles of conduits and channels, mainly the horseshoe-shaped tunnel using the rough model method (RMM). CRC is firmly based on the internal walls' absolute roughness of the channel, the liquid kinematic viscosity, the longitudinal slope, the discharge and the filling rate. Additionally, a simplified method is proposed to determine CRC with a restricted number of variables such as the kinematic viscosity, the absolute roughness, the slope of the conduit, and the discharge. Based on studying the variation of CRC as a function of the filling rate, another explicit expression is provided to compute this coefficient efficiently when its maximum value is reached. To demonstrate how Chezy's resistance coefficient can be calculated in a horseshoe-shaped tunnel, some examples of calculations are proposed.

Key words*: *Chezy's resistance coefficient, uniform flow, horseshoe-shaped tunnel, rough model method, simplified method.

RÉSUMÉ

Dans le calcul des écoulements uniformes en conduites et canaux à surface libre, le coefficient de résistance de Chézy (CRC) n'est pas *a priori* une donnée du problème et sa valeur est considérée de manière arbitraire, ce qui implique un calcul plutôt approximatif. Cet inconvénient majeur se retrouve dans tous les profils géométriques de conduites et de canaux. La connaissance de la valeur de ce coefficient est indispensable au dimensionnement d'un ouvrage, voire même pour le calcul de la profondeur normale. C'est dans ce contexte que s'inscrit l'objectif de notre recherche en orientant principalement nos travaux sur l'identification et l'établissement de la relation du coefficient de résistance à l'écoulement. En nous basant sur la

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méthode du modèle rugueux (MMR) destinée au calcul des conduites et canaux, nous pouvons établir la relation générale du coefficient de résistance d'une manière explicite dans le domaine de l'écoulement turbulent pour les différents profils géométriques, notamment pour la conduite en forme de fer à cheval. Il apparaît clairement que le coefficient de résistance de Chézy dépend fortement du taux de remplissage, du débit, de la pente longitudinale, de la rugosité absolue des parois internes de la conduite et de la viscosité cinématique du liquide. En outre, dans ce travail, une méthode simplifiée par rapport à celle citée précédemment est présentée afin de déterminer le coefficient de résistance de Chézy avec un nombre limité de données, à savoir, le débit, la pente de la conduite, la rugosité absolue et la viscosité cinématique. Enfin, après l'étude de la variation du coefficient de résistance de Chézy en fonction du taux de remplissage, une expression explicite est également donnée pour le calcul aisé de ce coefficient lorsqu'il atteint sa valeur maximale durant l'écoulement dans la conduite. Des exemples de calcul sont proposés pour montrer comment on peut calculer le coefficient de Chézy dans une conduite en forme de fer à cheval.

Mots-clés* : *coefficient de résistance de Chézy, écoulement uniforme, méthode du modèle rugueux, conduite en forme de fer à cheval, méthode simplifiée.

1. INTRODUCTION

Several researchers have been concerned with determining Chezy's resistance coefficient C formula since its first appearance in 1775 (CARLIER, 1972; CHOW, 1973; FRENCH, 1986) and expressions for this parameter have continuously been developed in artificial channels and conduits characterizing the open channel uniform flow. However, the findings of these studies were not actually persuading, particularly for artificial channels. To fill such gaps and to develop the bibliography as well, the present study aims at formulating an easy-to-use expression for determining Chezy's coefficient C. Founded on the rough model method (RMM) (ACHOUR and BEDJAOUI, 2006; ACHOUR and BEDJAOUI, 2012; ACHOUR, 2015a; ACHOUR, 2015b; LOUKAM *et al.*, 2018) established for channels and conduits, a general relation of resistance coefficient in its explicit form can be set, catering for the necessary hydraulic parameters such as the filling rate, the discharge, the longitudinal slope, the absolute roughness of the conduit internal walls and the liquid kinematic viscosity. In the horseshoe-shaped artificial tunnel (Figure 1), this relation is acceptable for all states of the turbulent flow. In the same context, another simplified method taken from the same theory of the RMM is proposed to calculate the Chezy's resistance coefficient C taking into consideration a restricted number of data as follows: the discharge Q, the slope i, the kinematic viscosity ν, and the absolute roughness ε. Also, to determine the maximum value of the resistance coefficient C within the same conduit, some calculation procedures are formulated. Examples of application with practical data to clarify the calculation phases of each method are provided.

2. LITERATURE REVIEW

Many efforts have been made to establish formulas expressing the coefficient of resistance of Chezy. The most frequently known will be briefly described in this section. One of the equations of the Chezy coefficient (C) is Prony's suggestion, and its formula is as follows (CARLIER, 1972):

$$\frac{1}{C^2}=\frac{0.000044}{V}+0.000309 \tag{1}$$

with V: flow velocity ($m{\cdot}s^{-1}$).

Tadini simplified Chezy's coefficient by giving a constant value equal to 50 (CARLIER, 1972). While GANGUILLET and KUTTER (1869) presented a different formula using more parameters such as the hydraulic radius R_h, the roughness coefficient n and the slope i, as displayed below:

$$C=\frac{23+\frac{0.00155}{i}+\frac{1}{n}}{1+\left(23+\frac{0.00155}{i}\right)\frac{n}{\sqrt{R_h}}} \tag{2}$$

This formula uses the international unit system and the values of the roughness coefficient are tabulated. The experience has revealed that for low slopes (less than 0.0001), the term $0.00155/i$ takes values too high, and the formula 2 becomes significantly less precise (CARLIER, 1972).

Contrary to Ganguillet's formula, Kutter provided another expression, very much used in sanitation tunnels, and more accessible to be applied, namely (CARLIER, 1972):

$$C=\frac{100\sqrt{R_h}}{b+\sqrt{R_h}} \tag{3}$$

where b is the roughness of the conduit internal wall. We can notice in these expressions that none of them account for the kinematic viscosity of the flowing liquid.

In a more straightforward form, MANNING (1895) suggested another formula where C can be determined by the hydraulic radius R_h and the roughness coefficient n, the latter being similar to that experienced in the Ganguillet-Kutter formula, or:

$$C = \frac{1}{n} R_h^{1/6} \tag{4}$$

BAZIN (1897) suggested an expression for the coefficient C depending on the hydraulic radius R_h and the pipe roughness coefficient γ, whose values are tabulated (CARLIER, 1972).

$$C = \frac{87}{1 + \frac{\gamma}{\sqrt{R_h}}} \tag{5}$$

In 1949, Thijsse expressed in an implicit formula the Chezy's coefficient including the Reynolds number Re defining the flow regime in the conduct, in addition to the absolute roughness ε and the hydraulic radius R_h (CARLIER, 1972).

$$C = -18 \log\left[\frac{\varepsilon}{12 R_h} + \frac{C}{3Re}\right] \tag{6}$$

In this formula, the implicit coefficient C is based on various hydraulic parameters such as the absolute roughness and the Reynolds number, where it depends on the kinematic viscosity and evidently on the hydraulic radius.

