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Content and Context of Kaṇakkatikāram Manuscripts
Pre-Modern Malayalam Elementary Mathematics

Roy Wagner and Arun Ashokan

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Article abstract

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Principal Contact:

Dominik Wujastyk, Editor, University of Alberta
Email: ⟨wujastyk@ualberta.ca⟩

Mailing Address:

History of Science in South Asia,
Department of History, Classics and Religion,
2–81 HM Tory Building,
University of Alberta,
Edmonton, AB, T6G 2H4
Canada

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Content and Context of Kaṇakkatikāram Manuscripts: Pre-Modern Malayalam Elementary Mathematics

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ETH Zurich

INTRODUCTION

KAṆAKKATIKĀRAM IS THE TITLE of elementary mathematical treatises that focus on measurements, calculation techniques and practical-recreational word problems (for an online edition see Ashokan et al. 2024). These treatises enjoyed substantial distribution in medieval and colonial Tamil Nadu and Kerala (Parameswara Iyer 1990: vol 2, 524–527). In this paper we will describe these treatises based on Malayalam manuscripts. We will discuss their content, linguistic and stylistic form, context of use, relation to actual professional practices, the cultural values that they express, and the political-economic reality that they reflect. Since the Tamil versions have already been analyzed by Senthil Babu (2022), here we focus on aspects that complement his analysis and on features that are unique to, or more salient in, the Malayalam versions.

WHAT IS KAṆAKKATIKĀRAM?

THE TITLE KAṆAKKATIKĀRAM is a compound of two words: *kaṇakku*, meaning "calculation," "accounting," "mathematics," or (in a compound) "calculator/accountant/mathematician," and *atikāram*, a Dravidian version of the Sanskrit *adhikāra*. In a literary context, the latter term means "topic" or "section of a treatise," but in a broader sense it refers to "rule" and "official authority." Thus, the title *Kaṇakkatikāram* means something like "topics (or rules) of calculation (or of accounting)," but in an oblique, idiosyncratic reading, could also signify "the authority of calculation (or of the accountant)."

The subject of the *Kaṇakkatikāram* can be loosely described as "practical mathematics" (e.g., Moosad 1980: 7) – but this category will be problematized later on in this paper. The manuscripts contain invocations, a "table of content" (a list enumerating the verses and subjects covered in the treatise, which includes

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land, gold, rice, volume, weight…), lists of fractions, decimal powers and measurement units, explanations of some mathematical operations (like multiplication of fractions and summation of some finite series) and mathematical word problems, often organized according to topic.

Library catalogues of manuscripts register a treatise as *Kaṇakkatikāram* based on its opening verse, which explicitly names it as such. For example, the Trivandrum ORIML catalogue lists fourteen manuscripts (Sam and Sreelekha 1996), and the Calicut Tunchan library lists at least five manuscripts (Azhikode and M. M. P. Nair 1985–86). But this is a rather arbitrary bibliographic convention: other treatises, which have similar content, but do not include this opening verse, may be catalogued under other titles, whereas those with the relevant opening verse may display substantial variance. Other related treatises have names such as *Kaṇakkusāram* ("essence of calculation"), *Kaṇakkuśāstram* ("science of calculation"), *Kaṇakkucōdyam* ("questions on calculation"), and *Gaṇitnūl* ("arithmetic treatise") (Parameswara Iyer 1990: vol 2, 524–527; Sarma 1972: 95–96, 104).

Kaṇakkatikāram treatises exist in both Tamil and Malayalam. In Tamil, they are usually attributed to *Kāri Nāyaṉār*, although some versions suggest other attributions. The Tamil *Kaṇakkatikāram* is tentatively dated to the fifteenth century based on linguistic-stylistic evidence, but is acknowledged to have many substantially different variants (Senthil Babu 2022: 42–47; for an edition with an English translation see Subramaniam and Sathyabama 2007; on page 38 the author describes the work as a compilation, rather than an original work).

In Malayalam, no manuscript that we know includes an attribution to an author. The versions display substantial variety in content and ordering. While there are some verses that have variants in all Malayalam versions – indeed, some even have phonetically similar Tamil counterparts in the Tamil *Kaṇakkatikāram* (Subramaniam and Sathyabama 2007) or even some echoes in the later *Kaṇitnūl* (Subramaniam and Sathyabama 1999) – most verses can be found only in some versions, and some manuscripts leave entire sections out. The most consistently repeated common verses are a couple of introductory verses and some lists of units of measurements. The least consistent are the word problems, which vary substantially across the various versions, and are simply absent in some manuscripts.

This variety means that we shouldn't consider the *Kaṇakkatikāram* as an authorial text, but as a genre of mathematical treatises. It is held together by a common theme and a few common verses, as well as an apparently pedagogical purpose and context of use.

SOURCES CONSULTED FOR THIS WORK

AMONG THE MALAYALAM MANUSCRIPTS listed in various library catalogues, we used the following five manuscripts for our edition (Ashokan et al. 2024) and for this paper:

- MS Calicut, Tunchan 1307B, a palm leaf manuscript.
- MS Calicut, Tunchan 817C, a palm leaf manuscript.
- MS Trivandrum, ORIML T365, a later paper transcript.
- MS Trivandrum, ORIML 14633A, a palm leaf manuscript.
- MS Trivandrum, ORIML 22444B, a palm leaf manuscript.

We found a single printed version of a *Kaṇakkatikāram*: Māvanān 1863. It was prepared by Māvanān Māppiḷa Seyitu Muhammadu in 1863.[1] This is an earlier edition of the 1880 edition quoted extensively in the review of Kerala mathematics by Moosad (1980).

We also consulted, for comparison purposes, the two print versions of the *Kaṇakkusāram* (Menon 1950*a*,*b*). This treatise states that it is based on the *Kaṇakkatikāram* and *Bhāskarācārya* II's famous *LīlāvatīI* (a canonical Sanskrit mathematical treatise of the twelfth century), but the relation to the *Līlāvatī* is weak and is more about name-dropping than actual borrowing. The *Kaṇakkatikāram* itself, despite some contrary claims, has almost nothing to do with the *Līlāvatī*, borrowing only very few, isolated problems.[2] The *Kaṇakkusāram* is more Sanskritized than the *Kaṇakkatikāram*, and the problems there are organized primarily around variations of the rule of three, rather than around the *Kaṇakkatikāram*'s domains of application.

THE CONTENT OF KAṆAKKATIKĀRAM

HERE WE LIST THE VARIOUS SECTIONS OF KAṆAKKATIKĀRAM MANUSCRIPTS and their content. The division into these sections is our own, and is not explicit in the manuscripts themselves. Recall also that only some of the sections mentioned here are available across all five manuscripts in our sample (namely: invocation, list of contents, and some number and measurement unit lists). Moreover, even these sections present substantial variety across the different manuscripts.

1 Stated as *kollam* year 1038 on page 61 of the book; the year of printing is not clear, and the publisher was CMS Kottayam (see Gōvi 1998: 164).

2 We should note, however, that the term *Līlāvatī* itself has a complex context. While we usually think of it as a single authorial text, when we consider popular reception, commentaries, and compilations, its designation becomes much more vague.

Invocation

The various invocations, whose diversity suggest that at least some of them are late additions (cf. similar practices documented by Thampi (1999: 40–41)), mention the main deities, such as Vishnu, Shiva, Brahma, Ganesh, Lakshmi and Sarasvati. The gods are credited as the origin of mathematical knowledge or invoked as aids to its understanding and recitation. This kind of attribution is also common for other areas of knowledge.

According to some invocations, mathematical knowledge was mediated by the celestial rishis, legendary figures like Agastyan (at least according to one possible reading), and unnamed masters and gurus. These divine and heroic figures are almost entirely absent from the rest of the treatise. The exceptions are a few word problems presented in a divine setting and sporadic invocations of gods within the verses.

