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# Abstraction and Existence

## A Study on St. Thomas : *In Boethii de Trinitate, Q.5, A.3*

Cornelius J. Kelly

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# Abstraction and Existence

A STUDY ON ST. THOMAS :
*IN BOETHII DE TRINITATE, Q.5, A.3*

Natural science is about things that are changeable, material, or in the words of St. Thomas, it is about things that are in matter and motion. This knowledge, which can be truly scientific, has its own mode of defining. Natural definitions include common sensible matter. Although these definitions, and the demonstrations that follow from them, abstract from the singular, from the here and now, the singular in nature serves nonetheless to illustrate them. When this science defines or demonstrates, such knowledge is applicable to the kind of things that exist in nature. To cite again the response to the fourth objection of the previous article : " Per universalem enim hominis rationem possum iudicare de hoc vel illo." What is true of snubnose is true of *this* one.

If we turn now to mathematics and consider some of the elements of geometry, we see that a different situation obtains. The geometer considers things like points, lines, triangles, solids, spheres, etc. These are what his science is about. But note that by sphere, for example, he means ' a three-dimensional continuum bounded by one surface which is at every point equidistant from a point within called its center.' So defined, sphere does not contain sensible matter. It is neither metal, nor wood, nor plastic, nor hard, nor colored. No sensible matter, neither individual nor common, is part of what it is.[1] The sphere of the geometer is defined with such exactness that nothing whose definition requires sensible matter could possibly verify it. That is to say, while metal or plastic spheres may resemble the geometric sphere, they are not instances of the latter. A weather balloon is not an instance of what the geometer is talking about. When a teacher proves that a triangle, any triangle, must have 180 degrees, he is not referring to the large white one drawn in the lower right-hand corner of the blackboard even though this may have been sketched to help the students " get the idea " of what a triangle is. To take again the example of snubnose, snubnose can be said of *this* nose. But curve, defined by the geometer, cannot be so predicated ; something extrinsic to the definition of curve, viz., sensible matter, must be added to it. But then it is no longer a curve as defined by the geometer. Curve applied to this nose is a natural curve, in sensible matter. It is against this background that St. Thomas confronts the problem regarding mathematics.

1. ARISTOTLE, *Metaph.*, XI, c.3, 1061 *a* 29-35, 1061 *b* 1.

Given that the mathematician's way of considering his subject is such as here briefly described, St. Thomas is concerned with the possibility of this consideration to provide the basis for a distinct speculative science. In the division proposed in article one, mathematics was designated as a science that considers without matter things which are in matter. The point of the present inquiry is to determine how such an abstraction can be accomplished and still be true. Science produces knowledge of the true and truth is defined as the conformity of the intellect with what is. Thus the things which a given science is about should be such as they are defined in that science. But as we have seen, things which are in sensible matter are not the subject of mathematics as such. The plastic sphere requires sensible matter in its definition, whereas the geometer defines sphere without sensible matter. There is nothing in nature to verify the mathematical definition of curve the way a nose verifies sensible curve. Thus it would seem that mathematics, considering things otherwise than they are outside the mind, is not true and consequently must relinquish its claim to the quality of science.

In approaching this difficulty St. Thomas is obliged to examine abstraction more closely. And while the express purpose of this examination is to clarify the abstraction proper to mathematics, it provides more than that. It is, in fact, an exposition of the manner in which the mind abstracts in each of the sciences ; hence its importance. That the mind does not accomplish abstraction in a unique way Aristotle makes clear in the *Physics*.[1] When the mathematician and the naturalist consider the same subjects they do not consider them in the same fashion. As St. Thomas points out : " Definitio substantiarum naturalium non tantum formam continet sed etiam materiam ; aliter enim definitiones naturales et mathematicae non differrent." [2] Both natural and mathematical definitions require abstraction, but the abstraction required in each case is not at all the same. This is to say that the word abstraction has more than one meaning.

1. *Operations of the mind.*

To elucidate this difference in meaning St. Thomas first shows how the word is applied to the different operations of the intellect. For this purpose he finds it sufficient to mention only the first two. (The third operation, though not mentioned here, is implied by the second.[3]) The first operation is called *intelligentia indivisibilium*. Later on in the article, and elsewhere it is also called *formatio*.[4] In

1. *Physics*, II, c.2.
2. *De Ente et Ess.*, c.2.
3. Cf. St. Thomas, *In Periherm.*, prooem., n. 1.
4. *De Spirit. Creat.*, q.un., a. 9, ad 6.

this operation, discussed in the preceding article, the intellect attains first of all, though vaguely, what something is, and then by division and composition, reaches a definition stating distinctly what something is. The second operation is that by which the intellect composes and divides objects apprehended, which composition or division is signified by an affirmative or negative enunciation.

Compositio quidem, quando intellectus comparat unum conceptum alteri, quasi apprehendens coniunctionem, aut identitatem rerum, quarum sunt conceptiones ; divisio autem, quando sic comparat unum conceptum alteri, ut apprehendat res diversas. Et per hunc etiam modum in vocibus affirmatio dicitur compositio, in quantum coniunctionem ex parte rei significat ; negatio vero dicitur divisio, in quantum significat rerum separationem.[1]

The necessity of more than one intellectual operation is a condition peculiar to the human way of knowing. Unlike angelic intellects, the human intellect cannot, at once, grasp everything that is contained in a given nature. The potentiality which marks the inception of human intellection continues throughout its development. A more perfect intellect, such as that of a separated substance, in one simple apprehension knows immediately whatever pertains or does not pertain to a given subject. The grasp of angelic intellection carries beyond specific to individual principles of the thing known and the disposition of the subject, attained in its concretion, is an adequate principle for knowing all that may or may not inhere in that subject. To attain the knowledge that superior intellects achieve immediately, the human intellect must proceed gradually, moving from vague to distinct apprehension. But it does not apprehend for the sake of apprehension. Irrespective of how distinct it may be, human apprehension does not have the nature of a term, inasmuch as truth is achieved only in the act of composition and division.

Intellectus autem noster, apprehendendo incomplexa, nondum pertingit ad ultimam suam perfectionem, quia adhuc est in potentia respectu compositionis . . . Sed veritas consequitur intellectum nostrum in sua perfecta cognitione, quando iam usque ad compositionem pervenit.[2]

Apprehension leaves us at a half-way house. We must make our way from here by composition and division. By knowing *what* man is, we do not, by that fact, have knowledge of the truth unless we assert that he is such or such. Whatever further knowledge we must acquire about man will be referred to the initial knowledge of ' what man is.' We do this by successive compositions and divisions.

1. St. Thomas, *In I Periherm.*, lect.3, n.5.

2. *Cont. Gent.*, I, c.59.

The reason for the debility of human knowing, as we have seen, lies in the fact that our intellect is posterior to things to be known and is measured by them. This means, of course, that our intellect is dependent on things for what it knows of them. It also means that what it knows of one thing does not necessarily imply the knowledge of something else actually inhering in it. As a result, the intellect is forced to take its knowledge where it can find it and must go from the knowledge of one thing to another.

Corresponding to the intellect's first operation is the nature of things, namely, what they are.[1] This nature may be something complete, a whole, like man or stone. Or it may be an incomplete nature, as a part or an accident. The actuality found in things includes more, however, than their essential nature. Other things can be attributed to this nature or denied of it. It has properties and accidents such as relations. To attain this actuality, the intellect, which in its first operation knows the nature at first vaguely and then distinctly according to what it is, must have recourse to a second and a third operation in order to know the way in which it is or is not.[2] Thus St. Thomas here in the text says : " Secunda vero operatio respicit ipsum esse rei . . ."

2. *Secunda vero operatio respicit ipsum esse rei.*

The difference between these two operations and that which they report about things is not without consequence for abstraction in general, and for mathematical abstraction in particular. St. Thomas brings this out by considering abstraction first in relation to

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1. The word ' nature ' (*natura*) appears throughout the course of this article. Since this too is a word of many significations, it is well to have in mind the sense in which it is used here. Its various meanings are discussed in several places by both Aristotle and St. Thomas. To highlight the meaning that is of present interest, the treatment in the response *ad quartum* of article one, q. 29 of the *Prima Pars* is sufficient. After briefly discussing, in order, some of its prior meanings, St. Thomas says : " Et quia per formam completur essentia uniuscuiusque rei, communiter essentia uniuscuiusque rei, quam significat eius definitio, vocatur natura."