The formulas from 1 to 6 are expressed in the international unit system, where R_h (m), i (m·m^{-1}), ε (m) and γ, n, b are coefficients of roughness with values provided by tables as a function of the type of the material forming the channel or the conduit.

POWELL (1950) expressed the Chezy's coefficient based on the works of KEULEGAN (1938), with an implicit formula like that of Thijsse:

$$C = -42 \log\left[\frac{\varepsilon}{R_h} + \frac{C}{4Re}\right] \tag{7}$$

where R_h is the hydraulic radius in feet, Re is the Reynolds number, and ε is a practically measurable roughness.

The calculation of the coefficient C by Thijsse's relation 6 and that of Powell 7 requires an iterative process.

SWAMEE and RATHIE (2004) suggested a new general formula 8 for C, which is similar to the Colebrook formula for tapping pipelines to account for all the parameters of the flow:

$$C = -2.457 \sqrt{g} \ln\left[\frac{\varepsilon}{12 R_h} + \frac{0.221 \nu}{R_h \sqrt{g i R_h}}\right] \tag{8}$$

where g is the acceleration due to gravity (m·s^{-2}), ν the kinematic viscosity (m^2·s^{-1}), R_h (m), i (m·m^{-1}) and ε (m).

This formula is valid for every shape of conduits and in all turbulent flow domains, be it smooth, rough, or of transition. However, when the conduit linear dimension is not provided, it may be implicit.

Several other studies have been carried out by researchers to define the Chezy coefficient. However, their uses remain very restricted. They include STREETER (1936), PERRY *et al.* (1969), MARONE (1970), PYLE and NOVAK (1981), NAOT *et al.* (1996), EAD *et al.* (2000), GIUSTOLISI (2004), etc.

3. METHODS

3.1 Horseshoe-shaped tunnel

The horseshoe-shaped tunnel profile is generally used in free-surface flows for evacuating rainwater and the drainage of sewage from cities, in the transport of supply and irrigation water. It has been designed and built for several hydraulic projects in some countries (HU, 1973; LV *et al.*, 2001; MERKLEY, 2005).

The geometric shape of the horseshoe-shaped tunnel can offer essential hydraulic qualities. Indeed, it rests on a much larger surface than the circular or egg-shaped conduit. The vertical blanks of the lower half allow a robust resistance to support tunnel loads and those of essential embankments. The bottom of the tunnel is large enough to allow easy access for maintenance and cleaning.

3.1.1 Geometrical characteristics

The horseshoe-shaped tunnel profile is displayed in figure1. In terms of geometry, it is defined by elements that follow (Figure 1):

- Section (FA): an arc of the circle with center (C) and diameter $2D$;
- Section (FE): an arc of the circle of center (B) and diameter $2D$;
- Section (BA): an arc of the circle with center (E) and diameter $2D$;
- Section (ECB): the semicircle of center (O) and diameter D;
- $\alpha+\beta = \pi/4$, $\alpha = 0.42403104$ radian and $\gamma = \pi/4$;
- $Y = 0.088562171D \approx 0.089D$, $Ym = D$.

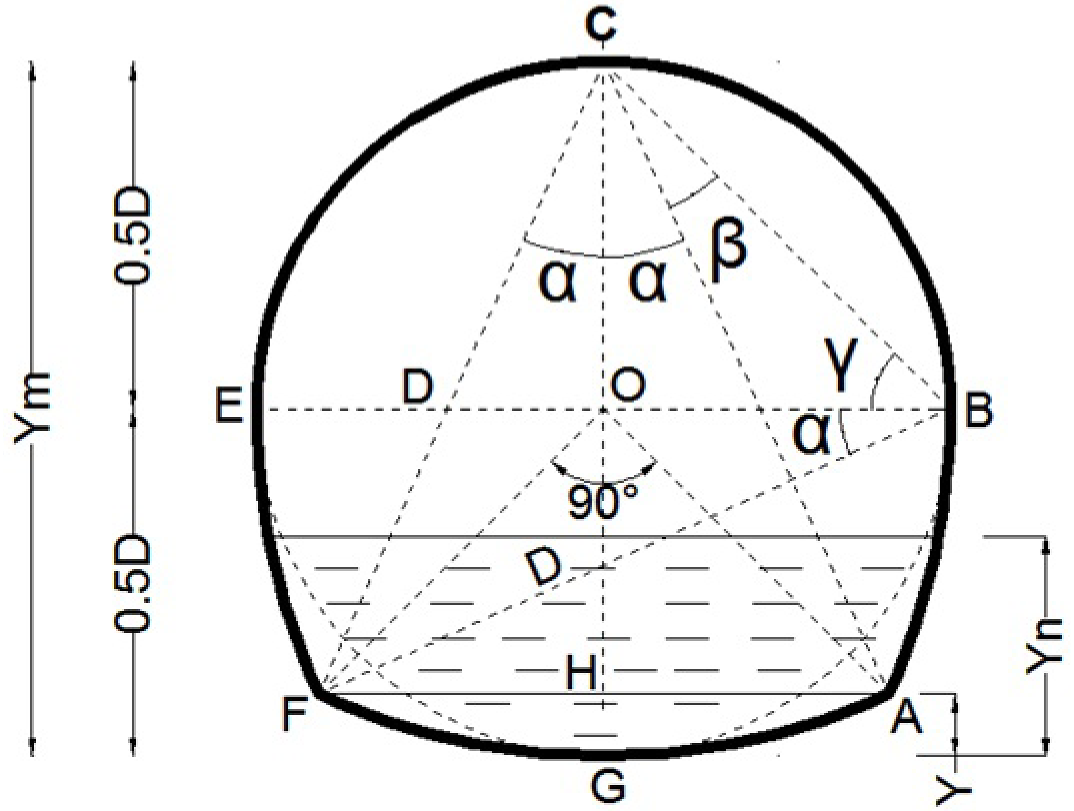

Figure 1. **Horseshoe-shaped tunnel profile (LENCASTRE, 1996; ACHOUR, 2007).**
Profil de la conduite en forme de fer à cheval (LENCASTRE, 1996; ACHOUR, 2007).

3.1.2 Hydraulic characteristics

In the conduit (Figure 1) and accordance with the normal depth *Yn*, three cases studies can be analyzed separately. Consequently, the hydraulic characteristics, specifically: the wetted perimeter *P*, the wetted cross-sectional area *A* and the hydraulic radius R_h can be formulated as a function of the filling rate $\eta = Yn/D$.