List of Contents

The list of the *Kaṇakkatikāram*'s contents mentions (with some variations) six subjects: land (*nilam*), gold (*pon*), paddy (*nel*), rice (*arici* = *ari*), volume (*kāl*, which might also be interpreted as referring to distance or time), and weight (*kallu*). It then enumerates the problems dedicated to each subject. It states that the total number of verses should be sixty.

In some versions, there is also a list of the main primary elements of measurement: the number one, followed by a basic units of weight, volume, distance, and time. These units are already inconsistent with the lists of subjects above, and inconsistencies in the description of the content within and between versions arise as well. Most importantly, the subjects and numbers of verses stated in the content verses do not fit any of the available problem-sections in our sample. This is a clear indication that the manuscripts are ad-hoc compilations,[3] rather than authorial creations.

Fractions and Large Numbers

These verses of the *Kaṇakkatikāram* name the standard fractions in the Tamil-Malayalam system. In this system, 1/2, 1/4, 1/20, 1/80, 1/320 are the "primitive" fractions, in the sense that the names of other fractions are derived from them. Composites are generated by addition (e.g., 1/20 + 1/80 = 1/16) and multiplication (e.g., 1/2 × 1/4 or 3 × 1/2 × 1/4). Then the same scale of fractions is applied to 1/320 and sometimes to 1/320 × 1/320. The last fraction of this scale is divided into 21 parts and then into 7 parts, yielding the fraction called *aṇu*. This is a Sanskrit term that designates tiny, sometimes indivisible or infinitesimal-like mag-

3 The high level of inconsistency cannot be attributed to copying errors alone, but errors obviously contributed to the inconsistencies, as discussed below.

nitudes; in the context of the Kerala school of mathematical astronomy, it is used in a manner similar to early modern European infinitesimals.

Then begin several alternate scales that further divide the *aṇu* (with variations and omissions in the different versions) into subunits according to the following ratios (each number represents the number of times that the next unit in the scale fits into the previous one):

- 21, 22, 32, 32
- 21, 25, 247, 306, 880
- 21, 21, 21, 51
- 21, 19, 17, 27, 9.

Some of these divisions include echoes of known Sanskrit terms, whereas others appear to be indigenous. None of these minute divisions are useful for calculations that describe anything remotely measurable (they begin *after* $1/320 \times 1/320 \times 1/21 \times 1/7 = 1/15,052,800$), and none are actually used in the problem sections (only the nineteenth century Māvanān uses one of these scales in a handful of calculations to demonstrate his virtuosity). Some of these scales seem to be designed explicitly to complicate, rather than enable calculation – they seem to deliberately make calculations difficult. We will try to explain this phenomenon later in this paper.

These lists are followed by the names of decimal powers reaching as far as 10^{126}. These are clearly related to some Sanskrit systems (especially Buddhist and Jain ones), but do not fit any of the systems we could find in the literature (cf. Hayashi 1995: 64–70). Note that the Malayalam number notation does not use zero, so large numbers have to be given specific names.[4]

Units of Measurement

All versions have verses designating scales of units of weight, volume, time and distance. These lists contain a realistic range, where the various versions tend to be more mutually consistent, and an exceptionally small, non-realistically measureable range, where the consistency within and between different versions is lower. In the context of time, the well-known scale of exceptionally large Hindu time units, organized around the notion of *yuga*, occurs as well. Some of these scales are echoed in Sanskrit sources, while others appear to be independent. The small unit scales tend to use more reasonable and regular division factors than the small fraction scales mentioned above. Even when restricting to the realistic range, only a few of the units mentioned in these sections are actually

4 Tracing down the origin of this list would require an analysis of lists of decimal powers in other manuscript sources prevalent in Kerala at the time. This holds for the other scales discussed in this paper as well.

used in the problem sections, and the problems sometimes refer to units that are not mentioned in the lists – yet another indication of the compilatory nature of these texts.

Some manuscripts include verses that provide conversion ratios for non-consecutive units of the same scale, converting several sub units to some canonical unit. Other verses provide conversion ratios between units of different scales for specific commodities (e.g., the weight of a given volume of rice, or the number of grains of a given food substance per standard volume unit, where the number is expressed as so many years (i.e., multiples of 360) and months (multiples of 30)). These verses are sometimes inconsistent between or within versions as well: values may change not only across manuscripts, but also across verses in the same manuscript and between a verse and its commentary. Once again, we will attempt to explain this situation below.

We note that there is hardly any reference in the manuscripts to the fact that both the actual content of named measures and the factors of their sub-unit scales vary across time and place. However, such diversity is common in non-centralized, pre-modern societies (see, e.g., Kula 1986; Lugli 2019).

Calculation Methods

The calculation-method verses of the *Kaṇakkatikāram* present some mathematical tools. They sometimes begin with the list of names of basic arithmetical operations: addition (*kūṭuka*), subtraction (*kaḷayuka*), multiplication (*perukkuka*), division (*īkkuka*, from the verbal root *ī*), squaring (*varkkuka*), cubing (*kanikkuka*) and their roots (*mūlam*), as well as the *saṃkalita*s (sums of finite series) described below (note that the verses offer many Malayalam synonyms for these operations). However, only some aspects of multiplication and the *saṃkalita*s are actually explained, whereas others are taken for granted, assuming some elementary arithmetical knowledge.

Multiplication of fractions is conducted, according to the relevant verses, by multiplying one fraction by an integer factor so that its product with the other fraction yields an integer, and then dividing the result by the aforementioned factor (e.g., to multiply 3/4 by 3/4, multiply $320 \times 3/4 = 240$ by 3/4, yielding 180, and then divide by 320). Decimal powers are multiplied by assigning each power a numbered place (1 for units, 2 for tens, etc.), adding the place-numbers, and subtracting one.

Another topic is the rule of three. In a schematic form, this rule answers the following question: if a (units of money or commodity) yield b (units of some other money or commodity), how much of the latter would c of the former yield? The answer, $(b \times c)/a$, is usually presented in terms of uniquely Malayalam terminology: *taḷḷa* (mother) for a, *piḷḷa* (son) for b, and *peṟuvāḷ* (expecting mother) for c. The "expecting mother" is then supposed to produce a "son" by analogy to the relation between the original "mother" and "son." This colorful terminology, however, may be a deliberate or accidental variation of more lackluster terms that simply mean something like "the taken," "the counterpart" and "that which will be given." Some of the manuscripts that present the rule of three follow it with examples that stand apart from the topically organized problem sections. In the *Kaṇakkusāram*, this form of organization accounts for the bulk of the treatise.

The two other topics mentioned in this section concern the summation of finite series. One variety, deriving from the Sanskrit study of *saṃkalita* (lit. "accumulation," "addition"), provides formulas for adding sequences of integers from 1 to n, 1^2 to n^2, and 1^3 to n^3, as well as the sum of the following sums: $1, 1+2, 1+2+3, \cdots, 1+2+\cdots+n$. These sums, which occur in some word problems, turned out to be important preliminaries for the achievements of the Kerala School of mathematical astronomy in the context of sine calculations and approximations of π and trigonometric functions (see Sarma et al. 2009: ch. 6.4 and the corresponding notes).

Another kind of summation, the *alakunila* (lit. "number-position," in the sense of the bottom line of a calculation), relates directly to the *eñcuvaṭi* tables. These tables are multiplication tables, memorized during early arithmetical training, which exceed the modern 10-by-10 multiplication table by including larger numbers, fractions and sometimes measures. The *alakunila* is the sum of all terms of the form $n \times m$, where m is a constant factor and n goes through the terms of the *eñcuvaṭi* table according to their typical order (e.g., $1, 10, 100, 2, 20, 200, \cdots, 9, 90, 900, 1000$). These sums, which can be appended at the end of each *eñcuvaṭi* column, serve as checksums: if the products do not sum to the appropriate checksum, the student knows that they made an error.

