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Gregory Gutin and Daniel Karapetyan

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Article abstract

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Generalized Traveling Salesman Problem Reduction Algorithms

Gregory Gutin and Daniel Karapetyan

Department of Computer Science, Royal Holloway University of London, Egham, Surrey TW20 0EX, UK

Abstract

The generalized traveling salesman problem (GTSP) is an extension of the well-known traveling salesman problem. In GTSP, we are given a partition of cities into groups and we are required to find a minimum length tour that includes exactly one city from each group. The aim of this paper is to present a problem reduction algorithm that deletes redundant vertices and edges, preserving the optimal solution. The algorithm's running time is $O(N^3)$ in the worst case, but it is significantly faster in practice. The algorithm has reduced the problem size by 15–20% on average in our experiments and this has decreased the solution time by 10–60% for each of the considered solvers.

Key words: Generalized Traveling Salesman Problem, Preprocessing, Reduction Algorithm

1. Introduction

The generalized traveling salesman problem (GTSP) is defined as follows. We are given a weighted complete undirected graph G on N vertices and a partition $V = V_1 \cup V_2 \cup \dots \cup V_M$ of its vertices; the subsets V_i are called *clusters*. The objective is to find a minimum weight cycle containing exactly one vertex from each cluster. There are many publications on GTSP (see, e.g., the surveys [4,6] and the references therein). The problem has many applications, see, e.g., [2,11]. It is NP-hard, since the traveling salesman problem (TSP) is its special case (when $|V_i| = 1$ for each i). The weight of an edge xy of G is denoted $\text{dist}(x, y)$ and will be often called the *distance* between x and y .

Various approaches to GTSP have been studied. There are exact algorithms such as branch-and-bound and branch-and-cut described in [3]. Another approach uses the fact that GTSP can be converted to an equivalent TSP with the same number of vertices [2,13–15] and then can be solved with some efficient TSP solver such as Concorde [1]. Heuristic GTSP algorithms have also been investigated, see, e.g., [7,8,10,18–21].

Different preprocessing procedures are often used for hard problems to reduce the computation time. There are examples of such approaches in integer and linear programming (e.g., [9,17]) as well as for the Vehicle Routing Problem [12]. In some cases preprocessing plays the key role in an algorithm (e.g., [5]). We intro-

duce preprocessing procedure for GTSP. A feature of GTSP is that not every vertex of a problem should be visited and, thus, GTSP may contain vertices that a priori are not included in the optimal solution and may be removed. We have a similar situation with edges.

The experimental results show that almost each GTSP instance tested in the literature can be reduced by the presented procedure at a very low cost and that this reduction is almost always beneficial for the GTSP solvers.

2. Vertex Reduction

Since GTSP solution covers only M vertices, up to $N - M$ vertices may be reduced without a change of the optimal solution. We present an approach to detect some of the redundant vertices in a reasonable time.

Definition 1. Let C be a cluster, $|C| > 1$. We say that a vertex $r \in C$ is *redundant* if for each pair x, y of vertices from distinct clusters different from C , there exists a vertex $s \in C \setminus \{r\}$ such that $\text{dist}(x, s) + \text{dist}(s, y) \leq \text{dist}(x, r) + \text{dist}(r, y)$.

In other words, if for each path xry there exists another path xsy , $s \in C \setminus \{r\}$, with the same or smaller weight, vertex r can be removed. Testing this condition for every vertex will take approximately $O(N^3 \cdot \overline{|V|})$, where $\overline{|V|} = N/M$ is the average cluster size. In the symmetric case of the problem there is an efficient heuristic that usually allows to reduce the preprocessing time significantly.

Let us take two distinct vertices r and s in some

Email: Gregory Gutin [G.Gutin@rhul.ac.uk], Daniel Karapetyan [Daniel.Karapetyan@gmail.com].

Table 1

Differences Table example.

| $s \setminus x$ | cl.2 v.1 | cl.2 v.2 | cl.2 v.3 | cl.3 v.1 | cl.3 v.2 | Negative # |
|-----------------|-------------------------|----------|----------|--------------------|----------|------------|
| v.2 | 2 | 0 | -1 | -3 | 4 | 2 |
| v.3 | -1 | -2 | -1 | 1 | 2 | 3 |
| max | 2 | 0 | -1 | 1 | 4 | |
| | $\min\{2, 0, -1\} = -1$ | | | $\min\{1, 4\} = 1$ | | |

cluster C . We can calculate the differences between the distances to r and s from each vertex $x \notin C$ ($\Delta_x^{r,s} = \text{dist}(x, r) - \text{dist}(x, s)$) and save this information to a *Differences Table* such as Table 1. Notice that in Table 1 we assume that clusters 1 and 2 have three vertices each and cluster 3 has two vertices, r belongs to the first cluster and it is the first vertex in the cluster, i.e., vertex s can be only the second and the third vertices of cluster 1.