Case 01: $\eta \leq 0.089$

$$P = 2D\sigma(\eta) \tag{9}$$

$$A = D^2\sigma(\eta)\varphi(\eta) \tag{10}$$

$$R_h = \frac{D}{2}\varphi(\eta) \tag{11}$$

where

$$\sigma(\eta) = \cos^{-1}(1-\eta) \tag{12}$$

$$\varphi(\eta) = 1 - \frac{(1-\eta)\sqrt{\eta(2-\eta)}}{\cos^{-1}(1-\eta)} \tag{13}$$

Case 02: $0.089 \leq \eta \leq 0.5$

$$P = \rho(\eta)D \tag{14}$$

$$A = D^2\chi(\eta) \tag{15}$$

$$R_h = D\frac{\chi(\eta)}{\rho(\eta)} \tag{16}$$

where

$$\rho(\eta) = 1.696 - 2\sin^{-1}\left(\frac{1}{2}-\eta\right) \tag{17}$$

$$\chi(\eta) = 0.937 - \sin^{-1}\left(\frac{1}{2}-\eta\right) - \left(\frac{1}{2}-\eta\right)\left[\sqrt{1-\left(\frac{1}{2}-\eta\right)^2}\right] - \eta \tag{18}$$

Case 03: $0.5 \leq \eta \leq 1$

$$P = D\omega(\eta) \tag{19}$$

$$A = D^2\lambda(\eta) \tag{20}$$

$$R_h = \frac{\lambda(\eta)}{\omega(\eta)} D \quad (21)$$

where

$$\omega(\eta) = 3.267 - \cos^{-1}(2\eta - 1) \quad (22)$$

$$\lambda(\eta) = 0.829 - \frac{1}{4}\cos^{-1}(2\eta - 1) + \left(\eta - \frac{1}{2}\right)\left[\sqrt{\eta(1-\eta)}\right] \quad (23)$$

3.2 Chezy's resistance coefficient: general expression

In free-surface uniform flows, Chezy's formula expresses the discharge Q as:

$$Q = CA\sqrt{R_h i} \quad (24)$$

The open channel flow design draws upon the discharge Q, the longitudinal slope i, the filling rate η, the absolute roughness ε of the internal wall of the conduit, and the kinematic viscosity v of the liquid.

However, the resistance coefficient to flow C in equation 24 changes as a function of the filling rate η. For this purpose, this coefficient is not an *a priori* constant for the problem, and thus it becomes the objective of the study at hand.

The discharge relationship of ACHOUR and BEDJAOUI (2006) accepted in all geometric profiles and set in all turbulent flow regimes (smooth turbulent, transitional and turbulent rough) shows that C depends on all flow parameters.

ACHOUR and BEDJAOUI (2006) formulate the discharge Q as follows:

$$Q = -4\sqrt{2g}A\sqrt{R_h i}\log\left[\frac{\varepsilon}{14.8R_h} + \frac{10.04}{Re}\right] \quad (25)$$

where ε is the absolute roughness of the conduit internal wall, Re: Reynolds number expressed by the following formula:

$$Re = 32\sqrt{2}\frac{\sqrt{giR_h^{\ 3}}}{v} \quad (26)$$

When the relationships 24 and 25 are combined respectively, C can be stated as follows:

$$C = -4\sqrt{2g}\log\left[\frac{\varepsilon}{14.8R_h} + \frac{10.04}{Re}\right] \quad (27)$$

Equation 27 shows that Chezy's resistance coefficient C depends on the absolute roughness, the hydraulic radius R_h, and the Reynolds number Re. According to equation 26, the last parameter is a function of the slope i, the liquid kinematic viscosity v, and the hydraulic radius R_h. Equations 11, 16 and 21 are taken into consideration where the hydraulic radius R_h depends on the filling rate η and the conduit diameter D.

In dimensional terms, equation 27 becomes:

$$\frac{C}{\sqrt{g}} = -4\sqrt{2}\log\left[\frac{\varepsilon}{14.8R_h} + \frac{10.04}{Re}\right] \quad (28)$$

Case 01: $\eta \le 0.089$

By equations 11 and 26, we obtain:

$$Re = 16\frac{\sqrt{giD^3}}{\nu}\left[\varphi(\eta)\right]^{3/2} \quad (29)$$

In the full state of the conduit where $\eta = 1$, and from the relations 21 and 26, we can write:

$$Re_f = 5.788\frac{\sqrt{giD^3}}{\nu} \quad (30)$$

The index f indicates the full state of the conduit.

Thus, we can also reformulate equations 29 and 30 as follows:

$$Re = 2.764\, Re_f\left[\varphi(\eta)\right]^{3/2} \quad (31)$$

Based on both equations 11 and 31, the relationship 28 can be rewritten as follows:

$$\frac{C}{\sqrt{g}} = -4\sqrt{2}\log\left[\frac{\varepsilon/D}{7.4\varphi(\eta)} + \frac{3.632}{Re_f\left[\varphi(\eta)\right]^{3/2}}\right] \quad (32)$$

Case 02: $0.089 \le \eta \le 0.5$

For this case, using equations 16 and 26, we have:

$$Re = 32\sqrt{2}\frac{\sqrt{g\,iD^3}}{\nu}\left[\frac{\chi(\eta)}{\rho(\eta)}\right]^{3/2} \quad (33)$$

Therefore, we can modify equations 30 and 33 with the following:

$$Re = 7.818\, Re_f\left[\frac{\chi(\eta)}{\rho(\eta)}\right]^{3/2} \quad (34)$$

From equations 16 and 34, 28 can be rewritten as follows:

$$\frac{C}{\sqrt{g}}=-4\sqrt{2}\log\left[\frac{\varepsilon/D}{14.8\,\dfrac{\chi(\eta)}{\rho(\eta)}}+\frac{1.284}{Re_f\left[\dfrac{\chi(\eta)}{\rho(\eta)}\right]^{3/2}}\right] \quad (35)$$

Case 03: 0.5 ≤ η ≤ 1

Equations 21 and 26 result in:

$$Re=32\sqrt{2}\,\frac{\sqrt{giD^3}}{\nu}\left[\frac{\lambda(\eta)}{\omega(\eta)}\right]^{3/2} \quad (36)$$

Consequently, we can also write from equations 30 and 36:

$$Re=7.818\,Re_f\left[\frac{\lambda(\eta)}{\omega(\eta)}\right]^{3/2} \quad (37)$$

From equations 21 and 37, 28 can be rewritten as follows:

$$\frac{C}{\sqrt{g}}=-4\sqrt{2}\log\left[\frac{\varepsilon/D}{14.8\,\dfrac{\lambda(\eta)}{\omega(\eta)}}+\frac{1.284}{\mathrm{Re}_f\left[\dfrac{\lambda(\eta)}{\omega(\eta)}\right]^{3/2}}\right] \quad (38)$$

3.3 Chezy's resistance coefficient (CRC) calculation via the rough model method (RMM)

To calculate Chezy's resistance coefficient, the diameter D can be an unknown parameter. The given parameters are the discharge Q, the conduit filling rate η, the longitudinal slope i, the absolute roughness ε, and the kinematic viscosity ν of the flowing liquid. The expressions 32, 35 and 38 will no extend for the explicit computing of C. Then, in this case, the rough model method (RMM) can be valid to determine this coefficient.