While it is likely that the *saṃkalita* calculations entered the *Kaṇakkatikāram* manuscripts from the more mathematically sophisticated, mostly Sanskrit, tradition, we should not rule out the possibility that the Kerala School advances in *saṃkalita* calculations might have been triggered by the more popular practice of *alakunila* calculations.

Problem Sections

The problem sections in our sample include the following topics:

- Commodity exchange and price calculations, based on the rule of three;
- Gold evaluation: relating weight, quality and value; mixtures of different kinds of gold;
- Wood measurement: relating the circumference, sides and volume of logs, box-shaped pieces of wood, the cost of sawing them, unit conversions;
- Land measurement: relating circumference, diameter, sides and areas of rectangular, triangular, circular and bow shaped areas, conversions of

units, relating land areas to sowing capacity;
- Rice-paddy conversions: problems on equitable exchange of rice; of different qualities (measured in terms of the paddy:rice ratio), conversions of volume measures, and mixtures of rice of different qualities;
- Quadratic problems: problems relating to the Pythagorean theorem, problems based on elementary quadratic identities, evaluating pearls (whose value is presented as proportional to the square of their weight);
- Interest calculations: relating the principal, time, interest rate and actual interest of loans
- Miscellaneous: problems typical of the "recreational problems" tradition, such as summations of finite series, "Chinese remainder" problems, linear combinations (e.g., purchase of various commodities in given proportions and a given total cost, problems of the type "if you give me so-and-so of your money, I'll have so-and-so much times what you have left," draining pools by several pipes, etc.), and other problems;

Most of the problems are presented in terms of everyday craft and commercial practices, including some verses that introduce relevant crafts (e.g., explaining how to assay gold or cut wood). Note however, that this does not mean that the problems should be understood as practical (see below for a discussion; see also Ashokan (forthcoming)). A smaller number of problems are presented in terms of religious and mythical Hindu settings, framing them as stories about gods or heroes. A handful of problems are presented as problems about numbers as such, without any practical, seemingly-practical or mythical context.

Altogether, the content of the problem sections is in line with the so-called practical-recreational mathematical treatises in India and beyond (ranging from Mesopotamian and Egyptian collections of problems to Chinese, Arabic and European collections – see Høyrup 1990). However, these problems are set within the specific socio-cultural and conceptual framework of India and Kerala mathematics.

LANGUAGE, STYLE AND PROVENANCE

THE STYLE OF THE KAṆAKKATIKĀRAM can be loosely categorized as *pāṭṭu* verses (Dravidian-style poems or songs) followed by prose commentaries. The level of poetic sophistication is limited. *Etuka* (assonant second syllables in every line) is usually, but not always, present. *Mōna* (assonant syllables at the beginning of the first and third feet of a line) is usually absent. The precise meter is usually, but not exclusively, an under-structured *primitive kēka* (to use the terminology of Krishna Warrior (2018: 28–30)). Note, however, that Malayalam meters depend on rhythms rather than on counting syllables and syllable lengths as in Sanskrit or Tamil, which sometimes hinders the reconstruction of metres and may lead to variant prosodic interpretations. This vagueness of meter is not uncommon in Malayalam folk songs (Thampi 2000: 30; Swiderski 1988: 130; Gamliel 2008: 49).

Some invocations of a woman addressee, which is a Tamil poetic convention, are scattered across some verses, whereas others appeal to "you" or to a vaguely defined expert or experts (the use of plural as honorific makes these difficult to distinguish). Otherwise, the language tends to be adorned mostly with standard poetic filler words and only rarely more sophisticated poetic expressions typical of more canonical Tamil and Malayalam poetry. The invocations and some of the problems framed in religious contexts tend to be more poetically refined, indicating the different poetic standards of different sources.

Generally, the verses show a marked Tamil influence. Phonologically, the verses tend to shy away from palatalization and nasalization where these are available in modern and late medieval Malayalam. In a sample survey of five of the verses most consistently present across our five manuscripts (which probably represent the oldest layer of the text), we found palatalization or nasalization only in a quarter of the words where these are expected in the modern language (the specific occurrences are not always consistent across the different versions). Personal endings in finite verbs – another feature that tends to disappear in late medieval Malayalam – are sometimes available (the same sample suggests that they are available in around half of the occasions where they could occur). Non-Dravidian consonants (vocalized stops, aspirated consonants and sibilants) are uncommon, but present. A large majority of Sanskrit words (which mostly occur in invocations or where Sanskrit provides religious terms, technical terms or units of measure) are expressed in a Dravidian orthography, as is common in *pāṭṭu* (e.g., Thampi 2000: 42). The prose commentaries are somewhat less Tamilized, tending towards more palatalization, nasalization and Sanskritization and towards less personal endings, but this trend is not universal. We note that we did not identify any markedly Arabic or European terms (except in the nineteenth century Muslim version by Māvanān (1863: 178–179), which refers to the Turks – *tuluṅkan* and *tūlukkar* – of the king of Kannada, although this might be a corruption of *tuḷu*; England also appears there as the source of a ship carrying horses (Māvanān 1863: 171–172)).

We also note that the language of *Kaṇakkatikāram* is difficult to understand. The many copying errors (or hearing errors, assuming an oral dictation) and idiosyncrasies of the language, as well as the presence of Tamil words and morphologies, often render the text quite perplexing. The fact that we had more than one version at hand for most verses was crucial for allowing us to create even a tentative translation. In fact, it seems that the problem of understanding the texts is not just ours as modern readers, but sometimes of the scribes themselves, whose corrupt copies indicate an incomplete understanding of some of the words or sentences they were inscribing onto the leaves. This is not as surprising as it may sound, given contemporary evidence. Both Thampi (1999: 16), in the context of the southern folk songs, and Swiderski (1988: 131), in the context of Christian wedding songs, testify to folk singers who do not quite understand the precise content of the songs they sing, due to outdated or Tamilized language.[5]

The above linguistic features suggest a link between *Kaṇakkatikāram* treatises and the linguistic-poetic environment of southern folk songs (Namboodiri 1985: 114, 119–120; Thampi 1999: 12–28; Gamliel 2008: 53),[6] or, perhaps, with a linguistic environment with strong Tamil influence, like the mountain passes of the Western Ghats. Another clue for the geo-linguistic origin of the text is verse 27 in the first part of our edition (Ashokan et al. 2024). This verse provides conversion rates between the *palam* and *paṇam* units of weight in different places in Kerala (a rare acknowledgment in the *Kaṇakkatikāram* of the local diversity of measures). The places mentioned in the verse are: Kochi, Kozhikode, Pallakad, Palayoor, Veliyancode and Kulamukku. This array of cities belongs to central-northern Kerala.

The same verse also provides us with some temporal information. Kochi became a major port only after the destruction of the Kodungallur port in the middle of the fourteenth century. Kulamukku seems to have lost its status as a secondary trade center by the end of the sixteenth century, when the Portuguese disrupted the Kerala horse trade. While this attests only to the origin of a single verse that appears in three of our five manuscripts, the evidence is strengthened by coins mentioned in a few of the miscellaneous problems, namely *tāram* and *accu*, which also indicate the same spatio-temporal environment (Mitchiner 1998: part 7; Vijayalekshmy 1997: 293; Augustine 2014: 292). Other verses, however, may have originated earlier, later or elsewhere.