A graduated schema of reality based on the perfections of forms from which definitions are derived can be thus established : " . . . Discurrenti per singula apparet unam speciem super aliam aliquem gradum perfectionis adiicere : sicut animalia super plantas, et animalia progressiva super animalia immobilia . . . Propter quod Aristoteles, in *VIII Metaph.*, dicit quod definitiones rerum sunt sicut numerus, in quo unitas subtracta vel addita speciem numeri variat : Per quem modum in definitionibus, si una differentia subtrahatur, vel addatur, diversa species invenitur." *Cont. Gent.*, II, c.95.

2. " . . . Intellectus humanus non statim in prima apprehensione capit perfectam rei cognitionem ; sed primo apprehendit aliquid de ipsa, puta quidditatem ipsius rei, quae est primum et proprium objectum intellectus ; et deinde intelligit proprietates et accidentia et habitudines circumstantes rei essentiam. Et secundum hoc, necesse habet unum apprehensum alii componere vel dividere ; et ex una compositione vel divisione ad aliam procedere, quod est ratiocinari." *Ia*, q.85, a.5, c.

the second operation. Since mathematical abstraction, as will be shown, belongs to the first operation, what is said here serves to manifest it negatively by showing what it is not.

In general, to abstract means to consider or to understand one thing without another. By abstraction we distinguish or understand one thing or one aspect apart from something else to which it may or may not be really united. Abstraction accounts for the devious ways in which the mind copes with the extreme complexity of objects confronting it. This mental process is not arbitrary. Nor is it fictive since every type of abstraction has its proper criterion based, at least remotely, on things outside the mind. In the case of the second operation, this criterion is things themselves in the way they are. In this operation which *respicit ipsum esse rei*, the mind can abstract truly only when it represents things as they are outside the mind. What, exactly, is meant by the affirmation that this second operation deals with the very being of the thing? If we hope to grasp the implications of this important article, the question cannot go unanswered.

In approaching this question, let us recall first of all that the immanence of knowledge mentioned earlier holds good for all three operations. Again by way of preparing the ground, a basic difference between cognitive and appetitive powers should be underscored. It is proper to the will, not the intellect, to incline towards things in their conditions outside the mind.[1] This much having been said, what then is the being (*esse*) in things which the intellect, in its own manner, attains by its second operation? Assuredly, it is not the ' act of existence,' as if, having grasped an essential nature, such as ' man,' by a prior apprehension, the intellect in this subsequent operation were to reveal that man exists the way Socrates does, and that the actuality of existence as thus found in the singular would be the subject of metaphysics. Appealing as it may be, the attempt to make of this operation the act of wisdom, as though by it alone we were able to extract what is most perfect in the real, is based on an assumption that is found nowhere in the teaching of St. Thomas and which is unsupported by experience.

That this last statement will be controverted is unavoidable. Our present intention is not to dispute but to discover what, in this context, St. Thomas means by *ipsum esse rei*. Actually we have not far to look. St. Thomas himself provides the straight forward answer :

Cum in re duo sint, quidditas rei, et esse ejus, his duobus respondet duplex operatio intellectus. Una quae dicitur a philosophis formatio, qua

1. ". . . Actus cognitivae virtutis perficitur per hoc quod cognitum est in cognoscente : actus autem virtutis appetitivae perficitur per hoc quod appetitus inclinatur in rem ipsam. Et ideo oportet quod motus appetitivae virtutis sit in res secundum conditionem ipsarum rerum : actus autem cognitivae virtutis est secundum modum cognoscentis." *IIa IIae*, q.27, a.4, c.

apprehendit quidditates rerum, quae etiam a Philosopho, in *III De Anima* dicitur indivisibilium intelligentia. Alia autem comprehendit esse rei, *componendo affirmationem*, quia etiam esse rei ex materia et forma compositae, a qua cognitionem accipit, consistit in quadam compositione formae ad materiam vel accidentis ad subjectum.[1]

*Ipsum esse* is the way a thing actually is, which results, in the case of natural things, from various kinds of composition, form and matter, substance and accident. It is the condition of the whole composite embracing all that it has rather than simply something which is had. To formulate the difference between this interpretation of *ipsum esse rei* and that of ' act of existence,' the former can be designated by ' mode of existence ' or simply the way things are, e.g., ' man *is* a rational animal ' which would be just as true even if there were no individual men.

The position espoused here brings us back to our earlier discussion of the intellect's different operations and therein finds its justification. In that discussion composition and division was recognized as the complement of apprehension inasmuch as it brings about the unification of the manifold concepts acquired in the first operation. Hence its function is not to replace the first operation by grasping aspects of the real that escape apprehension, but to organize its discontinuous knowledge. It reconstructs the original unity possessed by the thing in its mode of being outside the mind which, due to the weakness of our intellect, is not so grasped in a single operation, but only successively by operations differing in kind. To see, by contrast, how this unity can be known without a multiplicity of operations will help make the point.

" . . . [Angelus] non intelligit superaddendo praedicatum subiecto per modum compositionis et divisionis intellectus nostri, sed statim in simplici consideratione subiecti considerat ea quae subiecto conveniunt, vel quae ab eo removentur : Utriusque enim est eadem ratio, eo quod dispositio subiecti est principium cognoscendi inhaerentiam praedicati ad ipsum. Unde angelus per simplicem apprehensionem subiecti cognoscit esse vel non esse, sicut et nos componendo et dividendo.[2]

By its composition our intellect does not know more things ; rather it knows the same things as true or false. Although composition here implies a duality of subject-predicate, the predicate is as form to the subject, manifesting the aspect of inherence or existence in the subject.[3] The composition of the intellect is according to the

1. *In I Sent.*, dist. 38, q.1, a.3, sol.

2. *De Malo*, q.16, a.6, ad 19.

3. ". . . Intellectus noster diversas conceptiones format ad cognoscendum subiectum et accidens, et ad cognoscendum diversa accidentia ; et ideo discurrit de cognitione substantiae ad cognitionem accidentis, et iterum *ad hoc quod inhaerentiam unius ad alterum*

mode of the intellect, that is, according to the mode of identity, whose sign is the copula 'is.' Unlike composition in things themselves, wherein component parts are united in so far as they are different, the composition of the intellect is according to similitudes of the same thing.[1] And yet, that the intellect compose at all, or divide, is because of the composition in the things that are proportioned to our mind and the way that we come to know their truth. Exterior reality is the criterion whereby the validity of the intellect's composition is determined.[2] The repeated reference to composition in the foregoing discussion should not obscure the fact that the second operation is one of composition *and* division. "Esse autem, in quo consistit compositio intellectus, ut affirmatio, compositionem quamdam et unionem indicat : non esse vero, quod significat negatio, tollit compositionem, et designat pluralitatem et diversitatem."[3]

To return to abstraction, St. Thomas says that the mind in its second operation cannot truly abstract things unless that is the way they really are ; that is, if it were to represent as separate things which in reality are not separate the intellect would be in error. The immediate reason for this is that the adequation which defines truth is only accomplished when, by composing or dividing, the intellect enunciates that a thing *is* or *is not* such and such or so and so.

Similiter dico de veritate, quod habet fundamentum in re, sed ratio eius completur per actionem intellectus, quando scilicet apprehenditur eo modo quo est. Unde dicit Philosophus, *VI Metaph.*, quod verum et falsum sunt in anima ; sed bonum et malum in rebus. Cum autem in re sit

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*cognoscat*, componit alteram speciem cum altera, et unit eas quodammodo ; et sic in seipso enuntiabilia format." *De Ver.*, q.2, a.7, c.