The rough model is mainly characterized by $\overline{\varepsilon}/\overline{D}_h$ = 0.037 (ACHOUR, 2007) as the arbitrarily assigned relative roughness value, where $\overline{D}_h$ is the hydraulic diameter. Thus, the friction factor is $\overline{f}$ = 1/16 according to Colebrook-White relationship for $Re = \overline{Re}$ tending to an infinitely large value. For Re tending to infinity, Colebrook-White's relationship leads to the Nikuradse formula as follows (ACHOUR, 2007):

$$\overline{f}=\left[-2\log\left(\frac{\overline{\varepsilon}/\overline{D}_h}{3.7}\right)\right]^{-2} \quad (39a)$$

By introducing the value $\overline{\varepsilon}/\overline{D}_h$ = 0.037, we have:

$$\overline{f}=\left[-2\log\left(\frac{0.037}{3.7}\right)\right]^{-2}=4^{-2}=1/16 \quad (39b)$$

For a coefficient of friction $\overline{f}$ = 1/16 defined by the reference rough model (ACHOUR, 2007), the $\overline{C}$ can be written:

$$\overline{C}=\sqrt{8g/\overline{f}}=8\sqrt{2g}=\text{constant} \quad (39c)$$

The conduit in the rough model is defined by a diameter $\overline{D}$, a longitudinal slope $\overline{i}$, flowing a discharge $\overline{Q}$ for a liquid of kinematic viscosity $\overline{\nu}$ and a filling rate $\overline{\eta}$.

For this purpose, to express the C value, characterizing the flow in the conduit, we can adopt these conditions: $\overline{D}\neq D$; $\overline{Q}=Q$; $\overline{i}=i$; $\overline{\eta}=\eta$; $\overline{\nu}=\nu$.

Case 01: η ≤ 0.089

From equations 10 and 11, equation 24 becomes:

$$Q=\frac{1}{\sqrt{2}}\sigma(\eta)\varphi(\eta)^{\frac{3}{2}}\sqrt{C^2D^5i} \quad (40)$$

We set:

$$Q^*=\frac{1}{\sqrt{2}}\sigma(\eta)\varphi(\eta)^{\frac{3}{2}} \quad (41)$$

Then:

$$Q^*=\frac{Q}{\sqrt{C^2D^5i}} \quad (42)$$

The relative conductivity of the rough model in accordance with formula 42 gives the following:

$$Q^*=\frac{Q}{\sqrt{\overline{C}^2\overline{D}^5i}} \quad (43)$$

Then, according to formula 39c, equation 43 becomes:

$$\overline{Q}^*=\frac{Q}{\sqrt{128g\overline{D}^5i}} \quad (44)$$

For a rough model, equation 41 becomes:

$$\frac{Q}{\sqrt{128g\bar{D}^5 i}}=\frac{1}{\sqrt{2}}\sigma(\eta)\left[\varphi(\eta)\right]^{\frac{3}{2}} \quad (45)$$

Thus:

$$\bar{D}=\frac{1}{2}\left[\sqrt{2}\sigma(\eta)\right]^{-0.4}\left[\varphi(\eta)\right]^{-0.6}\left[\frac{Q}{\sqrt{gi}}\right]^{0.4} \quad (46)$$

For this purpose, if the parameters Q, i and η are known, equation 46 computes explicitly the diameter $\bar{D}$ of the rough model.

In the rough model and using equation 29, the Reynolds number $\overline{Re}$ characterizing the flow is:

$$\overline{Re}=16\frac{\sqrt{gi\bar{D}^3}}{\nu}\left[\varphi(\eta)\right]^{3/2} \quad (47)$$

or

$$\overline{Re}=2.764\overline{\mathrm{Re}_f}\left[\varphi(\eta)\right]^{3/2} \quad (48)$$

where

$$\overline{Re_f}=5.788\frac{\sqrt{gi\bar{D}^3}}{\nu} \quad (49)$$

Chezy's coefficient C is provided according to the RMM as follows (ACHOUR and BEDJAOUI, 2006):

$$C=\frac{\bar{C}}{\psi^{5/2}} \quad (50)$$

where ψ is a dimensionless parameter determined by the following expression (ACHOUR and BEDJAOUI, 2006; ACHOUR and BEDJAOUI, 2012):

$$\psi=1.35\left[-\log\left(\frac{\varepsilon/\bar{R}_h}{19}+\frac{8.5}{\overline{Re}}\right)\right]^{-\frac{2}{5}} \quad (51)$$

From equations 11 and 48, the relationship 51 becomes:

$$\psi=1.35\left[-\log\left(\frac{\varepsilon/\bar{D}}{9.5\varphi(\eta)}+\frac{3.075}{\overline{\mathrm{Re}_f}\left[\varphi(\eta)\right]^{3/2}}\right)\right]^{-\frac{2}{5}} \quad (52)$$

As of 39 and 50:

$$C=\frac{\bar{C}}{\psi^{5/2}}=\frac{8\sqrt{2g}}{\psi^{5/2}} \quad (53)$$

From equation 52, the relationship 53 becomes:

$$C=-5.343\sqrt{g}\log\left(\frac{\varepsilon/\bar{D}}{9.5\varphi(\eta)}+\frac{3.075}{\overline{\mathrm{Re}_f}\left[\varphi(\eta)\right]^{3/2}}\right) \quad (54)$$

In dimensionless terms, equation 54 can be rewritten as follows:

$$\frac{C}{\sqrt{g}}=-5.343\log\left(\frac{\varepsilon/\bar{D}}{9.5\varphi(\eta)}+\frac{3.075}{\overline{\mathrm{Re}_f}\left[\varphi(\eta)\right]^{3/2}}\right) \quad (55)$$