Observe that a vertex r is redundant if there is no pair of vertices from different clusters such that the sum of differences Δ (see above) for these vertices is negative for every s , i.e., r is redundant if for every x and y there exists $s \in C \setminus \{r\}$ such that $\Delta_x^{r,s} + \Delta_y^{r,s} \geq 0$, where x and y belong to distinct clusters. That is due to

$$\begin{aligned} \Delta_x^{r,s} + \Delta_y^{r,s} &= \text{dist}(x, r) - \text{dist}(x, s) + \text{dist}(y, r) - \text{dist}(y, s) \\ &= \text{dist}(x, r) + \text{dist}(r, y) - (\text{dist}(x, s) + \text{dist}(s, y)) \end{aligned}$$

Therefore we need to check every pair of columns (col_1, col_2) (except the pairs of columns corresponding to the same clusters) in the Differences Table $T_{row, col}$. If $T_{1, col_1} + T_{1, col_2} < 0$, we check the second row ($T_{2, col_1} + T_{2, col_2}$). If the result is still negative, we check the third row, etc. If all the rows are checked and each time we obtain a negative sum, the vertex r cannot be removed and the rest of the procedure may be skipped.

Example 1. In the example above (Table 1) it is necessary to perform up to 6 tests provided in Table 2.

The only test that does not allow us to declare the vertex r redundant is in the row 3 of the Table 2 (cl.2 v.2—cl.3 v.1) as both sums (for $s = v.2$ and for $s = v.3$) are negative. (Certainly, there is no need to calculate the sum for $s = v.3$ in rows 2, 4, and 6 in the example above, and the calculations may be stopped after the row 3.)

Removing redundant vertex may cause a previously irredundant vertex to become redundant. Thus, it is useful to check redundancy of vertices in cyclic order until

Table 2

Vertices pairs for the example.

| Pair | Sum for $s = v.2$ | Sum for $s = v.3$ |
|-------------------|-------------------|-------------------|
| cl.2 v.1—cl.3 v.1 | -1 | 0 |
| cl.2 v.1—cl.3 v.2 | 6 | -1 |
| cl.2 v.2—cl.3 v.1 | -3 | -1 |
| cl.2 v.2—cl.3 v.2 | 4 | 0 |
| cl.2 v.3—cl.3 v.1 | -4 | 0 |
| cl.2 v.3—cl.3 v.2 | 3 | 1 |

we see that, in the last cycle, no vertices are found to be redundant. However, in the worst case, that would lead to $\Theta(N^2)$ redundancy tests. (Recall that N is the total number of vertices in GTSP.) Our computational experience has shown that almost all redundant vertices will be found even if we restrict ourselves to testing each vertex of GTSP at most twice. Thus, we assume in the rest of the paper that each vertex is tested at most twice for redundancy.

2.1. Acceleration Heuristic

In some cases it is possible to determine faster that a vertex r is not redundant. If

$$\min_{x \notin Z} \max_{s \in C} \Delta_x^{r,s} + \min_{x \in Z} \max_{s \in C} \Delta_x^{r,s} < 0$$

for some cluster Z , then r cannot be reduced. This condition means that there exist two columns in the Differences Table corresponding to distinct clusters and the sum of these columns maxima is negative. This ensures that the sum for every row of these columns is also negative.

We can use an equivalent condition:

$$\min_{x \in \bigcup_{j < i} V_j} \max_{s \in C} \Delta_x^{r,s} + \min_{x \in V_i} \max_{s \in C} \Delta_x^{r,s} < 0$$

This condition can be tested during the Differences Table generation. For each column we calculate the maximum value:

$$\text{vertexmax}(x) = \max_{s \in C} \Delta_x^{r,s}$$

Also for each cluster Z , we have

$$\text{clustermin}(Z) = \min_{x \in Z} \text{vertexmax}(x)$$

We define $\text{totalmin}(i) = \min_{j < i} \text{clustermin}(V_j)$; if $\text{totalmin}(i) + \text{clustermin}(V_i) < 0$ for some i , we can conclude that vertex r is not redundant.

In the example above, the heuristic performs just one check for V_2 and V_3 . We have $\text{totalmin}(3) = \text{clustermin}(V_2) = -1$ and $\text{clustermin}(V_3) = 1$ and $-1 + 1 \geq 0$ so the acceleration heuristic does not reduce our computations in this case.

Another way to make the redundancy test faster is to order the rows of the Differences Table such that the row with the minimal number of negative values would be the first one. Notice that, if this row contains no negative values, it is obvious that r is redundant.

2.2. Algorithm Complexity

Let K_{\min} and K_{\max} be the minimum and the maximum number of tests (of vertices) for redundancy. Observe that $K_{\min} = N$, since we will perform only N tests if no vertex is detected to be redundant. Since we have assumed that no vertex is tested more than twice for redundancy, $K_{\max} = 2N - 1$.

Now consider how many operations are required for each redundancy test (with a fixed vertex r). The test requires table generation and table processing. Due to the acceleration heuristic, table generation can be aborted already after processing of two clusters. Thus, in the best case it takes $E_{\min} = (|C| - 1)(|X| + |Y|)$ operations where $r \in C$, and X and Y are some other clusters. The average size of a cluster can be estimated as N/M (recall that M is the number of clusters). Therefore, in the best case each redundancy test requires approximately

$$E_{\min}(N') \approx \left(\frac{N'}{M} - 1\right) \left(2 \cdot \frac{N'}{M}\right) \approx 2 \cdot \left(\frac{N'}{M}\right)^2$$

operations, where N' is the current number of vertices in the problem.