The intersection of the evidence in this verse and the linguistic evidence detailed above may lead to the conjecture that the Malayalam *Kaṇakkatikāram* originated around the fifteenth century, somewhere in the Palakkad region, per-

5 For a general discussion of Tamil influences Malayalam folk literature, see Choondal 1980: 12–14.

6 Consulting the *Dialect Survey of Malayalam* (Subramoniam 2007) suggests a not completely consistent tendency toward southern dialects. For example, *vaḻi* and *tāṟe*/*tāṟa* (as opposed to *vayi* and *tāye*/*tāya* [Frequency list items 14b, 58]) fits central and southern Kerala; *ila*, *iraṭṭi*, *viral*, *itaḷ.* (as opposed to forms with *e* in the first syllable [15, 16, 61, 63]) are popular only in Quillon and Travancore; *neñcu* (as opposed to *neññu* [237]) is more popular south of Ernakulam. On the other hand, the form *taṇṇīr* and its cognates [3] is documented only in Calicut and Cannanore, *irippatu* (as opposed to *iruppatu* [96]) is popular only in the Calicut region, and *mākāṇi*/*makāṇi* (as opposed to *māhāṇi*/*mahāṇi* [104]) are popular only in the north. Unfortunately, the caste specificity of the survey and its modern setting prevent us from giving it too much weight in reconstructing the geographical sources of our corpus.

haps in the context of merchants that connected the Malabar Coast with Tamil Nadu via the Pallakad Gap (more on that below). However, this may be a hasty conclusion. The distribution and variety of the manuscripts and the geographical spread of the educational and commercial networks that carried them do not compel us to assume a unified source or origin. Instead, we may surmise that different verses came from different times and places, and were constantly recombined, re-edited and renewed by local teachers and practitioners who created new *Kaṇakkatikāram* compilations.

This is supported by the stylistic variety in the manuscripts. Some verses are strongly embedded in a practical-artisanal context, demonstrate little poetic skill compared to more canonical poetry, and pay almost no heed to religious themes. However, some invocations and word problems (e.g., verses 24–27 of the second part of our Ashokan et al. 2024 edition) are embedded in divine settings and are poetically richer and more Sanskritized. The substantial gaps between the units of measure in the introductory parts and those used in the word problems also suggest a variety of sources.

Based on this evidence, it is clear that the *Kaṇakkatikāram* was not a lofty Brahmin composition. Nevertheless, the people who contributed to its verses appear to have spanned a wide range of poetic proficiency levels and most likely also a variety of caste-professional backgrounds (the different themes covered relate to the practices of different professional castes, although we should not assume that the practitioners were necessarily involved in composing the pedagogical verses). This fits the popular poetic practices of late medieval Kerala, which involve the adaptation of known verses to the varying circumstances of performance and the singer's choice of meter, relying on braiding together song lines (*keṭṭumuṟa*), adding filler words (*nikattumoḻi*), and forming assonance (*aṭukkumoḻi*; see Thampi 1999: 29–31).

Sanskrit and Tamil sometimes managed to preserve the content of treatises in contexts of de-centralized oral transmission by means of their higher level of grammatical codification and stricter prosodic templates. Malayalam, however, did not have recourse to such mechanisms. Malayalam is much younger than Sanskrit and Tamil, and lacked a strong philological tradition that would codify its grammar. Moreover, Malayalam meters depend on musical rhythms, and the performance conventions allow singers to change the length of syllables to fit a given meter (Panicker 1978: 79–81; Swiderski 1988: 130; Thampi 1999: 29). This renders Malayalam chants rather malleable and easy to adapt (or corrupt) in transmission.

There are further processes that may have contributed to the adaptation or corruption of the verses. First, verses may have migrated between various Tamil and Malayalam dialects, which reduced the intelligibility or readability of the chants or texts. Second, the transition from the older *kōleḻuttu* script to the modern script may have corrupted some passages, as the former distinguishes less consonants and vowels, and transcribing to the latter requires some interpretation. Third, we cannot exclude a temporal hiatus in copying, namely a period where the texts fell into disuse, only to be picked up again later (Parameswara Iyer (1990: 524–525) seems to suggest a nineteenth century revival, or at least expansion, of the use of *Kaṇakkatikāram*; this may be related to the transition from palm leaf to print, the emergent market for texts, and the colonial pressure for numerate education). By that time, the archaic language and poor physical conditions of surviving manuscripts may have posed substantial obstacles for new copyists, which led to the corruption of the texts.

Recall the estimates that the authorized version of the Tamil *Kaṇakkatikāram* traces back to the fifteenth century. Given that this may have already been an eclectic composition building on earlier scattered verses, we can assume that at least some verses go back to the fifteenth century or earlier. Considering Māvanān's Muslim *Kaṇakkatikāram* (Māvanān 1863, partly quoted in Moosad 1980), which provides "secular" versions of some of the problems that, in our manuscripts, are set in a Hindu context, we can assume that new or revised verses were still being produced well into the nineteenth century, keeping this tradition alive.[7] Collecting all the evidence above, we conclude that the *Kaṇakkatikāram* was the product of a decentralized and uncoordinated knowledge transfer network, which cuts across time, place, caste and (at least in later times) religion.

THE CONTEXTS OF KAṆAKKATIKĀRAM

(*a*) *Education*

Senthil Babu (2022: 91–92) places the Tamil *Kaṇakkatikāram* in the context of teaching in village schools. Indeed, the *Kaṇakkatikāram* appears to have been used in the second stage of a child's mathematical education, after they have mastered the *eñcuvaṭi* multiplication tables and some basic arithmetic already assumed in the *Kaṇakkatikāram*. This is corroborated by the fact that at least two of our manuscripts are bundled together with an *eñcuvaṭi*, and the only elementary mathematical operations explained in the *Kaṇakkatikāram* are fraction multiplication and the multiplication of higher decimal powers. Moreover, Moosad (1980: 74–75) claims that *Kaṇakkatikāram* problems were used to teach mathematics in the early twentieth century, as reflected in the memoir of E. V. Krishna Pillai.[8]

7 Muslim communities have a long history of trade in the area, which cut across different occupational communities, and therefore possibly also knowledge communities (see, e.g., Malekandathil 2007).

8 We note, as an aside, that actual calculation practices in Kerala combined mental calculations with the use of cowries as

The caste and professional composition of the students of village schools may have depended on the time and place. Radhakrishnan (1990: 7) brings evidence from the 1883 education commission's report for the exclusion of polluting class, whereas Dharampal (2000: 26 ff., 196–201) quotes reports form the 1820s suggesting that many non-varna students did study in such schools. The teaching profession was associated with such castes as *eḻuttacchan* and *paṇikkar* or *kaṇiyār* (teacher-astrologer castes, see Sreejith 2019: 113, 143). With the exception of Nair *paṇikkar*s, they belonged to the lower echelons of the *jāti* hierarchy or fell below it. They seem to have been considered polluting castes (at least in the early twentieth century, see Ananthakrishna Iyer 1909: vol I, ch. XI; vol. II, ch. VI).[9] Assuming that village school teachers were indeed actively creating or recreating *Kaṇakkatikāram* verses, the relatively low social status of its authors may explain why we do not find references to caste hierarchies in the *Kaṇakkatikāram*.

This diverse setting may also explain the variety of versions and corruptions of *Kaṇakkatikāram* texts. Teachers of various middle and lower castes may have excluded, revised or created verses to fit their pedagogical purposes (cf. Thampi 1999, 39–40). They probably dictated the verses and commentaries to students who may have had an imperfect understanding of their meaning and even of some of the words. The manuscript preparation may have served as memorization aid or as a sort of record of studies or proof of "graduation".

Moreover, village schools were highly independent institutions, not subject to any centralized hierarchy (Sreejith 2019: 100–101; for village schools in Tamil Nadu, see Senthil Babu 2022: ch. 3). The highly localized setting may have involved a focus on specific crafts, which led to the inclusion or exclusion of some families of problems and techniques. In such settings, the grammatical and semantic quality of the text might not have been terribly important, and long chains

tokens on counting boards. A counting board is similar to the abacus, but requires nothing but a flat surface that can be divided into (real or imagined) columns and pebble-like tokens that can be placed on it. The system of columns designates place values, and the tokens indicate the number of units per place value. Moving the tokens about allows to perform many arithmetic calculations efficiently. The cowries are attested in the use of astrologers (Sreedhara Menon 1979: 161–162). It is possible that the 23 "places" of the number-magnitude scale recorded by Senthil Babu (2022: 97) refer to the columns of a counting board.