Case 02: 0.089 ≤ η ≤ 0.5

From the equations 15 and 16, 24 becomes:

$$Q=\left[\chi(\eta)\right]^{\frac{3}{2}}\left[\rho(\eta)\right]^{\frac{-1}{2}}\sqrt{C^2D^5i} \quad (56)$$

We set:

$$Q^*=\left[\chi(\eta)\right]^{\frac{3}{2}}\left[\rho(\eta)\right]^{\frac{-1}{2}} \quad (57)$$

Then:

$$Q^*=\frac{Q}{\sqrt{C^2D^5i}} \quad (58)$$

The relative conductivity of the rough model in accordance with equation 58 will give the following:

$$Q^*=\frac{Q}{\sqrt{\bar{C}^2\bar{D}^5i}} \quad (59)$$

Then, according to equation 39c, 59 becomes:

$$\bar{Q}^*=\frac{Q}{\sqrt{128g\bar{D}^5i}} \quad (60)$$

For a rough model, equation 57 is written:

$$\frac{Q}{\sqrt{128g\bar{D}^5i}}=\left[\chi(\eta)\right]^{\frac{3}{2}}\left[\rho(\eta)\right]^{\frac{-1}{2}} \quad (61)$$

So:

$$\bar{D} = 0.379\left[\rho(\eta)\right]^{0.2}\left[\chi(\eta)\right]^{-0.6}\left[\frac{Q}{\sqrt{g\,i}}\right]^{0.4} \qquad (62)$$

For this purpose, if the parameters Q, i and η are known, equation 62 permits the explicit computing of the diameter $\bar{D}$ of the rough model.

In the rough model and using equation 33, the Reynolds number $\overline{Re}$ characterizing the flow is:

$$\overline{Re} = 32\sqrt{2}\,\frac{\sqrt{g\,i\bar{D}^3}}{\nu}\left[\frac{\chi(\eta)}{\rho(\eta)}\right]^{3/2} \qquad (63)$$

or

$$\overline{Re} = 7.818\,\overline{\mathrm{Re}_f}\left[\frac{\chi(\eta)}{\rho(\eta)}\right]^{3/2} \qquad (64)$$

where

$$\overline{Re_f} = 5.788\,\frac{\sqrt{g\,i\bar{D}^3}}{\nu}$$

From expressions 16 and 64, 51 becomes:

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon/\bar{D}}{19\dfrac{\chi(\eta)}{\rho(\eta)}} + \frac{1.087}{\overline{\mathrm{Re}_f}\left[\dfrac{\chi(\eta)}{\rho(\eta)}\right]^{3/2}}\right)\right]^{-\frac{2}{5}} \qquad (65)$$

According to 65, equation 53 becomes:

$$C = -5.343\sqrt{g}\,\log\left(\frac{\varepsilon/\bar{D}}{19\dfrac{\chi(\eta)}{\rho(\eta)}} + \frac{1.087}{\overline{\mathrm{Re}_f}\left[\dfrac{\chi(\eta)}{\rho(\eta)}\right]^{3/2}}\right) \qquad (66)$$

Then, in dimensionless terms:

$$\frac{C}{\sqrt{g}} = -5.343\log\left(\frac{\varepsilon/\bar{D}}{19\dfrac{\chi(\eta)}{\rho(\eta)}} + \frac{1.087}{\overline{\mathrm{Re}_f}\left[\dfrac{\chi(\eta)}{\rho(\eta)}\right]^{3/2}}\right) \qquad (67)$$

Case 03: 0.5 ≤ *η* ≤ 1

From equations 20 and 21, equation 24 becomes:

$$Q = \left[\lambda(\eta)\right]^{\frac{3}{2}}\left[\omega(\eta)\right]^{\frac{-1}{2}}\sqrt{C^2D^5i} \qquad (68)$$

We set:

$$Q^* = \left[\lambda(\eta)\right]^{\frac{3}{2}}\left[\omega(\eta)\right]^{\frac{-1}{2}} \qquad (69)$$

Hence:

$$Q^* = \frac{Q}{\sqrt{C^2D^5i}} \qquad (70)$$

The relative conductivity of the rough model in accordance with equation 70 gives the following:

$$Q^* = \frac{Q}{\sqrt{\bar{C}^2\bar{D}^5i}} \qquad (71)$$

Then, according to 39c, equation 71 becomes:

$$\bar{Q}^* = \frac{Q}{\sqrt{128\,g\bar{D}^5i}} \qquad (72)$$

For a rough model, the relationship 69 is written:

$$\frac{Q}{\sqrt{128\,g\bar{D}^5i}} = \left[\lambda(\eta)\right]^{\frac{3}{2}}\left[\omega(\eta)\right]^{\frac{-1}{2}} \qquad (73)$$

So:

$$\bar{D} = 0.379\left[\omega(\eta)\right]^{0.2}\left[\lambda(\eta)\right]^{-0.6}\left[\frac{Q}{\sqrt{gi}}\right]^{0.4} \qquad (74)$$

For this purpose, if the parameters Q, i and η are known, equation 74 permits the explicit computing of the diameter $\bar{D}$ of the rough model.

In the rough model and by the formula 36, the Reynolds number $\overline{Re}$ characterizing the flow is:

$$\overline{Re} = 32\sqrt{2}\,\frac{\sqrt{g\,i\bar{D}^3}}{\nu}\left[\frac{\lambda(\eta)}{\omega(\eta)}\right]^{3/2} \qquad (75)$$

or

$$\overline{Re} = 7.818\,\overline{Re_f}\left[\frac{\lambda(\eta)}{\omega(\eta)}\right]^{3/2} \qquad (76)$$

where

$$\overline{Re_f} = 5.788 \frac{\sqrt{gi\overline{D}^3}}{\nu}$$

From equations 21 and 76, 51 becomes:

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon/\overline{D}}{19\frac{\lambda(\eta)}{\omega(\eta)}} + \frac{1.087}{\overline{\mathrm{Re}_f}\left[\frac{\lambda(\eta)}{\omega(\eta)}\right]^{3/2}}\right)\right]^{-\frac{2}{5}} \tag{77}$$

According to 77, equation 53 becomes:

$$C = -5.343\sqrt{g}\,\log\left(\frac{\varepsilon/\overline{D}}{19\frac{\lambda(\eta)}{\omega(\eta)}} + \frac{1.087}{\overline{\mathrm{Re}_f}\left[\frac{\lambda(\eta)}{\omega(\eta)}\right]^{3/2}}\right) \tag{78}$$

Then, in dimensionless terms:

$$\frac{C}{\sqrt{g}} = -5.343\log\left(\frac{\varepsilon/\overline{D}}{19\frac{\lambda(\eta)}{\omega(\eta)}} + \frac{1.087}{\overline{\mathrm{Re}_f}\left[\frac{\lambda(\eta)}{\omega(\eta)}\right]^{3/2}}\right) \tag{79}$$

3.4 Simplified method

This paragraph is devoted to the presentation of a simplified method that originated from the rough model theory. Compared to the method presented previously in section 3.3, this method makes it easy to determine the coefficient C using relatively few data. To do so, the required parameters are limited to the discharge, the slope of the conduit, the absolute roughness, and the liquid kinematic viscosity.