1. ". . . Differt compositio intellectus a compositione rei ; nam ea quae componuntur in re, sunt diversa ; compositio autem intellectus est signum identitatis eorum quae componuntur. Non enim intellectus sic componit, ut dicat quod homo est albedo ; sed dicit quod homo est albus, idest habens albedinem : idem autem est subiecto quod est homo, et quod est habens albedinem." *Ia*, q.85, a.5, ad 3.

2. "Non enim ideo tu es albus, quia nos vere existimamus te esse album ; sed e converso, ideo existimamus te album, quia tu es albus. Unde manifestum est, quod dispositio rei est causa veritatis in opinione et oratione . . . Oportet enim veritatem et falsitatem quae est in oratione vel opinione, reduci ad dispositionem rei sicut ad causam. Cum autem intellectus compositionem format, accipit duo, quorum unum se habet ut formale respectu alterius : unde accipit id ut in alio existens, propter quod praedicata tenentur formaliter. Et ideo, si talis operatio intellectus ad rem debeat reduci sicut ad causam, oportet quod in compositis substantiis ipsa compositio formae ad materiam, aut eius quod se habet per modum formae et materiae, vel etiam compositio accidentis ad subiectum, respondeat quasi fundamentum et causa veritatis, compositioni, quam intellectus interius format et exprimit voce. Sicut cum dico, Socrates est homo, veritas huius enunciationis causatur ex compositione formae humanae ad materiam individualem, per quam Socrates est hic homo ; et cum dico, homo est albus, causa veritatis est compositio albedinis ad subiectum : et similiter est in aliis. Et idem patet in divisione." ST. THOMAS, *In IX Metaph.*, lect.11, nn.1897-1898.

3. *Ibid.*, n.1900.

quidditas eius et suum esse, veritas fundatur in esse rei magis quam in quidditate, sicut et nomen entis ab esse imponitur ; et in ipsa operatione intellectus accipientis esse rei sicut est per quamdam similationem ad ipsum, completur relatio adaequationis, in qua consistit ratio veritatis. Unde dico, quod *ipsum esse rei est causa veritatis, secundum quod est in cognitione intellectus.* Sed tamen ratio veritatis per prius invenitur in intellectu quam in re . . . Unde dico, quod verum per prius dicitur de veritate intellectus, et de enuntiatione dicitur inquantum est signum illius veritatis ; de re autem dicitur, inquantum est causa.[1]

Abstractions as performed in the first operation enjoy a certain freedom from the exigencies of truth. This freedom, as will appear shortly, is essential to mathematical abstraction. Truth, as the preceding citation states, is found first in the intellect, and more precisely, in the intellect composing and dividing, rather than in the apprehension of the first operation. The following text from the *De Veritate* explains most lucidly why this is so.

Veri enim ratio consistit in adaequatione rei et intellectus ; idem autem non adaequatur sibi ipsi, sed aequalitas diversorum est ; unde ibi primo invenitur ratio veritatis in intellectu ubi primo intellectus incipit aliquid proprium habere quod res extra animam non habet, sed aliquid ei correspondens, inter quae adaequatio attendi potest.

Intellectus autem formans quidditates, non habet nisi similitudinem rei existentis extra animam, sicut et sensus in quantum accipit speciem rei sensibilis ; sed quando incipit iudicare de re apprehensa, tunc ispsum iudicium intellectus est quoddam proprium ei, quod non invenitur extra in re. Sed quando adaequatur ei quod est extra in re, dicitur iudicium verum esse.

Tunc autem iudicat intellectus de re apprehensa quando dicit quod aliquid est vel non est, quod est intellectus componentis et dividentis ; unde et Philosophus dicit in *VI Metaph.*, quod compositio et divisio est in intellectu, et non in rebus.[2]

The composition and the division of the intellect is expressed by an *oratio* that signifies the truth, i.e., an enunciation, either affirmative or negative. Conformity, between the intellect and what is, obtains when an enunciation expresses what in fact is, or what in fact is not. However, if an enunciation asserts what is not or negates what is, the intellect is not in conformity with what is or is not, and its composition or division is false. If, to use the example of St. Thomas, I say ' man is not white,' what I signify in this negative enunciation is a separation made by my mind between man and whiteness. Since, in fact, there *are* white men, there is no conformity between my intellect and what is. If however, I say ' man is not an ass,' my intellect, in making this separation, is true because

1. *In I Sent.*, dist.19, q.5, a.1, c.
2. *De Ver.*, q.1, a.3, c.

man and ass are really separate. Accordingly, by this operation the intellect can abstract only those things which are really separate.

3. *The first operation has its criterion.*

By contrast, the first operation is less restricted inasmuch as it can abstract or separate in mind things which are not really separate. It cannot, however, do so indiscriminately. The reason is that anything, to be understood at all, must be intelligible in act. To know is to be another as other. ". . . Cognoscentia a non cognoscentibus in hoc distinguuntur, quia non cognoscentia nihil habent nisi formam suam tantum ; sed cognoscens natum est habere formam etiam rei alterius, nam species cogniti est in cognoscente." [1] To know is an actuality, a perfection. But it is a perfection that is proportioned to the one who knows. For this reason, it is more proper to say of the human intellect that to know is to become the other as other in a purely immaterial way. While our intellect has a capacity to know, it knows now in potency, now in act.[2] Since nothing acts as it is in potency but only in so far as it is in act, in order to know the intellect must be reduced from potency to act by something already actual.

The potency of the intellect is a potency for the actuality and the determination in things to which it is posterior and upon which it is dependent. Some of these things are most actual and, therefore, most knowable since, of themselves, they present no obstacle to the assimilative union that is knowledge. These are simple, wholly immaterial substances. So eminently intelligible are they, there can be no proportion between them and an intellect which, in order to know, is dependent upon and posterior to composite things. Even in regard to composite, sensible things which it knows naturally, there is an original disproportion between our intellect and its object. This disproportion, however, must be resolved if we are to know as we do.

Between the intellect and the sensible things it knows, a twofold relation obtains : one of act to potency, according to which the intellect is immaterial in act whereas sensible things existing in matter and its individuating conditions are intelligible in potency ; the other of potency to act, inasmuch as the intellect is in potency to the actual determination in sensible things. The intellect, accordingly, is compared to things to be known both as making them intelligible in act and as actually knowing them. Things are rendered intelligible when, through abstraction from individual matter and its conditions, they are assimilated to the immateriality of the intellect. To ac-

1. *Ia*, q.14, a.1, c.
2. *Ibid.*, q.79, a.2, c.

complish this it suffices that the intellect possess in act the basis of the similarity. We know in fact that there must be in us such a power of illumination. This power which is completely active, is called the agent intellect. To know actually, beyond assimilating many things to itself, the intellect must somehow be proportioned to them in order that it might have determinate knowledge of each. But this requires that it possess, in act, the forms of all these things. The nature of the intellect, however, is limited. Of itself unable to be assimilated to the natures of all the things it knows, it relies on the reception of something from outside itself. Owing to the aspect of passivity implied in this reception, the power in which it is accomplished is called the possible intellect. Through the joint cooperation of agent and possible intellect the disproportion obstructing the advance of knowledge is remedied.

. . . Cum intellectus possibilis sit in potentia ad intelligibilia, necesse est quod intelligibilia moveant intellectum possibilem. Quod autem non est, non potest aliquid movere. Intelligibile autem per intellectum possibilem non est aliquid in rerum natura existens, in quantum intelligibile est ; intelligit enim intellectus possibilis noster aliquid quasi unum in multis et de multis. Tale autem non invenitur in rerum natura subsistens, ut Aristoteles probat in *VII Metaphys.* Oportet igitur, si intellectus possibilis debet moveri ab intelligibili, quod huiusmodi intelligibile per intellectum fiat. Et cum non possit esse id quod est, in potentia ad aliquid factum ipsius, oportet ponere praeter intellectum possibilem intellectum agentem, qui faciat intelligibilia in actu, quae moveant intellectum possibilem. Facit autem ea per abstractionem a materia, et a materialibus conditionibus quae sunt principia individuationis. Cum enim natura speciei, quantum ad id quod per se ad speciem pertinet, non habeat unde multiplicetur in diversis, sed individuantia principia sint praeter rationem ipsius ; poterit intellectus accipere eam praeter omnes conditiones individuantes ; et sic accipietur aliquid unum.[1]

Sensible things then, existing in their proper nature, are incapable of acting on the intellect. They are intelligible only in potency. They can become intelligible in act through the intellect's ability to abstract from the phantasm the *what* of the sensible singular apart from the individuating conditions which render the singular a mere instance of that *what*. This may be called the *asbstrahibilitas* of the sensible nature.[2]

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1. *Q. D. de Anima*, q.un., a.4, c. See also a.5 of the same question ; *Cont. Gent.*, II, c.77 ; *Ia*, q.54, a.4, c. ; *Ia*, q.79, a.3.