In the worst case both the table generation and the further table inspection will be completed normally. Table generation will take $(|C| - 1)(N' - |C|)$ operations.

Table inspection takes about

$$\left(N' - |C| - \overline{|V|}\right)^2 (|C| - 1) / 2$$

operations in the worst case, where $\overline{|V|}$ is the average cluster size. Thus, we have the following number of operations per test in the worst case:

$$\begin{aligned} E_{\max}(N') &\approx (|C| - 1)(N' - |C|) \\ &\quad + \frac{\left(N' - |C| - \overline{|V|}\right)^2 (|C| - 1)}{2} \\ &\approx |C| \cdot N' + \frac{(N')^2 \cdot |C|}{2} \approx \frac{(N')^3}{2M}. \end{aligned}$$

The total number of operations in the worst case is

$$K_{\max} \cdot E_{\max}(N) \approx 2 \cdot N \cdot \frac{N^3}{2M} = \frac{N^4}{M}.$$

The total operation number in the best case is

$$K_{\min} \cdot E_{\min}(N) \approx N \cdot 2 \cdot \left(\frac{N}{M}\right)^2 = 2 \cdot \frac{N^3}{M^2}.$$

Since usually $M = \Theta(N)$, the algorithm complexity changes from $O(N)$ to $O(N^3)$. The experimental algorithm complexity is $\Theta(N^{2.4})$ (see Section 4.1.).

3. Edge Reduction

Definition 2. Let u, v be a pair of vertices from distinct clusters U and C respectively. Then the edge uv is *redundant* if for each vertex $x \in V \setminus U \setminus C$ there exists $v' \in C \setminus \{v\}$ such that $\text{dist}(u, v') + \text{dist}(v', x) \leq \text{dist}(u, v) + \text{dist}(v, x)$.

Testing this condition for every edge will work for both symmetric and asymmetric cases and will take approximately $O(N^3 \cdot \overline{|V|})$, where $\overline{|V|}$ is the average cluster size. We introduce an algorithm for edge reduction for the symmetric case of the problem; it proceeds as follows. Given a vertex $v \in C$, where $|C| > 1$, we detect redundant edges incident with v using the following procedure:

- (1) Select an arbitrary vertex $v'' \in C \setminus \{v\}$.
- (2) Set $P_x = \Delta_x^{v, v''}$ for each vertex $x \in V \setminus C$ (recall that $\Delta_x^{r, s} = \text{dist}(x, r) - \text{dist}(x, s)$).
- (3) Sort array P in non-decreasing order.
- (4) For each cluster $U \neq C$ and for each vertex $u \in U$ do the following:
 - (a) $\delta = \Delta_u^{v, v''}$

- (b) For each item $\Delta_x^{v,v''}$ of the array P such that $\Delta_x^{v,v''} + \delta < 0$ check the following: if $x \notin U$ and $\Delta_x^{v,v'} + \Delta_u^{v,v'} < 0$ for every $v' \in C \setminus \{v, v''\}$, the edge uv is not redundant, continue with the next u .
- (c) Edge uv is redundant, set $\text{dist}(u, v) = \infty$.

To prove that the above edge reduction algorithm works correctly, let fix some edge uv , $u \in U$, $v \in C$, $U \neq C$. The algorithm declares this edge redundant if the following condition holds for each $x \notin C$ (see 4b):

$$\begin{aligned} \Delta_x^{v,v''} + \Delta_u^{v,v''} &\geq 0 && \text{or} \\ \Delta_x^{v,v'} + \Delta_u^{v,v'} &\geq 0 && \text{for some } v' \in C \setminus \{v, v''\} \end{aligned}$$

This condition is equivalent to

$$\Delta_x^{v,v'} + \Delta_u^{v,v'} \geq 0 \quad \text{for some } v' \in C \setminus \{v\}$$

So the algorithm declares the edge uv redundant if for each $x \in V \setminus C \setminus U$ there exists $v' \in C \setminus \{v\}$ such that $\Delta_x^{v,v'} + \Delta_u^{v,v'} \geq 0$,

$$\begin{aligned} \text{dist}(x, v) - \text{dist}(x, v') + \text{dist}(u, v) - \text{dist}(u, v') &\geq 0 \\ \text{and} \\ \text{dist}(u, v) + \text{dist}(v, x) &\geq \text{dist}(u, v') + \text{dist}(v', x). \end{aligned}$$

Let us evaluate the algorithm's complexity. The edge reduction algorithm performs the following steps for every cluster C , $|C| > 1$ for each $v \in C$:

- Array P generation. This takes $\Theta(N)$ operation.
- Array P sorting. This takes $\Theta(N \log_2 N)$ operations.
- Edges uv testing. Each test takes $O(1)$ to $O(N \cdot |C|)$ operations and $\Theta(N)$ tests are performed.

Thus the complexity of the entire algorithm is $\Theta(N^2 \log_2 N)$ in the best case, and $\Theta(N^3 \cdot |C|)$ in the worst case.

As usually $|C| = \Theta(N)$, we may say that this algorithm's complexity varies from $\Theta(N^2 \log_2 N)$ to $\Theta(N^3)$. The experimental algorithm complexity is $\Theta(N^{2.6})$ (see Section 4.1.).