If it is supposed that $\eta \neq \overline{\eta}$ and when equation 74 is used for the rough model, we have the following:

$$Q^* = \left[\lambda(\overline{\eta})\right]^{\frac{3}{2}}\left[\omega(\overline{\eta})\right]^{\frac{-1}{2}} \tag{80}$$

where Q^* is the relative conductivity and is given using equation 72.

Consider the referential rough model with a diameter $\overline{D}$ equal to that of the full state of the tunnel corresponding to $\overline{\eta}$ = 1: for $\overline{\eta}$ = 1, the equations 22 and 23 will be: $\omega(\overline{\eta})$ = 3.267, $\lambda(\overline{\eta})$ = 0.829.

Thus, equation 80 becomes $Q^* = 0.133\pi$. For this same value of relative conductivity, equation 80 provides another value of the filling rate $\overline{\eta} \approx 0.8505$ different from $\overline{\eta}$ = 1.

The hydraulic radius $\overline{R_h}$ is provided in accordance with equation 21 for the full state of the tunnel:

$$\overline{R}_h = 0.3056\overline{D} \tag{81}$$

For $Q^* = 0.133\pi$, the diameter $\overline{D}$ of the full rough model is gotten by the formulation below:

$$\overline{D} = \left(0.133\pi\right)^{-0.4}\left(\frac{Q}{\sqrt{128gi}}\right)^{0.4} \tag{82}$$

C can easily be calculated by following the steps below:

1. Compute the diameter $\overline{D}$ corresponding to the full state of the conduit by equation 82.
2. Therefore, the hydraulic radius $\overline{R_h}$ is computed by equation 81.
3. Besides, equation 26 directly computes the Reynolds number of the rough model.
4. The dimensionless correction factor ψ is explicitly calculated using equation 51.
5. Lastly, using equation 53, C is obtained.

4. RESULTS AND DISCUSSION

Following equations 32, 35 and 38, the relative roughness ε/D the filling rate η of the conduit and the Reynolds number corresponding to the full state of the conduit Re_f are required for Chezy's resistance coefficient C. When these parameters are known, the same coefficient can explicitly be determined by equations 32, 35 and 38.

However, using the rough model method (RMM), C can be calculated using the parameters $\overline{Re_f}$, $\overline{D}$, ε and η, by equations 54, 66 and 78 without knowing the diameter D. The simplified method derived from the (RMM) can also determine the C, with inputs limited to the discharge Q, the slope i of the tunnel, the absolute roughness ε, and the kinematic viscosity ν.

In what follows, examples are provided after having discussed the variation of this coefficient as a function of the filling rate η.

4.1 Maximum of Chezy's resistance coefficient

According to equations 32, 35 and 38, C depends on variables η, ε/D and Re_f. Curves can be drawn presenting the variation of C as a function of the filling rate η for fixed values of the relative roughness ε/D and by varying values of the Reynolds number Re_f.

Figures 2a and 2b showed the variation of $C/\sqrt{g}$ as a function of η for both states of turbulent flow, the smooth state $\varepsilon/D = 0$, and the rough state $\varepsilon/D = 0.05$, where the Reynolds number Re_f varying between 10^4 and 10^7.

In these curves, $C/\sqrt{g}$ undergoes an upsurge along the variation of the filling rate η, the increase of $C/\sqrt{g}$ is remarkably rapid where η is lower than 0.3. Then, where $\eta > 0.3$, the increase becomes very slow until the $C/\sqrt{g}$ reaches a maximum value for the same filling rate $\eta = 0.8108$ in all curves. In the interval $0.8108 < \eta < 1$, a decrease in $C/\sqrt{g}$ seems remarkable by the decline of the curves. Notably, in the figure 2b the curves merge above the 10^5 value of the Reynolds number, which explains that in rough turbulent regime the variation of $C/\sqrt{g}$ depends only on the filling rate η.

For all curves, the C reaches the maximum at the same value of the filling rate $\eta \cong 0.8108$ in the case of the horseshoe-shaped tunnel. For this purpose, we can say that it does not depend upon the value of the Reynolds number or the state of the inner tunnel wall (smooth or rough).

$$\text{At } \eta \cong 0.8108, \quad \frac{C}{\sqrt{g}} = \frac{C_{max}}{\sqrt{g}} \tag{83}$$

Also, equations 22 and 23 give the following:

$$\omega(\eta = 0.8108) = 3.267 - \cos^{-1}(2\times 0.8108 - 1) = 2.367$$

$$\lambda(\eta = 0.8108) = 0.829 - \frac{1}{4}\cos^{-1}(2\times 0.8108 - 1) + \left(0.8108 - \frac{1}{2}\right)\left[\sqrt{0.8108(1 - 0.8108)}\right] = 0.726$$

By replacing these last two values in equation 38, we obtain:

$$\frac{C_{max}}{\sqrt{g}} = -4\sqrt{2}\log\left[\frac{\varepsilon/D}{4.54} + \frac{7.558}{Re_f}\right] \tag{84}$$

or

$$C_{max} = -4\sqrt{2g}\log\left[\frac{\varepsilon/D}{4.54} + \frac{7.558}{Re_f}\right] \tag{85}$$

Equation 85 permits the calculation of the maximum Chezy's resistance coefficient C_{max} if the relative roughness ε/D and the Reynolds number Re_f are known.

However, without knowing the diameter D, C_{max} can be calculated by attributing to η the value 0.8108 in equation 79.

Therefore, we can write:

$$\frac{C_{max}}{\sqrt{g}} = -5.343\log\left(\frac{\varepsilon/\overline{D}}{5.828} + \frac{6.398}{\overline{Re_f}}\right) \tag{86}$$

so

$$C_{max} = -5.343\sqrt{g}\log\left(\frac{\varepsilon/\overline{D}}{5.828} + \frac{6.398}{\overline{Re_f}}\right) \tag{87}$$

Example 1

For the following data: $Q = 0.8\ m^3 \cdot s^{-1}$, $i = 3 \times 10^{-4}$, $\varepsilon = 10^{-4}$ m, $\eta = 0.7$ and $\nu = 10^{-6}\ m^2 \cdot s^{-1}$, calculate C_{max}.