2. " Phantasma actu quidem habet similitudinem determinatae naturae ; sed illa similitudo determinatae speciei est in phantasmate in potentia *abstrahibilis* a materialibus conditionibus. In parte vero intellectiva est e converso ; nam non habet actu similitudines distinctarum rerum ; sed tamen actu habet lumen immateriale habens virtutem abstrahendi quae sunt *abstrahibilia* in potentia." *De Spir. Creat.*, a.10, ad 4.

Although only potentially intelligible, sensible things have an actuality of their own independently of the intellect. If they did not, they would not be knowable *qua* sensible in any way. They have this actuality through their form.

> Illa enim [similitudo] quae est in intellectu nostro, est accepta a re secundum quod res agit in intellectum nostrum, agendo per prius in sensu ; materia autem, propter debilitatem sui esse, quia est ens in potentia tantum, non potest esse principium agendi ; et ideo res quae agit in animam nostram agit solum per formam. Unde similitudo rei quae imprimitur in sensum, et per quosdam gradus depurata, usque ad intellectum pertingit, est tantum similitudo formae.[1]

That which of itself is not actual can offer no determination to the intellect and hence can only be known as related to what is actual. It is through its natural forms that prime matter is somehow knowable.[2]

' Things act upon the intellect,' ' the intellect receives its knowledge from things,' ' the possible intellect is moved by the intelligible,' these, and similar expressions, can give rise to a good deal of confusion unless correctly understood. It is true that *intelligere est quoddam pati*, but passivity is understood here in a very wide sense to mean the reception by the intellect of its intelligible species. But this is something prerequisite to the act of intellection and not intellection itself.[3] To know, our intellect requires a twofold information : one entitative and physical, according to which the intelligible species and the act of intellection are present in the intellect as accidents in a subject ; the other, intentional, by which the intellect becomes the thing known in an immaterial way. The first, while absolutely necessary, is a condition ancillary to the second which, formally speaking, constitutes the act of knowledge.[4] Knowledge, because of immateriality, is an immanent act perfecting the principle whence it proceeds. It is not a passion undergone, nor a perfection received from without. It is not even something which the agent performs upon itself.[5] So miscontrued, knowledge would be of the effects that things cause in the knower but not of things themselves. The determination which the thing known affords the act of intellection, as such, is in the line of formal, not efficient, causality—the intellect in act is

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1. *De Ver.*, a.2, a.5, c.

2. St. Thomas, *In I Phys.*, lect.13, n.9.

3. " . . . Moveri ab obiecto non est de ratione cognoscentis inquantum est cognoscens, sed inquantum est potentia cognoscens." *Ia*, q.56, a.1, c.

4. Cf. *De Ver.*, q.8, a.6, c.

5. As perfecting the intellect by securing its object, intellection is an act that pertains to the predicamental genus of quality rather than of action. Cf. John of St. Thomas, *Cursus Philosophicus*, t.III, q.11, a.1.

the thing understood in act. Form exercises its causality by perfecting that which it informs : ". . . forma inquantum huiusmodi, habet esse in perficiendo illud in quo est, et quiescendo in ipso." [1] Since the intelligible species of the intellect is the representative species of the thing itself understood in act, *intelligens* and *intellectum* constitute one principle (*unum quid*) of the act of understanding. This recalls a text previously cited : " Nam intelligere importat solam habitudinem intelligentis ad rem intellectam ; in qua nulla ratio originis importatur, sed solum informatio quædam in intellectu nostro, prout intellectus noster fit in actu per formam rei intellectæ." [2] The possibility then of the intellect's attaining an intelligible nature independently of something else to which it is conjointed depends upon the actuality which that nature possesses. " . . . Et hoc est illud, ex quo unaquæque natura suam rationem sortitur."

In the preceding article, St. Thomas used the word *ratio* to designate a nature signified by its definition in the sense that the form from which the definition is taken is called *ratio*. Here, it seems, he uses the word according to a prior imposition signifying the definition itself. "Nam ratio quam nomen significat est definitio rei." [3] The change in meaning is understandable. In article two there was question of that which is known. Here, in considering abstraction, the emphasis is on the way or the manner in which something is known. This becomes clearer when, from what he has just established as a requisite to definition, St. Thomas arrives at the following conclusion : " Quando ergo secundum hoc, per quod constituitur ratio naturæ et per quod ipsa natura intelligitur, natura ipsa habet ordinem et dependentiam ad aliquid aliud, tunc constat quod natura illa sine illo alio intelligi non potest . . ."

When the actuality that a nature must have in order to be understood implies an order to or a dependence upon something else, the intellect in defining that nature cannot abstracted from this ' something else.' It is to be noted again that the definable nature may be complete, as a whole, or incomplete, as a part or an accident. Even though only the intellect is capable of recognizing it, this essential order or dependence obtains independently of the intellect's consideration.

St. Thomas verifies this conclusion by applying it to the various ways according to which things can be conjoined one to another. A nature can be joined to something else as part to whole, or as form to matter. The verification of the conclusion in regard to things conjoined as part to whole is exemplified by what a foot is. A foot

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1. *De Ver.*, q.2, a.14, c.
2. *Ia*, q.34, a.1, ad 3.
3. St. Thomas, *In IV Metaph.*, lect.16, n.733.

cannot be defined unless animal be included in its definition. The reason is that ' what a foot is ' is to be part of an animal. A foot that is not the foot of an animal is so called only by equivocation. A foot is defined by its function : an organic member providing the power to walk.[1]

Applying the same conclusion to things united as form to matter, St. Thomas gives an example of the union between accident and subject. The curve of the snubnose, being a sensible accident, can only be defined by including its subject, namely, flesh. " Simum enim nihil aliud est quam nasus curvus vel concavus."[2] The conclusion can further be applied in regard to things existing separately from each other. The relations of paternity and filiation exist in distinct persons and yet, father as father can only be defined in relation to child, and child in relation to father.

Inversely, when a nature has actual intelligibility without dependence upon something else, it can be abstracted by the intellect so as to be understood without this other. This holds good not only when the intellect apprehends one thing without understanding something else from which it is really separate, but is also true of things actually conjoined. In the case of things conjoined as whole and part, the intellect can, at times, know the part without knowing the whole. For instance, it can grasp a letter without the syllable of which the letter is an element. The converse, however, is not true. Sometimes the intellect can grasp the whole without the part as, for example, animal can be understood without foot, but as seen above, the converse is not true. Whiteness as an accident in man is such that it can be known without understanding man, and man is understood without it. It is not necessary to be white to be a man, nor are men all the white things there are.

Having examined each of the operations, St. Thomas goes on to determine the precise meanings of abstraction. Since this consists in restricting the common meaning, viz., to know one thing apart from something else, he calls this " distinction"[3] to avoid confusion with the proper signification he is about to establish. Thus he says

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1. " . . . Quia destructo toto homine, non remanet pes neque manus nisi aequivoce, eo modo quo manus lapidea posset dici manus. Et hoc ideo, quia talis pars corrumpitur corrupto toto. Illud autem, quod est corruptum, non retinet speciem, a qua sumitur ratio definitiva. Unde patet, quod non remanet eadem ratio nominis, et sic nomen aequivoce praedicatur. Et quod pars corrumpatur corrupto toto, ostendit per hoc quod omnis pars definitur per suam operationem, et per virtutem qua operatur. Sicut definitio pedis est, quod sit membrum organicum habens virtutem ad ambulandum. Et ideo, ex quo iam non habet talem virtutem et operationem, non est idem secundum speciem sed aequivoce dicitur pes." St. Thomas, *In I Pol.*, lect.1, n.38.