After the search for redundant edges has been completed, the edge reduction algorithm finds redundant vertices using the following observation: if after the edge reduction procedure some vertex has finite distance edges to at most one cluster, then this vertex can be declared redundant.

This reduction takes $O(N^2)$ operations.

4. Experiments

We tested the reduction algorithms on the standard GTSP instances (see, e.g., [2,18–20]) which were generated from some TSPLIB [16] instances by applying the clustering procedure of Fischetti, Salazar and Toth [3]. The algorithms were implemented in C++ and tested on a computer with AMD Atlon 64 X2 Core Dual processor (3 GHz frequency).

We have tested three reduction algorithms: the Vertex Reduction Algorithm (see Section 2.), the Edge Reduction Algorithm (see Section 3.), and the Combined Algorithm which first applies the Vertex Reduction Algorithm and then the Edge Reduction Algorithm.

4.1. Experimental Results

Each test was repeated ten times. The columns of the table are as follows:

- *Instance* is the instance name. The prefix number is the number of clusters of the instance; the suffix number is the number of vertices (before any preprocessing).
- R_v is the number of vertices detected as redundant.
- R_e is the number of edges detected as redundant. For the Combined Algorithm R_e shows the number of redundant edges in the already reduced by the Vertex Reduction Algorithm problem.
- T is the preprocessing time in seconds.

The results of the experiments show that the preprocessing time for the Vertex Reduction is negligible for all the instances up to 212u1060, i.e., for almost all TSPLIB-based GTSP instances used in the literature. The average percentage of detected redundant vertices for these instances is 14%, and it is 11% for all considered instances. The experimental algorithm complexity is about $O(N^{2.4})$.

The Edge Reduction is more time-consuming than the Vertex Reduction. The running time is negligible for all instances up to 115rat575. Note that in most of the GTSP literature, only instances with $N < 500$ are considered. The average per cent of the detected redundant edges for these instances is about 27%, and it is 21% for all instances in Table 3. The experimental algorithm's complexity is $O(N^{2.6})$.

4.2. Algorithms Application Results

Certainly, one can doubt the usefulness of our reduction algorithms since they may not necessarily decrease

Table 3

Test results of the Reduction Algorithms.