9 An origin story of the *eḻuttacchan* claims that they are the descendants of Tamil Brahmins who were expelled from the Chola kingdom in the fifteenth century due to their support of Buddhism (Ananthakrishna Iyer 1909: vol. 2, 104). If we take this story seriously, this may indicate a migration of the *Kaṇakkatikāram* from Tamil Nadu to Kerala and from a higher to a lower caste context. If we do not, this origin myth may serve two parallel functions: from the point of view of the *eḻuttacchan*, it provides them with a lofty origin. From the point of view of Brahmin ideology, it confirms the idea that knowledge, even low caste vernacular knowledge, originates from Brahmins. The *kaṇiyār* also have an origin story that ascribes their knowledge of astrology to a fallen Brahmin (Ananthakrishna Iyer 1909: vol. 2, 186–188).

of uncoordinated teacher-student transmission may account for the diversity and sometimes poor quality of the texts.

(b) Commercial practice

Another context for understanding *Kaṇakkatikāram* is that of commerce. Recall verse 27 discussed above, which names six Kerala trading centers. These centers form a network that connects the largest ports (Kochi, Kozhikode and Ponnani-Veliyancode, the latter marked by its large Muslim community) via secondary backwaters trading centers (Palayoor with its important Christian community and the horse trading center in Kulamukku, having strong trade ties with Karnataka, see Nayaka 1999; Vijayalekshmy 1997: 239–241), upward to the Palakkad Gap, which leads to Tamil Nadu.

This list of trading centers may have been selected for this verse because it designated the nodes of the trajectory of an itinerant trader with contacts in various trade communities. This putative trader would travel along the Bharatapuzha River from the Palakkad Gap via Kulamukku to Ponnani-Veliyancode, and via the backwaters or the coast to Palayoor, Kochi and Kozhikode (see Malekandathil 2018: 168–169 for a description of the Bharatapuzha trade network). Such a trader may belong to various castes and religious groups. With the collapse of the large trade guilds around the thirteenth century (Vijayalekshmy 1997: ch. 3, 166–178), the trader communities became more diverse and dispersed (assuming they were ever as coherent as the mainstream historiography claims), and one may assume that training and education followed these decentralizing trends as well. This may explain the emergence, distribution and diversity of *Kaṇakkatikāram* manuscripts in these contexts.

We also note that the folk song entitled *Payyannūr-Pāṭṭu* describes the knowledge that a trader's son must master, which includes calculations related to gold, silver, money, loans, measurements, gems, land and seeds (Anthony 1994: 11–12, verses 32–35) – all of which are dealt with in our *Kaṇakkatikāram* measurement units and problems.

Another figure related to this commercial context is the *kaṇakku pillai*, the accountant, working in the service of land owners, temples or political administration, charged with managing inventories and rent or tax collection, sometimes rising to the rank of influential proprietor (related titles include *atikāri*, *mēnōn*, *pārvattukār*, and *kōlkāran*). Beyond general mathematical education and the training of merchants, *Kaṇakkatikāram* may have been particularly important for the training of accountants (Senthil Babu 2022: 86–100), and (although this is less probable, due to the mixed content of the treatise) merchants of various specific goods, such as gold or wood. The different scopes covered by the different *Kaṇakkatikāram* manuscripts may reflect the needs of these different contexts. Note, however, that we could only find references to rent or taxation in a handful of problems, which may indicate that the manuscripts were more oriented to commerce and general teaching than to administration.

(c) *Performance*

Yet another relevant context is that of literary performances and challenges. Verses that purport to share knowledge were not only vehicles of the preservation and dissemination of knowledge, they were also vehicles of proving one's literary virtuosity. Indeed, competitions of knowledge and poetic skill were part of late medieval Kerala culture, the most famous of these being the Zamorin sponsored *Revati Pattathanam* competitions (Sreedhara Menon 1987: 93–97).

While the *Kaṇakkatikāram* was of too low status to be part of such a prestigious, Sanskrit-oriented competition, in other, more modest contexts, it might have served to prove the performer's knowledge and proficiency. In the context of another *śāstra-pāṭṭu* (science poem), that of martial arts songs, Thampi (1999: 20) documents precisely this phenomenon: the ability to chant some arcane technical verses would prove the performer's proficiency. In the specific context of the *Kaṇakkatikāram*, Moosad (1980: 75–76) claims that some of the problems are more about the teacher expressing prestigious, arcane knowledge in order to gain veneration than about actually teaching the students. We note that the inclusion of arcane and partly esoteric knowledge by teachers, as well as a culture of challenges between mathematicians and students (Malayalam examples of which can be found in Moosad 1980: vi, 2; further discussion of the Malayalam riddle culture is available in Choondal 1980: 69–72), was also part of late medieval and Renaissance Italy (the Cardano-Tartaglia debate is a famous example, Høyrup (2008) discusses other interesting instances).

Indeed, several *Kaṇakkatikāram* verses are stated as a challenge to the listener, using formulas akin to "if you are a good calculator, say…." This phenomenon is attested also in Sanskrit mathematical verses, for example in the *Līlāvatī*. At least one version of one verse (verse 19 in part I of our edition, Ashokan et al. 2024) explicitly suggests that being able to recite conversion rates between capacity and weight for various substances is a proof of one's status or official position (*tānamānam*, the Dravidianized form of *sthānamānam*).

The dimension of performance highlights the tensions between the written and oral text. It invokes improvisation (adapting the arithmetical knowledge to different contexts), which may lead to new or revised verses in the written texts. On the other hand, lacking a canonical version, consensus among different practitioners and performers is jeopardized (cf. Swiderski 1988, 127–128), which may have undermined the status of *Kaṇakkatikāram* as authoritative knowledge. Without a widespread agreed version, the text cannot be used as a test or proof of erudition, and thus has only a limited value outside local contexts.

WHAT IS A "PRACTICAL" PROBLEM?

SINCE KAṆAKKATIKĀRAM is usually framed in the context of practical mathematics, it is important to be careful about the articulation of this category. First, we emphasize that this framing is not available in the texts themselves – the *Kaṇakkatikāram*s we consulted do not mention any practice/theory divide. They simply present the knowledge under the category of *kaṇakku*, meaning "accounting" or "calculations".

If we use "practical" as an analytic, rather than as an actor's category, we should note that it can be interpreted in different ways. First, "practical" may refer to calculations that are presented in terms of craft or commercial practices, regardless of whether they are actually used while pursuing one's trade. Consider, for example, a problem where one is asked to calculate the sides of a rectangular field, when given its area and the sum of one of its side and its diagonal. In a realistic setting, a surveyor would never have access to the "givens" of the problem without already having access to the actual sides. However, starting from practical problems (such as calculating the area or diagonal given the sides), the motivated practitioner might contemplate non-applied, inverted problems, motivated by mathematical curiosity or an interest in systematization. So this interpretation of "practical mathematics" may be too wide, and encroach on what we might prefer to designate as "theory." On the other hand, such a categorization would leave out calculations that are presented in general terms (pure geometry or numbers) but intended for use in specific, applied situations. This same interpretation is therefore also too narrow.