2. St. Thomas, *In VI Metaph.*, lect.1, n.1157.

3. " . . . In ratione distinctionis est negatio : distincta enim sunt quorum unum non est aliud." *Cont. Gent.*, I, c.71.

that the intellect in the first operation distinguishes one thing from another differently than it does in the second operation. Distinction is accomplished in the second operation when the intellect understands that one thing is *not* in another. This distinction asserts that, according to the way things are, what is signified by the predicate in an enunciation does not belong to or is not in that which is signified by the subject. In the first operation the intellect distinguishes one thing from something else by understanding what the one is while understanding nothing of the other. That is, the intellect does not consider the one *to be* without the other but simply considers the one and not the other. When the two, of which one is considered without the other, are actually united, the operation which so distinguishes them is properly called abstraction. Thus abstraction properly so called pertains to the first operation of the intellect. It supposes the union of things, one of which is understood without the other. It likewise presupposes that the one so understood is notionally independent from the other. Distinction in the second operation, wherein things are considered not merely separately but as actually separate, is rightly called separation.

4. *Abstraction and separation as employed by the sciences.*

Given the difference between separation and abstraction properly so called, the original difficulty is not thereby solved. Yet the direction of its resolution is indicated. It remains to verify the significance of these precisions in regard to science. The role of distinction in science is to aid the intellect in attaining the intelligible aspect of things. In our attempt to know reality the first contact with it is had in sense experience whose objects are mixed with matter and the conditions of matter. But matter and its conditions are the causes or the roots of unintelligibility. Hence the ' something else ', the ' other ' from which we abstract is matter. The different kinds of distinction or abstraction are all in the line of mental exclusion or separation from matter as an obstacle to intelligibility. When the mathematician considers without sensible matter a curve existing in sensible matter he abstracts. Actually what is thus understood is inside the mind in a way different from what it is in sensible matter. In spite of this difference, the mathematician is exempt from falsity since in abstracting he does not affirm that what is abstracted is still that way outside the mind. He restricts his consideration to what is abstracted and neglects that from which it has been abstracted. The natural scientist also abstracts, for in his consideration he disregards the individuating conditions that attend the nature as it actually exists. He does not assert that man, as defined with common sensible matter, is that way outside the mind.[1]

1. " Non est autem possibile, quod abstrahatur a materia individuali realiter, sicut Platonici posuerunt. Non enim est homo naturalis, id est realis, nisi in his carnibus, et

Both the mathematician and the naturalist avail themselves of this ability of the mind to abstract. In this they are similar. But that does not mean that mathematical abstraction and that of natural philosophy are identical. It is true that in each science certain things are known apart from certain other things to which they are actually conjoined. But that which is known is not at all the same in both cases. In the one, things so understood are defined with sensible matter, whereas in the other, they are known without sensible matter. This difference on the part of what is known (*id quod intelligitur*) stems from a difference on the part of the intelligible species (*id quo intelligitur*) which render the intellect in act, similar to the thing to be known.[1] Sometimes the species is a similitude of something as existing outside the mind, and sometimes a similitude of that which is the result of the intellects' leaving aside something of the thing that is outside the mind.

Sic etiam et intellectus intelligit lineam in materia sensibili existentem, absque materia sensibili : licet et cum materia sensibili intelligere possit. Haec autem diversitas accidit secundum diversitatem specierum intelligibilium in intellectu receptarum : quae quandoque est similitudo quantitatis tantum, quandoque vero substantiae sensibilis quantae.[2]

That there is question here of radically different abstractions is borne out by the fact already noted that the definitions in natural science, which are of universals, are applicable to the things from which they have been abstracted, whereas mathematical definitions are not. Natural definitions are applicable to existent singulars known through sense experience.

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in his ossibus, sicut probat Philosophus in *Septimo Metaphysicae*. Relinquitur igitur, quod natura humana non habet esse praeter principia individuantia, nisi tantum in intellectu. Nec tamen intellectus est falsus, dum apprehendit naturam communem praeter principia individuantia, sine quibus esse non potest in rerum natura. Non enim apprehendit hoc intellectus, scilicet quod natura communis sit sine principiis individuantibus ; sed apprehendit naturam communem non apprehendendo principia individuantia ; et hoc non est falsum." St. Thomas, *In II de Anima*, lect.12, nn.378-379.

1. " Habet se igitur species intelligibilis recepta in intellectu possibili in intelligendo sicut id quo intelligitur, non sicut id quod intelligitur . . . Id vero quod intelligitur, est ipsa ratio rerum existentium extra animam." *Cont. Gent.*, II, c.75.

Sometimes, however, to distinguish this *quod*, which is an intention formed within the mind, from the thing itself as existing outside the mind St. Thomas calls the former a *quo* also.

" . . . In intellectu speculativo videmus quod species, qua intellectus informatur ut intelligat actu, est *primum quo* intelligitur ; ex hoc autem quod est effectus in actu, per talem formam operari iam potest formando quidditates rerum et componendo et dividendo ; unde ipsa quidditas formata in intellectu, vel etiam compositio et divisio, est quoddam operatum ipsius, per quod tamen intellectus venit in cognitionem rei exterioris ; et sic est *quasi secundum quo intelligitur*." *De Ver.*, q.3, a.2, c. Cf. *Cont. Gent.*, I, c.53.

2. *Cont. Gent.*, II, c.75.

5. *The existence of mathematical subjects.*

The absence of any such verification in experience seems to undermine the very foundation of mathematics. One is prompted to ask : Do mathematical subjects exist? If the question refers to the way natural things exist, the answer is negative. Are they pure fictions? The answer again is negative. What then is the value of its alleged definitions? Are its assertions true? When the natural scientist defines man as ' rational animal,' with all that this implies, he can point to an actual man to illustrate what he is talking about. His definition is of something that has individual instances in nature, and what he demonstrates of man in virtue of his definition will be true of every instance. And if we started from centaur, i.e., half man, half horse, we would have to ascertain that there is or is not, could be or not, such a thing in nature. But for the geometer defining circle as ' a closed plane curve such that its circumference is at every point equidistant from the point within called its center,' there is no need to show, nor could there be, that there are instances of that figure in nature.

Although the geometer cannot and need not enlist the aid of experience to verify the kind of subject he is talking about, he does not improvise since he does guarantee, through construction, that there are such things in abstraction ; this he does, for example, when from some elements of his science given in abstraction, he demonstrates that there is such a thing as an equilateral triangle. When this subject, and not merely the meaning of its name, is thus positively known, then and then only, can he proceed to demonstrate its properties.[1]

But in what sense does the geometer prove that there *is* a triangle with three equal sides? What does ' is ' (as well as ' being ' and ' existence ') mean in relation to the subjects of mathematics? Certainly it does not have the same meaning that it has in regard to this man when it is said ' John Smith is.' In this latter case ' is ' signifies that an individual called John Smith exists in nature. Though this latter meaning is the one which we are first inclined to think of, it is not the only one. In evidence of this, it need only be recalled that animal *is* a genus and that this is true even though neither animal nor genus as such exists in nature. Here ' is ' signifies the truth of a composition made by the mind, i.e., ' is ' means that the proposition is true. ' Is ' or ' to be ' meaning that something is true is a sense that is subsequent to the same words understood of what is or can be in nature.[2]

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1. Cf. St. Thomas, *In I Post. Anal.*, lect.2, n.5.