| Instance | Vertex reduction | | | Edge reduction | | | Combined reduction | | |
|------------|------------------|-----------|-----|----------------|-------|-----|--------------------|-----------|-----|
| | R_v | $R_v, \%$ | T | $R_e, \%$ | R_v | T | $R_v, \%$ | $R_e, \%$ | T |
| 4ulysses16 | 9 | 56.3 | 0.0 | 62.0 | 4 | 0.0 | 56.3 | 23.5 | 0.0 |
| 4gr17 | 11 | 64.7 | 0.0 | 35.8 | 3 | 0.0 | 64.7 | 23.0 | 0.0 |
| 5gr21 | 8 | 38.1 | 0.0 | 48.7 | 3 | 0.0 | 38.1 | 45.0 | 0.0 |
| 5ulysses22 | 11 | 50.0 | 0.0 | 44.3 | 2 | 0.0 | 50.0 | 39.5 | 0.0 |
| 5gr24 | 13 | 54.2 | 0.0 | 33.1 | 3 | 0.0 | 54.2 | 10.4 | 0.0 |
| 6fri26 | 13 | 50.0 | 0.0 | 28.7 | 3 | 0.0 | 50.0 | 20.3 | 0.0 |
| 6bayg29 | 12 | 41.4 | 0.0 | 37.9 | 5 | 0.0 | 41.4 | 33.6 | 0.0 |
| 9dantzig42 | 6 | 14.3 | 0.0 | 36.2 | 0 | 0.0 | 14.3 | 24.9 | 0.0 |
| 10att48 | 15 | 31.3 | 0.0 | 41.5 | 7 | 0.0 | 31.3 | 25.3 | 0.0 |
| 10gr48 | 18 | 37.5 | 0.0 | 27.0 | 4 | 0.0 | 37.5 | 25.5 | 0.0 |
| 10hk48 | 6 | 12.5 | 0.0 | 34.2 | 3 | 0.0 | 12.5 | 32.3 | 0.0 |
| 11berlin52 | 15 | 28.8 | 0.0 | 36.1 | 1 | 0.0 | 28.8 | 35.0 | 0.0 |
| 11eil51 | 9 | 17.6 | 0.0 | 32.6 | 3 | 0.0 | 17.6 | 28.8 | 0.0 |
| 12brazil58 | 14 | 24.1 | 0.0 | 24.5 | 3 | 0.0 | 24.1 | 29.0 | 0.0 |
| 14st70 | 12 | 17.1 | 0.0 | 36.5 | 3 | 0.0 | 17.1 | 24.6 | 0.0 |
| 16eil76 | 12 | 15.8 | 0.0 | 28.8 | 2 | 0.0 | 15.8 | 28.6 | 0.0 |
| 16pr76 | 2 | 2.6 | 0.0 | 29.0 | 1 | 0.0 | 2.6 | 29.7 | 0.0 |
| 20gr96 | 13 | 13.5 | 0.0 | 25.8 | 3 | 0.0 | 13.5 | 20.6 | 0.0 |
| 20rat99 | 11 | 11.1 | 0.0 | 23.7 | 3 | 0.0 | 11.1 | 23.2 | 0.0 |
| 20kroA100 | 16 | 16.0 | 0.0 | 20.9 | 2 | 0.0 | 16.0 | 18.8 | 0.0 |
| 20kroB100 | 8 | 8.0 | 0.0 | 28.1 | 2 | 0.0 | 8.0 | 25.0 | 0.0 |
| 20kroC100 | 19 | 19.0 | 0.0 | 27.2 | 2 | 0.0 | 19.0 | 24.2 | 0.0 |
| 20kroD100 | 19 | 19.0 | 0.0 | 27.9 | 2 | 0.0 | 19.0 | 19.8 | 0.0 |
| 20kroE100 | 21 | 21.0 | 0.0 | 26.4 | 1 | 0.0 | 21.0 | 20.2 | 0.0 |
| 20rd100 | 11 | 11.0 | 0.0 | 32.1 | 2 | 0.0 | 11.0 | 28.8 | 0.0 |
| 21eil101 | 14 | 13.9 | 0.0 | 35.5 | 1 | 0.0 | 13.9 | 31.5 | 0.0 |
| 21lin105 | 9 | 8.6 | 0.0 | 35.4 | 3 | 0.0 | 8.6 | 32.4 | 0.0 |
| 22pr107 | 9 | 8.4 | 0.0 | 35.6 | 0 | 0.0 | 8.4 | 35.9 | 0.0 |
| 24gr120 | 15 | 12.5 | 0.0 | 28.4 | 4 | 0.0 | 12.5 | 29.6 | 0.0 |
| 25pr124 | 17 | 13.7 | 0.0 | 32.5 | 3 | 0.0 | 13.7 | 22.2 | 0.0 |
| 26bier127 | 2 | 1.6 | 0.0 | 21.5 | 1 | 0.0 | 1.6 | 19.7 | 0.0 |
| 26ch130 | 16 | 12.3 | 0.0 | 25.9 | 3 | 0.0 | 12.3 | 21.2 | 0.0 |
| 28pr136 | 14 | 10.3 | 0.0 | 22.4 | 1 | 0.0 | 10.3 | 26.3 | 0.0 |
| 28gr137 | 10 | 7.3 | 0.0 | 19.9 | 1 | 0.0 | 7.3 | 17.0 | 0.0 |
| 29pr144 | 19 | 13.2 | 0.0 | 33.2 | 2 | 0.0 | 13.2 | 31.1 | 0.0 |
| 30ch150 | 22 | 14.7 | 0.0 | 19.9 | 2 | 0.0 | 14.7 | 18.1 | 0.0 |
| 30kroA150 | 20 | 13.3 | 0.0 | 22.5 | 6 | 0.0 | 13.3 | 19.5 | 0.0 |
| 30kroB150 | 14 | 9.3 | 0.0 | 23.8 | 2 | 0.0 | 9.3 | 23.4 | 0.0 |
| 31pr152 | 34 | 22.4 | 0.0 | 37.5 | 7 | 0.0 | 22.4 | 26.6 | 0.0 |
| 32u159 | 33 | 20.8 | 0.0 | 23.5 | 3 | 0.0 | 20.8 | 15.1 | 0.0 |
| 35si175 | 45 | 25.7 | 0.0 | 27.4 | 5 | 0.0 | 25.7 | 17.5 | 0.0 |
| 36brg180 | 97 | 53.9 | 0.0 | 57.9 | 51 | 0.0 | 53.9 | 16.9 | 0.0 |
| 39rat195 | 12 | 6.2 | 0.0 | 22.2 | 1 | 0.0 | 6.2 | 20.4 | 0.0 |
| 40d198 | 7 | 3.5 | 0.0 | 23.1 | 4 | 0.0 | 3.5 | 24.2 | 0.0 |
| 40kroA200 | 16 | 8.0 | 0.0 | 20.3 | 2 | 0.0 | 8.0 | 20.6 | 0.0 |

Table 3

Test results of the Reduction Algorithms.