Alternatively, "practical" may refer to calculation techniques that are actually used by artisans, accountants or merchants for the purposes of their trades. This definition seems to be straightforward, but is in fact quite problematic too. Consider, for example, calculations that are only extremely rarely applied in a the above professional settings. Should they be considered "practical"? What about calculations that are presented only to impress prospective clients with the practitioner's virtuosity? The latter may not appear practical, but they could still be inherent to the practice of teachers and accountants, whose livelihood may depend on demonstrating such virtuosity (cf. a similar discussion in the Mesopotamian and European contexts respectively, Høyrup 2007: 263–271; 2008). Moreover, when one presents calculations in terms of pure numbers, it may be impossible to know whether they are or are not practical in the above sense.

Subbarayalu and Rajagopal (2001: 35–36) noted an interesting related example: the occasional use of tiny fractions (of the standard Tamil scale) in Tamil inscriptions recording land measurements, reaching as far as subdivisions of a square inch. These displays of calculation prowess (which may have resulted from dividing known land areas by factors that yield non-round results) are devoid of mensurational meaning, and yet they are part of the accountant's ongoing practice. This shows how difficult it is to draw a line between aspects of mathematics relevant to realistic measurement and evaluation and displays of virtuous precision that are effectively ignored in measurement and evaluation.

Another aspect which makes the category of "practical mathematics" problematic is the fact that some advanced knowledge (say, advanced calculations or some number theory) may help the practitioner master and creatively adjust the knowledge they actually need to apply in their trade, even if this advanced knowledge does not directly fit into realistic measurement and evaluation. To what extent should we consider such knowledge, which is indirectly helpful to the practitioner, as practical?

This ambiguity is even more pronounced where the knowledge has to do with the occult or religious meaning of numbers. Such knowledge, while not facilitating any particular calculation, allows practitioners (architects, sculptors, astrologers, etc.) to have a meaningful relation to the measurements and numbers that they use. They may then consider this knowledge inseparable from their daily practice (we are excluding from this argument instances such as the calculation of auspicious times and proportions in Kerala astrology and architecture – these calculations form the core practice of the astrologer, even if we cannot reconstruct what guides them).

All the ambiguities above can be found in the *Kaṇakkatikāram* literature: calculations that are really applied (or at least seem to be applied) in craft and trade measurement and evaluation situations; problems that are set in practical terms, but would not be encountered in realistic trade scenarios; purely arithmetical techniques (such as multiplication of fractions and *saṃkalita*s) that may or may not be applied in practice; problems that are meant to demonstrate the practitioner's virtuosity; and arcane knowledge concerning tiny fractions and measurements, with links to the sublime or occult. We can characterize this as the mathematics which, to varying extents, was of interest to teachers, merchants and accountants in medieval Kerala, or at least the part of this mathematics that they considered worthy of codification in verses and manuscripts. It is part of their intellectual-professional practice, whether we would consider it "practical" or not.

We can complicate the question of "practical" further by addressing the problem of the *use value* and *exchange value* of the knowledge exhibited in the *Kaṇakkatikāram*. The use value of mathematical knowledge commonsensically depends on the use of that knowledge in the production and exchange of goods by artisans, accountant and merchants. But as we noted above, very small and very large units are not used in the problem sections of the *Kaṇakkatikāram*, and some units used in the problem sections are not referred to in the unit sections. Moreover, some problems presented in the language of everyday craft and commercial practices do not actually reflect the practical situations of commerce and crafts (see Ashokan forthcoming). So the value of this knowledge is not captured by the notion of use value.

Being limited in terms of use value, the value of the *Kaṇakkatikāram* knowledge may be thought of as a sort of exchange value (loosely adopting Marxist terminology) or symbolic capital (Bourdieu's terminology). Teachers, scholars, accountants and merchants used the knowledge of the *Kaṇakkatikāram* to present virtuosity, which could be "exchanged" for prestige, respect or preferential treatment. Put differently, teachers, scholars, accountants and merchants used the knowledge of the *Kaṇakkatikāram* as symbolic capital for attaining positions of authority and power in education, commerce or administration. However, caste segregation limits the opportunities of such communication, and so could substantially limit the exchange value or symbolic capital of this knowledge, and so the extent of its actual use for that purpose remains an open question.

This issue leads to the possible social segregation between readers and writers of texts on the one hand and practitioners on the other. As in the context of architecture (*vāstu-vidyā* and *śilpa śāstra*), there are substantial doubts whether the available manuals were actually consulted by practitioners, or belonged more strictly to a scientific sphere detached from the actual design and construction of homes and temples (Otter 2016: ch. 2). Another example is the *Kṛṣi-Gīta*, a Malayalam agricultural scientific poem (Kumar 2008). On the one hand, it is set as a conversation between Brahmins and an avatar of Vishnu, and lists dozens of varieties of crops that belong to many different regions. This would suggest an intellectual-scientific milieu. On the other hand, it states "you should plow the lands yourself" (Kumar 2008: 63) and provides some highly specific information concerning cultivation, which relates directly to those who actually work the land, or at least actively manage the work. It is therefore unclear who was the intended audience of this poem, and to what extent its content corresponds to actual agricultural practice (most likely, this is the result of combining several sources that originate from different knowledge communities). We should bear in mind a similar tension between the text of the *Kaṇakkatikāram* and the students, artisans, merchants and accountants who were its purported target audience.

WHAT IS A NUMBER?

In this section, we would like to raise the question of the natures of numbers as reflected in the *Kaṇakkatikāram*. We note several phenomena. First, in the introductory verse that lists the basic units of measures of several kinds of magnitudes, the number "one" opens the list, followed by canonical units of weight (*kaḻañcu*), volume (*kalam*), short distances (*kōl*), long distances (*kātam*), and time (day). This suggests an ambiguous situation, where numbers are not a supervening genus for specific magnitudes, but a sort of privileged measure. In calculations, fractions of units of measure are sometimes designated by subunits and sometimes by numerical fractions. However, the numerical fractions are not as "pure" as they may appear: they seem to have emerged from the context of land measurement units (Subbarayalu and Rajagopal 2001: 31–37).

The emergence of abstract numbers, dissociated from what they measure, is usually considered a higher stage of mathematical-cultural evolution, but this has to be problematized. It makes perfect practical sense to measure different commodities according to different scales, which reflect the quantities of the commodity that are typically relevant in everyday life situations. For example, rice can be measured in terms of the daily consumption of a person, or the amount that a person can carry in a bag, or the amount required for planting a standard field; the measurement scale can be derived from these measures. Coming up with a unified scale appropriate for all measures (like the decimal scale) is thus not universally beneficial, but makes sense for a person who has to manage many different goods at once, as occurs in the context of trade or accounting (this indeed seems to be the origin of the sexagesimal scale as a universal scale for Mesopotamian scribes, see Robson 2008: 75–83). The *Kaṇakkatikāram* appears to adhere both to specific measurements and to a universalizing number scale, but without fully conceding primacy to the latter. Universal or unified systems of numeration are not a natural or inevitable development, and should be explained in terms of specific social contexts. In particular, the tensions between the particular (occupation specific) and universal (administrative) systems of numeration may be very salient in a caste society.

The next thing to note is the diversity of minute fractions and measures. Scales of minute fractions and huge numbers are not unique to the *Kaṇakkatikāram*. Indeed, they appear in various classical and scientific compositions in ancient and medieval India (see Hayashi 2017 for time units, Misra 2003: 169–170 for space), and are eclectically borrowed for some of the *Kaṇakkatikāram* scales. Other unit names derive from more mundane terms, such as "smell," "smoke," "milk," "ghee," "water," "particle" and "drop" for unrealistically small capacity measures (the initial terms in the list presumably refer to particles of those substances, and may arise from the context of distilling ghee). Small realistic weight units are named after seeds and nuts, whereas realistic capacity measures have names that signify measures such as "handful," "pot" and "drum". However, many names on the *Kaṇakkatikāram* scales are unintelligible. The origin of the minutest measures is sometimes attributed to "divine division" and sometimes to what appear to be some supposedly natural particles (see the notes to verses 22 and 7 respectively in part I of our edition, Ashokan et al. 2024). These choices may reflect the various layers of the text.