2. " Ponit alium modum entis, secundum quod *esse* et *est*, significant compositionem propositionis, quam facit intellectus componens et dividens. Unde dicit, quod esse significat veritatem rei. Vel sicut alia translatio melius habet ' quod esse significat ' quia aliquod dictum est verum . . . Sciendum est autem quod iste secundus modus comparatur

Yet this in no way diminishes its validity, nor does it prevent its having a wider application than does the prior signification. Taken in the sense of what is true, being can be said of anything about which a true proposition can be formulated. In this sense it is opposed, not to what is in nature, but to what is false. Thus blindness can be said *to be* from the fact that the proposition ' some men *are* blind ' is true, whereas ' the diagonal of the square *is not* commensurate with the side ' means that it is false to say that it is. Propositions can be formed not only about things which *are* in nature, but also in regard to relations discovered in knowing beings in this first sense, privations and negations, or even fictions — anything that the mind can in some way consider.

. . . Secundum Avicennam, tract. *II Metaph.*, cap. 1, de eo quod nullo modo est, non potest aliquid enuntiari ; ad minus enim oportet quod illud de quo aliquid enuntiatur, sit apprehensum ; et ita habet aliquod esse ad minus in intellectu apprehendente ; et ita constat quod semper veritati respondet aliquod esse ; nec oportet quod semper respondeat sibi esse in re extra animam, cum ratio veritatis compleatur in ratione animae.[1]

While truth is in the intellect, the intellect does not cause it. The intellect alone is not responsible for the truth of the propositions that it can form. Their truth is caused by the things (in the wide sense of the term) that the intellect knows in so far as it is conformed to them. " Rationes autem intellectæ habent duplicem firmitatem : scilicet firmitatem sui esse, et hanc, habent ab intellectu, sicut alia accidentia a suis subjectis ; et firmitatem suæ veritatis, et hanc habent ex re cui conformantur. Ex eo enim quod res est vel non est, locutio et intellectus veritatem vel falsitatem habet." [2] What the mind apprehends of blindness is obviously quite different from what it understands of something which is a mere fiction of the imagination. Hence when it said, and truly, that ' blindness *is* a privation ' and that ' golden mountain *is* a fiction,' the truth of these propositions is not of the same kind. They are true on condition that *to be* as said of privation and fiction does not mean the same way. *A fortiori* must this be realized of the true propositions about things that are or that

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ad primum sicut effectus ad causam. Ex hoc enim quod aliquid in rerum natura est, sequitur veritas et falsitas in propositione, quam intellectus significat per hoc verbum *Est* prout est verbalis copula. Sed, quia aliquid, quod est in se non ens, intellectus considerat ut quoddam ens, sicut negationem et huiusmodi, ideo quandoque dicitur esse de aliquo hoc secundo modo, et non primo. Dicitur enim, quod caecitas est secundo modo, ex eo quod vera est propositio, qua dicitur aliquid esse caecum ; non tamen dicitur quod sit primo modo vera. Nam caecitas non habet aliquod esse in rebus, sed magis est privatio alicuius esse." ST. THOMAS, *In V Metaph.*, lect.9, nn.895-896.

1. *In I Sent.*, dist.19, q.5, a.1, ad 5.

2. *In I Sent.*, dist.2, q.1, a.3, ad 5.

have at least a foundation in nature. It is in reference to the latter that we speak of the existence, in the sense of truth, of mathematical subjects.[1] In virtue of a demonstration the geometer proves that there *is* an equilateral triangle. The force of his proof lies in the fact that he actually constructs such a triangle. This construction is a technique which derives from the nature of the human mind and the way things are in the mind. The mathematician is not concerned with construction for the sake of constructing, but for the sake of producing a subject about which he can demonstrate properties. Granted that without such a construct the nature would not be known, existence in the mind is not therefore part of the definition of triangle. The mind does not form the nature of equilateral triangle. " Speculativarum vero scientiarum materiam oportet esse res quæ a nostro opere non fiunt . . ." [2] Starting from particulars, existing in individual and common sensible matter, as from a remote principle,[3] the intellect is able by abstraction, to attain abstract quantity from whence it effects its construction, as in the example used here of triangle. Triangle thus revealed as a definable nature has actuality, but actuality that follows upon the way it is known and which is impossible of verification in sensible experience.

Though both mathematics and natural philosophy abstract, to appreciate the essentially different ways in which these abstractions are performed is to recognize how mathematics can consider, without sensible matter, things which are in matter in such a way that the mathematician is not obliged to maintain that what is so separated in mind can, or should, be also separable in reality. Since abstraction properly so called refers to things which are conjoined, the difference in the two modes of abstraction corresponds to the two different modes of union mentioned above, viz., the union of whole and part and the union of matter and form. Hence the distinction, previously

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1. " . . . Ipsa conceptio intellectus tripliciter se habet ad rem quae est extra animam.

[*a*] Aliquando enim hoc quod intellectus concipit, est similitudo rei existentis extra animam, sicut hoc quod concipitur de hoc nomine ' homo ' ; et talis conceptio intellectus habet fundamentum in re immediate . . .

[*b*] Aliquando autem hoc quod significat nomen non est similitudo rei existentis extra animam, sed est aliquid quod consequitur ex modo intelligendi rem quae est extra animam ; et huiusmodi sunt intentiones quas intellectus noster adinvenit ; sicut significatum huius nominis ' genus ' . . . et huiusmodi intentionis licet proximum fundamentum non sit in re, sed in intellectu, tamen remotum fundamentum est res ipsa . . . Et simile est de omnibus aliis qui consequuntur ex modo intelligendi, *sicut est abstractio mathematicorum et huiusmodi.*

[*c*] Aliquando vero id quod significatur per nomen, non habet fundamentum in re, neque proximum, neque remotum, sicut conceptio chimerae : quia neque est similitudo alicuius rei extra animam, neque consequitur ex modo intelligendi rem aliquam vere." *In I Sent.*, dist.2, a.1, a. 3, c.

2. St. Thomas, *In Boeth. de Trin.*, q.5, a.1, c.

3. St. Thomas, *In I Post. Anal.*, lect.30, n.5.

alluded to, between the abstraction of the universal from the particular and the abstraction of form from matter. Considering first the abstraction of form from matter, St. Thomas observes that the intellect can consider form separately from matter only in so far as this matter is not required for the definition of such a form. Again the terms ' matter ' and ' form ' can be a source of confusion unless it be realized that they are used analogously. What St. Thomas is saying is that form can be abstracted from some matter but it cannot be abstracted from all matter. He goes on to distinguish sensible and intelligible matter.

6. *Sensible and intelligible matter.*

All accidents are related to substance as form to matter, as determining to the determinable. But since an accident cannot be defined without its subject,[1] no accident as form can be abstracted from substance as from matter. An accident cannot be considered or conceived as though it were not an accident. " Passionis autem esse et cuiuslibet accidentis est inesse subjecto." [2] However, accidents befall substance in a certain order ( of nature but not in time). Quantity inheres in substance prior to quality, and after quality come action and passion, etc. Quantity can then be grasped as a form in matter prior to understanding such matter as the subject of the sensible qualities by reason of which it is called *sensible matter*. Quantity can be grasped without sensible matter but not without substance. Substance, as necessary to the understanding of quantity, is called intelligible matter. Being the subject of quantity, known as prior to the sensible qualities that attend it in reality, substance can only be attained by the intellect. It is quantity thus abstracted from sensible matter but not from intelligible matter that constitutes the subject of mathematics. Or, in the words of St. Thomas, mathematics considers *quantities*, i.e., both discreet and continuous, as well as certain qualities that are consequent to quantity.[3]

Perhaps the subjects of mathematics can be clarified by investigating that from which they are first abstracted, i.e., sensible matter. Sensible matter has been explained above in reference to the senses as the subject of sensible qualities ; it is that which is apprehended as the subject of qualities which affect the senses in such a way that these qualities are called sensible *per se*, either proper or common. This subject is not attained by the senses *per se*. It is said to be sensible only *per accidens* because it is apprehended by the mind while *per se* sense experience occurs. As subject of the sensible qualities of a

1. St. Thomas, *In II de Anima*, lect.1, n. 213.

2. St. Thomas, *In I Post. Anal.*, lect.2, n.3.

3. Cf. St. Thomas, *In V Metaph.*, lect.16, nn.989-992.

material being, sensible matter is not the whole material being. That is, it is not the same as substance in every way. Rather it is material substance *qua* subject to sensible accidents. It is that of which the thing is made, e.g., the flesh and bones of man.