| Instance | Vertex reduction | | | Edge reduction | | | Combined reduction | | |
|------------|------------------|-----------|-----|----------------|-------|------|--------------------|-----------|-----|
| | R_v | $R_v, \%$ | T | $R_e, \%$ | R_v | T | $R_v, \%$ | $R_e, \%$ | T |
| 40kroB200 | 7 | 3.5 | 0.0 | 19.1 | 1 | 0.0 | 3.5 | 18.5 | 0.0 |
| 41gr202 | 4 | 2.0 | 0.0 | 18.8 | 1 | 0.0 | 2.0 | 18.5 | 0.0 |
| 45ts225 | 40 | 17.8 | 0.0 | 20.0 | 2 | 0.0 | 17.8 | 11.2 | 0.0 |
| 45tsp225 | 12 | 5.3 | 0.0 | 20.5 | 2 | 0.0 | 5.3 | 17.1 | 0.0 |
| 46pr226 | 12 | 5.3 | 0.0 | 29.6 | 1 | 0.0 | 5.3 | 28.4 | 0.0 |
| 46gr229 | 1 | 0.4 | 0.0 | 22.0 | 0 | 0.0 | 0.4 | 21.6 | 0.0 |
| 53gil262 | 16 | 6.1 | 0.0 | 21.8 | 3 | 0.0 | 6.1 | 18.9 | 0.0 |
| 53pr264 | 11 | 4.2 | 0.0 | 21.5 | 1 | 0.0 | 4.2 | 20.7 | 0.0 |
| 56a280 | 20 | 7.1 | 0.0 | 19.4 | 1 | 0.0 | 7.1 | 16.1 | 0.0 |
| 60pr299 | 15 | 5.0 | 0.0 | 16.2 | 0 | 0.0 | 5.0 | 14.7 | 0.0 |
| 64lin318 | 13 | 4.1 | 0.0 | 20.5 | 2 | 0.0 | 4.1 | 20.8 | 0.0 |
| 64linhp318 | 13 | 4.1 | 0.0 | 20.5 | 2 | 0.0 | 4.1 | 20.8 | 0.0 |
| 80rd400 | 11 | 2.8 | 0.0 | 14.8 | 1 | 0.1 | 2.8 | 13.0 | 0.0 |
| 84fl417 | 43 | 10.3 | 0.0 | 28.3 | 5 | 0.1 | 10.3 | 22.7 | 0.1 |
| 87gr431 | 0 | 0.0 | 0.0 | 17.2 | 0 | 0.3 | 0.0 | 17.2 | 0.3 |
| 88pr439 | 10 | 2.3 | 0.0 | 14.7 | 1 | 0.2 | 2.3 | 15.0 | 0.1 |
| 89pcb442 | 24 | 5.4 | 0.0 | 11.9 | 0 | 0.1 | 5.4 | 9.7 | 0.1 |
| 99d493 | 4 | 0.8 | 0.0 | 17.8 | 1 | 0.2 | 0.8 | 19.4 | 0.2 |
| 107att532 | 21 | 3.9 | 0.0 | 20.5 | 2 | 0.3 | 3.9 | 18.1 | 0.3 |
| 107ali535 | 29 | 5.4 | 0.1 | 16.6 | 2 | 0.5 | 5.4 | 14.3 | 0.5 |
| 107si535 | 96 | 17.9 | 0.0 | 26.5 | 9 | 0.3 | 17.9 | 17.9 | 0.1 |
| 113pa561 | 147 | 26.2 | 0.1 | 31.3 | 5 | 0.3 | 26.2 | 22.6 | 0.1 |
| 115u574 | 11 | 1.9 | 0.0 | 14.4 | 1 | 0.2 | 1.9 | 14.0 | 0.2 |
| 115rat575 | 18 | 3.1 | 0.0 | 11.2 | 2 | 0.2 | 3.1 | 10.9 | 0.1 |
| 131p654 | 88 | 13.5 | 0.1 | 32.6 | 2 | 0.8 | 13.5 | 28.2 | 0.5 |
| 132d657 | 8 | 1.2 | 0.0 | 10.8 | 0 | 0.3 | 1.2 | 9.6 | 0.3 |
| 134gr666 | 0 | 0.0 | 0.0 | 11.6 | 0 | 1.0 | 0.0 | 11.6 | 1.0 |
| 145u724 | 34 | 4.7 | 0.1 | 10.1 | 3 | 0.5 | 4.7 | 8.8 | 0.4 |
| 157rat783 | 25 | 3.2 | 0.0 | 9.8 | 2 | 0.4 | 3.2 | 8.4 | 0.3 |
| 200dsj1000 | 8 | 0.8 | 0.1 | 9.6 | 1 | 2.4 | 0.8 | 9.4 | 1.5 |
| 201pr1002 | 20 | 2.0 | 0.1 | 9.2 | 2 | 3.0 | 2.0 | 8.7 | 1.6 |
| 207si1032 | 85 | 8.2 | 0.2 | 12.1 | 12 | 1.2 | 8.2 | 10.2 | 0.9 |
| 212u1060 | 36 | 3.4 | 0.1 | 14.4 | 1 | 1.7 | 3.4 | 11.2 | 2.0 |
| 217vm1084 | 241 | 22.2 | 0.6 | 24.0 | 8 | 2.3 | 22.2 | 8.9 | 1.3 |
| 235pcb1173 | 11 | 0.9 | 0.1 | 8.2 | 0 | 1.5 | 0.9 | 8.2 | 1.3 |
| 259d1291 | 48 | 3.7 | 0.2 | 12.4 | 2 | 2.3 | 3.7 | 9.8 | 1.7 |
| 261rl1304 | 19 | 1.5 | 0.2 | 7.9 | 2 | 2.6 | 1.5 | 7.2 | 2.0 |
| 265rl1323 | 23 | 1.7 | 0.2 | 7.8 | 1 | 4.1 | 1.7 | 7.0 | 2.9 |
| 276nrw1379 | 11 | 0.8 | 0.2 | 7.4 | 1 | 3.7 | 0.8 | 7.1 | 2.6 |
| 280fl1400 | 23 | 1.6 | 0.9 | 17.4 | 0 | 6.5 | 1.6 | 17.5 | 5.3 |
| 287u1432 | 33 | 2.3 | 0.2 | 7.7 | 1 | 3.2 | 2.3 | 6.6 | 2.6 |
| 316fl1577 | 44 | 2.8 | 0.4 | 10.3 | 2 | 5.0 | 2.8 | 9.2 | 4.5 |
| 331d1655 | 14 | 0.8 | 0.2 | 6.7 | 1 | 3.7 | 0.8 | 6.7 | 3.7 |
| 350vm1748 | 285 | 16.3 | 2.5 | 19.8 | 2 | 11.4 | 16.3 | 11.0 | 5.5 |
| 364u1817 | 5 | 0.3 | 0.1 | 6.2 | 0 | 4.9 | 0.3 | 5.8 | 4.5 |