We note further that the Sanskrit literature never presents, as far as we know, scales of purely numerical tiny fractions without assigning them to specific kinds of magnitudes, like space or time. One explanation is that the general numerical fraction system popular in the Sanskrit literature is based on numerator-denominator pairs. This means that every number can be turned into a corresponding fraction, and there's no need to define specialized fraction scales as in the Malayalam-Tamil fraction system. Indeed, the few echoes of Sanskrit terms in the numerical tiny fractions scales are derived from distance or time measurements (see the note to verse 8 in part I of our edition, Ashokan et al. 2024).

We already noted that some of the tiny fractions do not seem to enable calculation, but appear to be designed to defy calculation – making calculation so difficult that only an expert could handle it. We also note that we have several fraction scales, and that the tiny measurement unit scales suffer from inconsistencies between versions, and sometimes even between a verse and its commentary in the same manuscript. Inconsistencies occur in the realistic scales as well, but they tend to be less frequent.

The question that arises is then why the *Kaṇakkatikāram* would provide readers with such useless, odd and inconsistent information, which is not even considered in the problem sections. The most readily available explanations are corruption in transmission, regional variation and eclecticism. However, looking at the various tiny scales, their disparity and arbitrariness appear to be intentional, even if some values and terms have been corrupted. Regional variety was indeed present (Subbarayalu and Rajagopal 2001: 31–37; Selvakumar 2017; Vijayalekshmy 2003), but if this is the reason for the variation and inconsistencies, why is regional variation hardly ever explicitly discussed as such? This is indeed done in verse 27 of part I of our edition (Ashokan et al. 2024) and in verse 16 of the *Kaṇakkusāram* (Menon 1950*b*: 4), but those are rare exceptions. Finally, a tendency to eclecticism rather than unification begs the questions: why would authors adopt an eclectic tendency?

We do not have definitive answers to these questions, but would like to suggest some possible explanations. At present, we cannot evaluate their respective explanatory value, but perhaps future researchers could.

- Mnemonic training: Memorization was an integral part of education (e.g., Sreejith 2019: 115, 125). The fraction verses are the first subject presented in most treatises. Listing arbitrary numbers and names, they could serve to train the student's memorization skills. Alternatively, they could serve as a selective entry bar: those students who fail to memorize them would not be allowed to continue their training. This would relate arithmetic to mnemonic virtuosity that exceeds what was already required from students in the earlier stage of mastering the *eñcuvaṭi* tables. In addition, the strange naming of fractions may have been part of a language oriented pedagogy, rather than numerosity as such.

- Aesthetic virtuosity: This leads us to recall that the *Kaṇakkatikāram* is not simply a text, but also the performance of a chant. Being able to master the performance of such apparently meaningless verses and present them in a skillful and pleasing manner forms a challenge, that can prove the performer's poetic virtuosity.
- Development/demonstration of calculation skill: Moosad (1980: 10) claims that numbers above *kōṭi* and below lower-*muntiri* (1/320 ×1/320) are not used in practice, and serve to demonstrate mathematical skill alone. We note that Māvanān indeed performs a few calculations involving the arcane tiny fractions, and the demonstration of skill is indeed a likely motivation. Relating to the last two points, we note that all the regular factors in the Malayalam fraction scale are composed of factors of 2 and 5. A division by 3 may terminate only when reaching *immi* (1/21 of the last fraction of the regular scale). So there's an aesthetic component to some of the small fractions: they may allow to terminate a division without rounding off.
- The arbitrariness of number: It is possible that the implicit purpose of the divergent and apparently arbitrary lists of tiny fractions and measures was to convey the idea that enumeration and scaling are arbitrary issues. This is important in order to allow the student to accept the facts of regional diversity and the sometimes unexplainable measurement scales encountered in real life. The purpose would then be to prepare the student to the fact that measurement scales can come in all shapes and forms, and are an arbitrary human creation. This relates arithmetic to the arbitrariness of worldly traditions and powers. A variant of this explanation is an explicit valuation of diversity, rather than uniformity, in the context of knowledge: multiple parallel alternative systems may have been considered a better approach to knowledge than an authoritative uniform scale.
- Autonomy and abstraction: Creating their own fraction scales, teachers gain an authority that is independent of the knowledge of "mere practitioners." These scales may represent the added value of *Kaṇakkatikāram* beyond what a person can learn on the job. Moreover, the abstraction of tiny fractions and measures from tangible magnitudes opens the way for an abstract treatment of number, unencumbered by the restrictions of actual practice. This allows mathematics to evolve in ways that are not subject to the needs of practitioners. This, in turn, legitimizes the autonomy and abstraction of arithmetic with respect to realms of practical knowledge. However, this depends on the actual status of teachers. In some South Indian contexts, village teachers are poor and ridiculed, and it would be difficult for them to assert autonomy by forming unrealistic abstractions (Senthil

Babu 2022: 202–208).

- Integration of the small and the large: In pre-modern measurement systems, there are often different kinds of units used for smaller and larger scales, which are usually not combined or related in a single scale (a "day's walk" may be converted to "feet," but in pre-modern societies it might make no sense to figure out or use such a conversion). However, scales that combine the large and the small were sometimes forced together in order to homogenize the world of magnitude and invoke an air of rationality or of mastery over it (Schemmel 2016: 39–40).
- Esoteric knowledge: Having to face unintelligible lists of fractions and lists of intangible magnitudes generates a realm of unworldly, perhaps divine or secret knowledge. Presenting these lists so enigmatically leaves the teacher with the power and authority to explain them or refuse their explanation.[10] This echoes the esoteric numerology inherent in astrology and the Kerala versions of *vāstu-vidyā* and *śilpa śāstra* architecture, and relates arithmetic to the arcane and occult. One may interpret this aspect of the *Kaṇakkatikāram* in two ways: either as an attempt to emulate Sanskrit numerological knowledge, which is outside the reach of the *Kaṇakkatikāram* authors, or as an attempt to compete with this knowledge by indigenous creations (it might well be that the former guides the discussion of time and space, whereas the latter guides the capacity scale in our manuscripts).
- "Infinitesimal" calculus: The Kerala school of astronomical mathematics used methods that foreshadowed European infinitesimal calculus, reaching as far as equivalents of power series for trigonometric functions and the value of π (for the question of transmission of this knowledge to Europe, see Joseph 2009). The interpretation of the Kerala school infinitesimals-like entities – whether absolute indivisibles, arbitrarily small dynamic magnitudes, or just limits of precision – is still unsettled, and perhaps cannot be settled (in Europe, this tension was salient in mathematical practice for centuries). The tiny numerical fractions of the Malayalam *Kaṇakkatikāram* may have been related to the more advanced and mostly Sanskrit culture of mathematical astronomy. If we were to accept such a relation, we open the question of which tradition triggered this development in the other (in Europe, for example, a theological-philosophical discussion of infinities and infinitesimals preceded the mathematical development of the calculus, see Thakkar 2009).

10 In chapter 12 of the *Lalitavistara*, for example, the Buddha proves his mastery of mathematical and cosmological knowledge with respect to a competitor by naming large and small numbers and units (Dahl et al. 2024).

We see here that what appears as an odd mathematical practice may in fact highlight mathematical values such as virtuosity, skill, arbitrariness, abstraction, autonomy, and esoteric knowledge, and may be related to advanced mathematical developments. The actual extent that any of the above were the intended or accomplished effect of the study of *Kaṇakkatikāram* cannot be clearly established by the evidence we have gathered so far.

NUMBERS AND AUTHORITY

The last perspective that we would like to consider here concerns the authority of calculations and the authority implied by mastering calculations. In order to understand the *Kaṇakkatikāram* (a title that contains the word *atikāram*, which may be read as "authority") in its cultural context, we need to understand its relation with these forms of authority.