Quantity, when understood as prior to sensible qualities, is likewise understood as prior to the *per accidens* sensible subject of these qualities and is related to it as accident to subject only in the sense of that from which it is abstracted. Sensible matter is not part of what mathematics abstracts ; it is the subject that is abstracted from. Such abstraction does not leave aside the subject *qua* subject of quantity, but *qua* subject of sensible qualities. Taken as sensible subject, i.e., sensible matter, it is subject of both sensible qualities and quantities. But this is not the business of the mathematician.

> . . . Materiale dicitur non solum id, cuius pars est materia, sed etiam illud, quod in materia esse habet, secundum quem modum linea sensibilis materiale quoddam dici potest. Unde per hoc non prohibitur quin linea sine materia intelligi possit. Non enim materia sensibilis comparatur ad lineam sicut pars, sed magis sicut subiectum, in quo esse habet, et similiter est de superficie et corpore.[1]

If mathematics, as distinguished from natural philosophy, abstracts from sensible matter, it does not abstract from all matter. ". . . Mathematici non abstrahunt ab omni materia sed a sensibili tantum, ut in *VIII Metaphys.*, text. 2, 3, et 15. Unde non oportet, ubicumque est materia, quod sit motus . . ." [2] The matter from which mathematics does not and cannot abstract is intelligible matter. What is intelligible matter? In the expression of St. Thomas, it is substance *qua* subject to quantity : " materia vero intelligibilis dicitur substantia secundum quod subiacet quantitati." [3] Substance so considered is likewise related to quantity as matter to form but in a different way than sensible matter is. Intelligible matter is attained by the mind, without proximate dependence upon sense experience. Due to the common source of all knowledge, mathematical quantity is remotely dependent on what is known in sensation, including sensible quantity. But it is not a mere elaboration or refinement of the latter. Sensible quantity is more a term or measure of natural bodies than pure dimension. That is, it is the quantity of something known through sensation. Now mathematical quantity does not result by simply leaving aside that 'something.' It must be apprehended by a totally different approach. Impervious to the senses, abstract quantity is comprehensible to the intellect inasmuch as the intellect is capable of recognizing its natural priority to other acci-

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1. St. Thomas, *In Boeth. de Trin.*, q.5, a.2, ad 2.
2. *In II Sent.*, dist.2, q.2, a.2, ad 4.
3. *Ia*, q.85, a.1, ad 2.

dents. But even when considered as prior to other accidents, abstract quantity does not cease to be an accident and cannot be conceived without substance as though it were not an accident. Quantity is the order of the part of material substance and hence cannot be considered apart from that of which it is the order.

Just as quantity considered as such is dependent upon intelligible matter to be understood, so likewise are its species. "Aliae vero scientiae mathematicae de nulla substantia considerant, sicut de arithmetica patet, quae est circa numeros, et de geometria quae circa magnitudines est : numerus autem et magnitudo, sunt accidentia." [1] In geometry, whose subject is magnitude, all definitions include the continuum as intelligible matter. This may be seen in the particular subjects as attained by way of construction, viz., the sides of the triangle or the surfaces and depth of the cube. A proportion can be established between intelligible matter in mathematics and sensible matter in nature. Allowing for their radical differences, we can say that a curve is to mathematics what snubnose is to nature. " Rectum enim mathematicum est, simum autem naturale. Ratio enim recti est cum continuo, sicut ratio simi cum naso. Continuum autem est materia intelligibilis, sicut simum materia sensibilis." [2] As in natural philosophy, the definitions of mathematics contain more than that which is in the mode of form. They also include intelligible matter because the " what it is " of mathematical subjects is dependent upon something more than form.[3]

Not only do the *mathematicalia* have their form in matter, but there can also be individuals of the same species, e.g., many equal lines, many circles. These numerically different individuals can be identified by a symbol. This means that there is a principle of individuation in mathematics, analogous to that in nature, which distinguishes individual *mathematicalia* from that which is directly signified by the definition.

Ratio autem huius est, quia materia, quae principium est individuationis, est secundum se ignota, et non cognoscitur nisi per formam, a qua sumitur ratio universalis. Et ideo singularia non cognoscuntur in sua absentia nisi per universalia. Materia autem non solum est principium individuationis in singularibus sensibilibus, sed etiam in mathematicis.[4]

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1. St. Thomas, *In XII Metaph.*, lect.9, n.2563.

2. St. Thomas, *In III de Anima*, lect.8, n.714.

3. " Unde sive in sensibilibus, sive in mathematicis, semper oportet quod sit in definitionibus aliquid quasi materia et aliquid quasi forma. Sicut in hac definitione circuli mathematici, Circulus est figura superficialis, superficies est quasi materia, et figura quasi forma. Eadem est ratio quare definitio mathematica est una, et quare definitio naturalis . . . quia utrobique alterum est sicut materia, et alterum sicut forma." St. Thomas, *In VII Metaph.*, lect.5, n.1761.

4. St. Thomas, *In VII Metaph.*, lect.10, n.1496.

In order to consider without sensible matter things that are in sensible matter, mathematics abstracts a form both from individual and common sensible matter, and also from individual intelligible matter — for mathematics as such does not deal with this circle A, but merely uses it, and any *this* would make do. From the preceding it should be clear that form here does not mean substantial form. Substantial form demands its appropriate matter. Just as matter can neither be nor be understood without form, knowledge of the substantial form would require knowledge of the proper matter of which such a form is the act. Form in mathematics is the accidental form of quantity.[1]

7. *Abstractio totius : partes speciei et partes materiæ.*

The abstraction that is common to all the sciences, and which natural philosophy employs in its own fashion, is the abstraction of the universal from the particular. It corresponds to things united as whole and part. The terms ' whole ' and ' part ' are essentially correlative. And just as form cannot be abstracted from all matter, so that which is as a whole cannot be abstracted in natural philosophy from all things which are related to it as parts. In *Metaphysics V*, Aristotle explains the various meanings of the word ' part.'[2] It first means that into which a thing is divided according to quantity, and this in two ways : whatever smaller quantity into which a greater quantity is divided is called a part ; or the smaller quantity is called a part of the greater only when it is in some way a measure of the latter. The name ' part ' is thence applied to that into which something is divided apart from quantity, as species are called parts of the genus. ' Part ' has a further meaning of that into which a whole is divided or of which it is constituted, the whole being either a species or that which has the species. Aristotle goes on to give a fourth meaning, i.e., the part of a definition. But it is the third meaning that is of interest to us. This third sense of part implies two kinds of whole : either the species itself, and corresponding to it, its proper parts, as triangle has three sides ; or the species as in an existing subject (*habens speciem*) with its parts, as semicircle is a part of this circle. In commenting on the previous chapter where Aristotle distinguishes the ways in which one thing is said to come from another, St. Thomas designates these parts as parts of the species and parts of the matter respectively.

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1. Although an accident, quantity as the foregoing has shown, is a unique accident and it is this uniqueness that guarantees the authenticity of mathematics : " . . . Quantitas dimensiva secundum suam rationem non dependet a materia sensibili, quamvis dependeat secundum suum esse ; ideo in praedicando et subjiciendo accipit modum substantiae et modum accidentis ; unde lineam dicimus et quantitatem et quantam, et magnitudinem et magnam." *In IV Sent.*, dist.12, q.1, a.1, qu[a] 3, ad 2.