Table 3

Test results of the Reduction Algorithms.

| Instance | Vertex reduction | | | Edge reduction | | | Combined reduction | | |
|--------------|------------------|-----------|------|----------------|-------|--------|--------------------|-----------|--------|
| | R_v | $R_v, \%$ | T | $R_e, \%$ | R_v | T | $R_v, \%$ | $R_e, \%$ | T |
| 378rl1889 | 17 | 0.9 | 0.7 | 7.3 | 3 | 10.9 | 0.9 | 6.8 | 7.2 |
| 421d2103 | 8 | 0.4 | 0.2 | 6.7 | 1 | 2.9 | 0.4 | 6.6 | 2.7 |
| 431u2152 | 10 | 0.5 | 0.3 | 5.2 | 0 | 7.8 | 0.5 | 5.0 | 6.6 |
| 464u2319 | 24 | 1.0 | 0.6 | 3.9 | 0 | 10.3 | 1.0 | 3.8 | 9.7 |
| 479pr2392 | 33 | 1.4 | 0.9 | 5.9 | 1 | 15.4 | 1.4 | 5.3 | 13.4 |
| 608pcb3038 | 29 | 1.0 | 1.4 | 4.7 | 1 | 45.4 | 1.0 | 4.7 | 36.2 |
| 759fl3795 | 21 | 0.6 | 4.9 | 6.4 | 0 | 127.2 | 0.6 | 6.5 | 94.5 |
| 893fnl4461 | 22 | 0.5 | 3.4 | 3.1 | 0 | 80.2 | 0.5 | 2.9 | 46.7 |
| 1183rl5915 | 28 | 0.5 | 7.9 | 2.4 | 2 | 258.1 | 0.5 | 2.3 | 114.1 |
| 1187rl5934 | 38 | 0.6 | 9.4 | 3.0 | 2 | 308.3 | 0.6 | 2.7 | 139.6 |
| 1480pla7397 | 196 | 2.6 | 31.5 | 4.6 | 1 | 2147.9 | 2.6 | 3.6 | 1001.3 |
| 2370rl11849 | 37 | 0.3 | 40.7 | | | | | | |
| 2702usa13509 | 21 | 0.2 | 98.7 | | | | | | |

the running time of GTSP solvers. Therefore, we tested the improvement of the running time of the following GTSP solvers:

- (1) Exact algorithm (**Exact**) based on a transformation of GTSP to TSP [2]; the algorithm from [4] was not available. The algorithm that we use converts a GTSP instance with N vertices to a TSP instance with $3N$ vertices in polynomial time, solves the obtained TSP using the **Concorde** solver [1], and then converts the obtained TSP solution to GTSP solution also in polynomial time.
- (2) Memetic algorithm from [19] (**SD**). A memetic algorithm (**MA**) is a combination of a genetic algorithm with local search.
- (3) MA from [7] (**GKK**).
- (4) MA from [18] (**SG**).
- (5) A modified version of MA from [8], the state-of-the-art GTSP memetic solver, (**GK**).

Each test was repeated ten times. The columns of the tables not described in Section 4.1. are as follows:

- T_0 is the initial problem solution time.
- B is the time benefit, i.e., $(T_0 - T_{pr})/T_0$, where T_{pr} is the preprocessed problem solution time; it includes preprocessing time as well.

The experiments show that the Vertex Reduction, the Edge Reduction and the Combined Reduction Technique significantly reduce the running time of the **Exact**, **SD** and **GKK** solvers. However, the Edge Reduction (and because of that the Combined Reduction Technique) is not that successful for **SG** (Table 7) and the origi-

Table 7

Time benefit for SG.

| Instance | T_0, sec | Vertices Red. | |
|----------------|-------------------|---------------|---------|
| | | $R_v, \%$ | $B, \%$ |
| 84fl417.gtsp | 4.5 | 10.3 | 12 |
| 87gr431.gtsp | 8.3 | 0.0 | 6 |
| 88pr439.gtsp | 10.2 | 2.3 | -3 |
| 89pcb442.gtsp | 11.5 | 5.4 | 0 |
| 99d493.gtsp | 20.0 | 0.8 | 7 |
| 107att532.gtsp | 25.1 | 3.9 | 11 |
| 107si535.gtsp | 16.9 | 17.9 | 34 |
| 107ali535.gtsp | 29.1 | 5.4 | 20 |
| 113pa561.gtsp | 14.5 | 26.2 | 31 |
| Average | | 8.0 | 13 |

nal version of **GK**. That is because not every algorithm processes infinite edges well.