The first dimension of authority that we encounter in *Kaṇakkatikāram* verses is that of authoritative knowledge and mutual challenge. The invocations attribute the verses to a divine origin, mediated by legendary sages. Some verses mention that they are (or should be) recited by the most knowledgeable scholars (e.g., verse 19 in part I of our edition, Ashokan et al. 2024). The tone is apodictic, often ending with a command to know, recite and understand. The word problems are often phrased as explicit challenges to the listener, and a successful solution would prove one's greatness or proficiency (e.g., verse 26 of part II of our edition, Ashokan et al. 2024). All this fits the discussion of the performative aspect of the *Kaṇakkatikāram* and its relation to virtuosity, echoing the discussion of the martial arts songs mentioned above by Thampi (1999: 20).

But we should also recall that calculations have another kind of authority: they are used to determine measures, prices and rent or taxes enforced in real life. The work of Kula (1986) and Lugli (2019) are important references in this context. They demonstrate how, in medieval and modern Europe, measuring was an act of power. The use of measuring devices was often contested: not only in the context of determining the right standard for a given measure, but also in questions such as whether the grain in a standard-container should be heaped or leveled (an example of changing vessels as a means of usury documented in the Travancore state manual is quoted by Rammohan (2006: 55–56)). New rulers would sometimes impose new measurement standards, but their subjects often kept unofficial, sometimes illegal, local measurements alive. The practice of measuring was therefore far removed from the image of an objective scientific practice that we tend to have in mind today. All this made measurement a highly contested act of political and economic power.

The situation in medieval Kerala appears to have been similar or even more extreme. As was already mentioned, variety was the rule – both for the value of a specific measure unit name, and, as we see by comparing different *Kaṇakkatikāram* versions, for the ratios between measures in a given scale. Moreover, some measures were defined relative to the bodily proportions of people. In the Kerala architectural treatise *Manuṣyālayacandrika*, for example, the measures were relative to the body of the home-owner (Ramakrishnan 1998: chapter II, verse 15). In administrative and economic contexts, it was the accountant who had the authority to decide how to interpret measurements, supported by publicly recorded standards, such as marking the size of the local rod on temple walls (Selvakumar 2017).

This situation may explain why the *Kaṇakkatikāram* shuns the explicit mention of local variation. The universalizing tone of the lists of measurement may have been meant to impose a standard, or at least provide a semblance of a universal standard, which the accountant would supposedly enforce. This portrays the accountant's measurements or estimates as more objective and general than the conflicting standards available to lay-persons. This is similar to the attempt of rulers to impose measurement unification reforms, except that, lacking a strong central authority, the *Kaṇakkatikāram* could only simulate textually the authority implicit in such a 'reform' (on the Tamil side, centralized authoritative standards did exist, but they did not displace local standards, only added another layer above them, see Subbarayalu and Rajagopal 2012).

In reality, the image of the knowledge of the accountant as universal and authoritative stands in stark opposition to lived reality. For example, the rule of three states that if 1 banana costs 1 rupee, then a 100 bananas should cost 100 rupees – but in real life bulk purchases tend to be discounted. Calculating areas of irregular shapes by multiplying an average length with an average width (as suggested in some verses) is only a crude approximation. Translating an area into a sowing capacity depends on many factors, not just a simple multiplication of an area by some numerical factor as suggested in some problems.

Actual practices of rent, taxation or land value estimation in Kerala in the late middle ages confirm this disparity. Historiographically, the late middle ages are presented as an era of decentralization between the more centralized authority of the Cheras (if the mainstream historiography is to be trusted) and the renewed centralization imposed by colonial rulers and their proxies (Sreedhara Menon 2007: chs 11–14; K. K. Nair 2012: ch. 4). At the time, Kerala was highly fragmented between various principalities, nominal and actual local authorities, temples, religious minorities, and other privilege holders. Governance was weak and decentralized. Even the trader guilds had already dissipated.

As a result, rent and tax administration was not properly systematized (Ganesh 1987: 32; Narayanan 2003: 141; Mathews 1996: 78). The well reasoned system of land evaluation and taxation that seems to arise from Tamil inscriptions (Subbarayalu and Rajagopal 2001: 53–59, 65–81) is at odds with the actual confusion and arbitrariness due to the fluidity of measures and the arbitrary or violent extraction of surplus value under the guise of rent or taxation (Ganesh 1987 has many examples, e.g.: 39, 258–259, 307, 326–329, 361–363, 366; Narayanan 2003: 147; Rammohan 2006: chs VIII, XI). Indeed, Ganesh's account of the middle ages is one of the rise, due to this extraction of surplus, of a class of accountants and merchants mediating between farmers and rulers.

Even in peaceful situations, property value was determined by a council of four evaluators, rather than by standardizes calculation (Narayanan 2003: 94, 132; Davis 1999: 176; Mathews 1996: 98, fn. 136). Note also that while the accountants were charged with estimates and calculations, they did not actually perform land measurements (Senthil Babu 2022: 170; Kerala architecture treatises also show a similar division of labor, e.g., Ramakrishnan 1998: 140–141, ch. 1, verses 12–14 of the translation from Sanskrit; a Malayalam version includes this information in verses 8–12, see Paramesvara Menon 1928: 4–6). This means that the relation between calculation, evaluation and measurement is further complicated and mediated.

We believe that both the *Kaṇakkatikāram*'s attempt to project an image uniformity and stability and its failure to achieve this goal reflect a culture of highly distributed, rather arbitrary and sometimes violent practices of measurement and evaluation in medieval Kerala. Accountants, who were sometimes effectively robber-barons, are imagined as vehicles for transmitting well regulated top-down authority, while often making it up as they went along. The *Kaṇakkatikāram*, with its host of oddities and contradictions, can be viewed as the ideological reflection or template of this reality.

This is further reflected by the Malayalam riddle "*aṭṭuppin tiṇṇamēl kaṇakkapiḷḷa*." The riddle asks to identify "the accountant on the threshold of the hearth." The answer, a cat, suggests that like a cat, the accountant takes whatever he can steal from the household, rather than his deserved allotted share (Choondal 1980: 70).

CONCLUSION

In this paper, we have surveyed the content and context of *Kaṇakkatikāram* manuscripts. Their language, context and style suggest a mixed origin and a process of transmission that include substantial creative variation and reformation. As for their sometimes odd content, in order to understand it, we need to relate the *Kaṇakkatikāram* culture to a thick network of social roles.

The *Kaṇakkatikāram* clearly has to do with actual practical calculations, such as those related to the work of the goldsmith and carpenter, but it also has to do with esoteric knowledge, echoing divine rule and classical Sanskrit treatises. It is a mathematics textbook of sorts for non-elite communities, covering some of the skills required of merchants and accountants, but it also bestows on the teacher, student and performer the symbolic capital of knowledge that is not reducible to practice. It serves to display virtuosity in memorization, calculation and poetic skill, but also asserts the authority of the accountant.

We would like to conclude with one more possible interpretation – one that is even more difficult to confirm, but that we would like to suggest nevertheless. Perhaps the *Kaṇakkatikāram* is not simply an attempt to establish the authority of numeracy, but, in its most perplexing verses, also a critical or even satirical reflection or a challenge, to authority. Perhaps the arbitrary and unintelligible lists of units are an affirmation of diversity in numeracy as a cultural good. Perhaps they even pose a subtle critique against the authoritative presumptions to represent law and order in the realm of numbers, while in fact their feigned channelling of rational and divine numeracy covers over nothing but an arbitrary extraction of wealth.

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Please write to ⟨wujastyk@ualberta.ca⟩ to file bugs/problem reports, feature requests and to get involved.

The History of Science in South Asia • Department of History and Classics, 2–81 HM Tory Building, University of Alberta, Edmonton, AB, T6G 2H4, Canada.