2. ARISTOTLE, *Metaph.*, V, c.25, 1023 *b* 11-25.

Sunt enim partium, quaedum partes speciei, et quaedam partes materiae. Partes quidem speciei dicuntur, a quibus dependet perfectio speciei, et sine quibus esse non potest species. Unde et tales partes in definitione totius ponuntur . . . Partes vero materiae dicuntur ex quibus species non dependet, sed quodammodo accidunt speciei . . . Unde huiusmodi partes non ponuntur in definitione totius speciei, sed potius e converso . . .[1]

The whole which is the species cannot be abstracted from those parts which are parts of the species because the ' what it is ' of such a whole is to be composed of these parts. When defining the whole these parts must be considered, as letters in relation to a syllable. Material parts, on the other hand, do not make up the whole as such but are merely incidental. " Quaedam vero partes sunt quae accidunt toti, inquantum huiusmodi . . ." It is accidental to the circle as such that it have parts such as two equal semicircles. A circle can be constructed and understood without the intervention of semicircle, for circle enters into the definition of half-circle, and not vice versa ; this is not the case of the three straight lines which are essential to the triangle. The independence of the circle from semicircle is explained by the fact that semicircle is not a part of the species as such but is a part of the subject in which the species is received.

Sed incisiones circuli sunt partes non circuli secundum speciem accepti, sed huius circuli particularis, vel horum circulorum, sicut materia in qua fit species circuli . . . Quod autem circulus sit actu divisus in semicirculos, hoc accidit circulo, non inquantum est circulus, sed inquantum est hic circulus, cuius haec linea dividitur quae est pars eius ut materia. Unde patet, quod semicirculus est pars circuli secundum materiam individualem.[2]

What is true of the letter in relation to the syllable and lines as parts of the triangle is true also of rational soul and organized matter in regard to man, i.e., these parts belong *per se* to man. Parts like finger, foot or hand are not essential to the definition and so man can be understood without them, though incompletely, for, as St. Thomas says, these parts are *post intellectum hominis* which reaffirms what was said previously about the definition of foot, i.e., what a foot is, is to be part of an animal.

Prior to his consideration of the various meanings of the word ' part,' Aristotle discusses the ways in which one thing is said to come from another *proprio* and *primo*.[3] The third way enumerated is at first baffling inamuch as parts are said to come from the whole, as a

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1. St. Thomas, *In V Metaph.*, lect.21, n.1089.
2. St. Thomas, *In VII Metaph.*, lect.9, nn.1474-1475.
3. Aristotle, *Metaph.*, V, c.24, 1023 *a* 25ff.

verse is said to come from the *Iliad* or a stone from a house. In his commentary St. Thomas points out that this is *in via resolutionis*. Because it possesses the form as the term of generation, the whole is, therefore, a perfection in relation to the parts. The *via resolutionis* is a resolution of the perfect whole into parts that are what they are in dependence upon the whole. ". . . Et hae partes dicuntur partes materiae, quae non ponuntur in definitione totius, sed magis e converso . . .' This citation from the *De Trinitate* should warn against identifying material parts, also called parts of the individual, with the individual as such. Although they require the whole for their definition, these parts can be defined while the individual cannot. In conjunction with the whole, material parts can have actual intelligibility (as in the case of semi-circle) while the individual as material individual can be no more than potentially intelligible. Material parts are so called because they are not understood as constitutive of the whole such as a species, meaning that they are not parts of the definition, i.e., of what the *definitum* is. Plato and Socrates are material parts of the species man, and so are their parts, such as their flesh and bones.

Material parts are thus accidental to the species, whether these parts be universal, as semicircle in relation to circle, or individual, as this flesh and this bone in relation to man. For this man Socrates to exist, he must possess this soul and this body of these bones and this flesh. And if it were possible to define Socrates, these parts would be part of his definition. But these parts, as parts of this man, add nothing to the species itself since it is accidental that the species be in this particular man rather than in another. Hence the intellect can abstract the species from these parts. This is the abstraction of the universal from the particular.

St. Thomas summarizes abstraction properly so-called in the following way :

> Et ita sunt duae abstractiones intellectus. Una quae respondet unioni formae et materiae vel accidentis et subiecti, et haec est abstractio formae a materia sensibili. Alia quae respondet unioni totius et partis, et huic respondet abstractio universalis a particulari, quae est abstractio totius, in quo consideratur absolute natura aliqua secundum suam rationem essentialem, ab omnibus partibus, quae non sunt partes speciei, sed sunt partes accidentales.

Knowledge that is acquired by way of abstraction is directly of things according to what is abstracted and not according to that from which the abstraction is made. What is attained in mathematical abstraction is still known as form implying matter, in spite of the fact that it has been abstracted from sensible matter. Quantity as studied by the mathematician is not known in reference to what is experienced through sensation. Its intelligibility is reached in another way. It is

nevertheless a form and, as such, is different from the natural whole known by the scientist. The latter likewise, having been separated in mind from its accidental parts, is still known as a whole, whose parts are form and common sensible matter. This is a nature considered absolutely.

8. *Parts cannot be abstracted from the whole nor matter from form.*

As a means of putting the notion of abstraction into relief, St. Thomas points out that while the intellect can abstract form from sensible matter and whole from parts, the opposite is not true. The intellect cannot abstract a part from the whole. If it be a question of material parts, the part cannot be abstracted because the whole must be included in its definition. If, on the other hand, there be question of parts of the species, then the part can *be* without the whole, as line without triangle and letters without syllable. And for things that can *be* separately, the understanding of one without the other is called separation more properly than abstraction. Nor can the intellect abstract matter from form. Form as abstracted from matter is the accidental form of quantity. Sensible matter, however, cannot be abstracted from quantity because, as has been seen, the sensible qualities which render matter sensible inhere in material substance through quantity and hence cannot be understood without it.

Having analyzed the meanings of abstraction in terms of the operations of the intellect and its manner of distinguishing, St. Thomas synthesizes the foregoing by relating it to the sciences under consideration.

Sic ergo in operatione intellectus triplex distinctio invenitur. [*a*] Una secundum operationem intellectus componentis et dividentis, quae separatio dicitur proprie ; et haec competit scientiae divinae sive metaphysicae. [*b*] Alia secundum operationem, qua formantur quidditates rerum, quae est abstractio formae a materia sensibili ; et haec competit mathematicae. [*c*] Tertia secundum eandem operationem quae est abstractio universalis a particulari ; et haec competit etiam physicae et est communis omnibus scientiis, quia in scientia praetermittitur quod per accidens est et accipitur quod per se est.

The intellect distinguishes in a threefold manner : *a*) One, according to the operation by which it composes and divides. This operation, by which the intellect understands that one thing *is* not in another, is properly called separation. And, St. Thomas tells us here, this pertains to metaphysics. In science, distinction is always ordered to separating something from matter in mind, and truth requires that this operation report things the way they are, which implies a distinction meaning that some things *are* separate from all matter individual and common sensible matter and all intelligible matter as well. If

things about which this distinction is asserted are that way, the mind is true ; if they are not, it is in error.

The intellect distinguishes in a different fashion according to its first operation which — and we have seen the reason — is at times designated as *formatio*. *b*) Sometimes it abstracts a form from sensible matter, though not from all matter. This sort of distinction pertains to mathematics, defining as it does with intelligible matter but without sensible matter ; such definitions are of things which would need sensible matter to form a complete, composite nature, whereas without sensible matter, quantity is taken as a subject that follows merely from our mode of understanding and therefore not outside the mind. *c*) Finally, a third kind of distinction is accomplished when the intellect, in its first operation, abstracts a universal from the particular. This is found also in natural science, seeing that this science defines, and definitions are of the universal only ; but it defines with common sensible matter abstracting only from individual matter. On the other hand, abstraction of the universal from the particular is common to all the sciences. If we can speak of this abstraction as characteristic, it will be by way of appropriation and not of exclusion.

This detailed examination of abstraction was prompted by a difficulty, one which did not seem to be too imposing, namely, that mathematics considers without matter things that are in matter. For those who wonder at the poor and sometimes hostile reception accorded St. Thomas, the foregoing is most revealing. St. Thomas, even in his most matter-of-fact statements, is extremely difficult. This is not due to what he says. He is always simple and to the point. Nor is it due to his manner of presentation. It is due rather to the complex structure of reality which St. Thomas analyzes in a most penetrating fashion. To recognize difficulties in the preceding discussion is not to confess embarrassment. Much less is it an attempt at some sort of justification. It is simply to acknowledge the light that has been shed on the complexity of human knowing in science.

Cornelius J. KELLY.