Solution

For the filling rate $\eta = 0.7$, equation 74 permits the explicit calculation of the rough model diameter $\overline{D}$ with the known parameters Q, η and i.

So,

$$\omega(\eta = 0.7) = 3.267 - \cos^{-1}(2\times 0.7 - 1) = 2.108$$

$$\lambda(\eta = 0.7) = 0.829 - \frac{1}{4}\cos^{-1}(2\times 0.7 - 1) + \left(0.7 - \frac{1}{2}\right)\left[\sqrt{0.7(1 - 0.7)}\right] = 0.631$$

$$\overline{D} = 0.379\left[\omega(\eta)\right]^{0.2}\left[\lambda(\eta)\right]^{-0.6}\left[\frac{Q}{\sqrt{gi}}\right]^{0.4} = 0.379\left[2.108\right]^{0.2}\left[0.631\right]^{-0.6}\left[\frac{0.8}{\sqrt{9.81\times 0.0003}}\right]^{0.4} = 1.701\text{ m}$$

Using equation 49, we can compute the full state Reynolds number $\overline{Re_f}$:

$$\overline{Re_f} = 5.788\frac{\sqrt{gi\overline{D}^3}}{\nu} = 5.788\times\frac{\sqrt{9.81\times 3\times 10^{-4}\times(1.701)^3}}{10^{-6}} = 696812.553$$

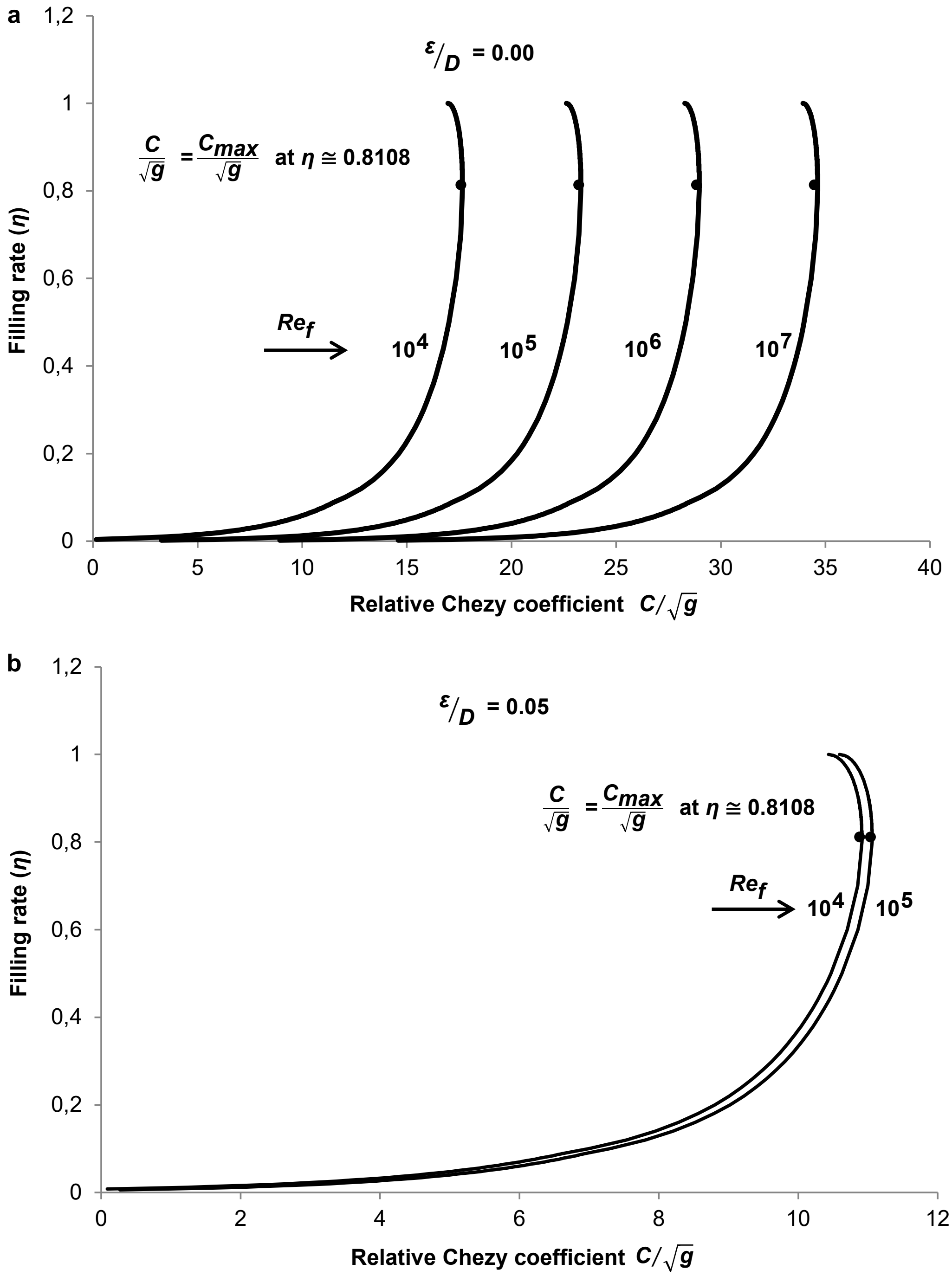

Figure 2. **Variation of $C/\sqrt{g}$ as a function of the filling rate η according to the equations 32, 35 and 38 for fixed values of the relative roughness and the Reynolds number Re_f: a) ε/D = 0 and b) ε/D = 0.05.**

Variation de $C/\sqrt{g}$ en fonction du taux de remplissage η selon les relations 32, 35 et 38 pour des valeurs fixées de la rugosité relative et du nombre de Reynolds Re_f : a) ε/D = 0 et b) ε/D = 0.05.

C_{max} can be easily calculated using equation 87 without knowing the diameter D of the tunnel:

$$C_{max} = -5.343\sqrt{g}\log\left(\frac{\varepsilon/\overline{D}}{5.828}+\frac{6.398}{\overline{Re_f}}\right)$$

$$= -5.343\sqrt{g}\,\log\left(\frac{0.0001/1.701}{5.828}+\frac{6.398}{696812.553}\right)$$

$$= 78.908\ m^{0.5}\cdot s^{-1}$$

4.2 Calculation of Chezy's resistance coefficient by the rough model method

The expressions 54, 66 and 78 are established using the rough model method (RMM) to direct calculating C as a function of the parameters $\overline{Re_f}$, $\overline{D}$, ε and η without knowing the diameter D of the tunnel. This calculation can be clearly shown in example 2.

Example 2

By the rough model method, calculate Chezy's resistance coefficient from the following data: $Q = 0.9\ m^3\cdot s^{-1}$, $i = 3 \times 10^{-4}$, $\varepsilon = 10^{-4}$ m, $\eta = 0.6$ and $\nu = 10^{-6}\ m^2\cdot s^{-1}$.

Solution

For $\eta = 0.6$, the calculation will be made by equations 22 and 23. From the known parameters Q, η and i, equation 74 permits the explicit calculation of the diameter $\overline{D}$ of the rough model.

So, at $\eta = 0.6$

$$\omega(\eta{=}0.6) = 3.267\text{-}\cos^{-1}(2\times0.6\text{-}1) = 1.897$$

$$\lambda(\eta{=}0.6) = 0.829\text{-}\frac{1}{4}\cos^{-1}(2\times0.6\text{-}1)+\left(0.6\text{-}\frac{1}{2}\right)\left[\sqrt{0.6(1\text{-}0.6)}\right]$$

$$= 0.536$$

Equation 74 gives the diameter $\overline{D}$:

$$\overline{D} = 0.379[\omega(\eta)]^{0.2}[\lambda(\eta)]^{-0.6}\left[\frac{Q}{\sqrt{g\,i}}\right]^{0.4}$$

$$= 0.379[1.897]^{0.2}[0.536]^{-0.6}\left[\frac{0.9}{\sqrt{9.81\times0.0003}}\right]^{0.4} = 1.926\ m$$

Using equation 49, we can calculate at full state, the Reynolds number $\overline{Re_f}$:

$$\overline{Re_f} = 5.788\times\frac{\sqrt{g i\overline{D}^3}}{\nu} = 5.788\times\frac{\sqrt{9.81\times3\times10^{-4}\times(1.926)^3}}{10^{-6}}$$

$$= 839517.558$$

Lastly, C is calculated using equation 78:

$$C = -5.343\sqrt{g}\left[\log\left(\frac{\varepsilon/\overline{D}}{19\dfrac{\lambda(\eta)}{\omega(\eta)}}+\frac{1.087}{\overline{Re_f}\left[\dfrac{\lambda(\eta)}{\omega(\eta)}\right]^{3/2}}\right)\right]$$

$$= -5.343\sqrt{9.81}\left[\log\left(\frac{0.0001/1.926}{19\times0.282}+\frac{1.087}{839517.558\times0.282^{3/2}}\right)\right]$$

$$= 79.282\ m^{0.5}\cdot s^{-1}$$

Example 3 is proposed to confirm the usefulness of the simplified method for straightforwardly determining the Chezy's resistance coefficient.

Example 3

According to the data of example 2: $Q = 0.9\ m^3\cdot s^{-1}$, $i = 3 \times 10^{-4}$, $\varepsilon = 10^{-4}$ m and $\nu = 10^{-6}\ m^2\cdot s^{-1}$, compute Chezy's resistance coefficient by the simplified method.

Solution

Chezy's resistance coefficient can be calculated by following the steps below:

1. Diameter $\overline{D}$ of the full rough model is computed by equation 82:

$$\overline{D} = (0.133\pi)^{-0.4}\left(\frac{Q}{\sqrt{128gi}}\right)^{0.4} = (0.133\pi)^{-0.4}\left(\frac{0.9}{\sqrt{128\times9.81\times3\times10^{-4}}}\right)^{0.4}$$

$$\overline{D} = 1.652\ m$$

2. The hydraulic radius $\overline{R_h}$ is calculated by equation 81:

$$\overline{R_h} = 0.3056\overline{D} = 0.3056\times1.652 = 0.505\ m$$

3. Using equation 26 the Reynolds number $\overline{Re}$ of the rough model is calculated as follows:

$$\overline{Re} = 32\sqrt{2}\times\frac{\sqrt{gi\overline{R_h}^3}}{\nu} = 32\sqrt{2}\times\frac{\sqrt{9.81\times3\times10^{-4}\times(0.505)^3}}{10^{-6}}$$

$$= 880916.328$$

4. The dimensionless correction factor ψ is explicitly obtained by equation 51:

$$\psi = 1.35\left[-\log\left(\frac{\varepsilon/\overline{R}_h}{19}+\frac{8.5}{\overline{\overline{Re}}}\right)\right]^{-\frac{2}{5}}$$

$$\psi = 1.35\left[-\log\left(\frac{10^{-4}/0.505}{19}+\frac{8.5}{880916.328}\right)\right]^{-\frac{2}{5}} = 0.727$$

5. Finally, Chezy's resistance roefficient C is calculated using equation 53:

$$C = \frac{8\sqrt{2g}}{\psi^{5/2}} = \frac{8\sqrt{2\times 9.81}}{(0.727)^{5/2}} = 78.607\ \mathrm{m}^{0.5}\cdot\mathrm{s}^{-1}$$

The value of C calculated by the simplified method (78.607) is less than that calculated in example 2 by the rough model method (79.282). The relative error rate between both values is around 0.8%.

5. CONCLUSION

In this research paper, Chezy's resistance coefficient in the horseshoe-shaped tunnel is expressed by various equations. According to the geometry shape of the tunnel and the position of the normal depth of the flowing liquid in the conduit, three cases of flow were distinguished. For every case, the hydraulic characteristics, wetted perimeter P, wetted cross-sectional area A and hydraulic radius R_h were determined as a function of filling rate η. More importantly, general equations 32, 35 and 38 have been established to explicitly determine the Chezy's resistance coefficient C depending on the following parameters: the filling rate η, the relative roughness ε/D and the Reynolds number at the full state of the tunnel Re_f. Whereas, equations 54, 66 and 78 are developed using the rough model method (RMM) to compute C as a function of the parameters $\overline{Re_f}$, $\overline{D}$, ε and η without knowing the diameter D of the tunnel.

Furthermore, a simplified method based on (RMM) is suggested in the current study, to explicitly compute the coefficient C. This method has the advantage of using relatively few data, precisely, the discharge Q, the slope i, the absolute roughness ε and the kinematic viscosity v. To conclude, curves (Figures 2a and 2b) are established to show the variation of Chezy's resistance coefficient as a function of the filling rate, this has been performed through attributing fixed values to the relative roughness $\varepsilon/D = 0$ and $\varepsilon/D = 0.05$, where the Reynolds number varying between 10^4 and 10^7. All these curves uncover that Chezy's resistance coefficient attains a maximum at the filling rate $\eta = 0.8108$. Therefore, in both cases where the diameter of the tunnel can be known or not, equations 85 and 87 are successively formulated to express for each case the maximum of Chezy's resistance coefficient C_{max}.

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