Next we show that a solver can be adjusted to work better with preprocessed instances. For this purpose we modified **GK** as follows:

- The 2-opt heuristic [8] was extended with the cluster optimization. For every iteration of 2-opt, where edges v_1v_2 and v_3v_4 are removed, instead of replacing them with v_1v_3 and v_2v_4 we replace them with v'_1v_3 and v'_2v_4 , where $v'_1 \in \text{cluster}(v_1)$ and $v'_2 \in \text{cluster}(v_2)$ and v'_1 and v'_2 are selected to minimize the solution objective value. (Here $\text{cluster}(v)$

Table 4

Time benefit for Exact

| Instance | T_0 , sec | Vertices Red. | | Edge Red. | | Combined Reduction | | |
|------------|-------------|---------------|---------|-----------|---------|--------------------|-----------|---------|
| | | R_v , % | B , % | R_e , % | B , % | R_v , % | R_e , % | B , % |
| 5gr21 | 0.8 | 38.1 | 40 | 48.7 | 52 | 38.0 | 45.0 | 56 |
| 5ulysses22 | 1.7 | 50.0 | 60 | 44.3 | 48 | 50.0 | 39.5 | 79 |
| 5gr24 | 0.2 | 54.2 | 74 | 33.1 | 53 | 54.1 | 10.4 | 81 |
| 6fri26 | 0.9 | 50.0 | 67 | 28.7 | 18 | 50.0 | 20.3 | 74 |
| 6bayg29 | 6.0 | 41.4 | 19 | 0.0 | 59 | 41.3 | 33.6 | 70 |
| 10gr48 | 16.1 | 37.5 | 57 | 27.0 | 2 | 37.5 | 25.5 | 55 |
| 10hk48 | 52.7 | 12.5 | 16 | 34.2 | 6 | 12.5 | 32.3 | 22 |
| 11eil51 | 32.8 | 17.6 | 37 | 32.6 | 17 | 17.6 | 28.8 | 42 |
| 14st70 | 150.4 | 17.1 | 43 | 36.5 | 17 | 17.1 | 24.6 | 50 |
| Average | | 35.4 | 45.9 | 31.7 | 30.2 | 35.3 | 28.9 | 58.8 |

Table 5

Time benefit for GKK

| Instance | T_0 , sec | Vertices Red. | | Edge Red. | | Combined Reduction | | |
|------------|-------------|---------------|---------|-----------|---------|--------------------|-----------|---------|
| | | R_v , % | B , % | R_e , % | B , % | R_v , % | R_e , % | B , % |
| 89pcb442 | 60.7 | 5.4 | 4 | 11.9 | 17 | 5.4 | 9.7 | 35 |
| 99d493 | 85.2 | 0.8 | 14 | 17.8 | 19 | 0.8 | 19.4 | 29 |
| 107att532 | 101.2 | 3.9 | 9 | 20.5 | 20 | 3.9 | 18.1 | 20 |
| 107ali535 | 99.3 | 5.4 | 0 | 16.6 | 47 | 5.4 | 14.3 | 51 |
| 107si535 | 166.1 | 17.9 | 12 | 26.5 | 14 | 17.9 | 17.9 | 41 |
| 113pa561 | 101.8 | 26.2 | 15 | 31.3 | 21 | 26.2 | 22.6 | 47 |
| 115u574 | 103.6 | 1.9 | -3 | 14.4 | 12 | 1.9 | 14.0 | 28 |
| 115rat575 | 219.3 | 3.1 | 38 | 11.2 | 36 | 3.1 | 10.9 | 45 |
| 131p654 | 165.4 | 13.4 | 21 | 32.6 | 12 | 13.4 | 28.2 | 38 |
| 132d657 | 189.1 | 1.2 | 10 | 10.8 | 22 | 1.2 | 9.6 | 24 |
| 134gr666 | 224.8 | 0.0 | 26 | 11.6 | 36 | 0.0 | 11.6 | 57 |
| 145u724 | 232.9 | 4.6 | 25 | 10.1 | 29 | 4.6 | 8.8 | 55 |
| 157rat783 | 392.7 | 3.1 | 1 | 9.8 | 16 | 3.1 | 8.4 | 29 |
| 200dsj1000 | 898 | 0.8 | 6 | 9.6 | 52 | 0.8 | 9.4 | 51 |
| Average | | 6.3 | 12.7 | 16.8 | 25.2 | 6.3 | 14.5 | 39.3 |

is the cluster corresponding to the vertex v : $v \in cluster(v)$.) Thereby, while the initial 2-opt heuristic could decline some good 2-opt if $w(v_1v_3) = \infty$ or $w(v_2v_4) = \infty$, the extended 2-opt will pass round the infinite edges.

- Direct 2-opt heuristic [8] is excluded from the Local Search Procedure.
- Every time before starting the Cluster Optimization [8] we remove all vertices that cannot be included in the solution, i.e., if a fragment of the

solution corresponds to clusters C_1 , C_2 and C_3 and there is no edge from C_1 to $v \in C_2$ or there is no edge from v to C_3 then v can be excluded for the current Cluster Optimization run.

- Since the modified Local Search Procedure is more powerful than the previous one, we reduced the number of solutions in a generation and the termination condition is also changed (now $r = 0.2G + 0.03M + 8$ while previously $r = 0.2G + 0.05M + 10$ and $I_{cur} \geq \max(1.5I_{max}, 0.025M + 2)$ instead of

Table 6

Time benefit for SD.

| Instance | T_0 , sec | Vertices Red. | | Edge Red. | | Combined Reduction | | |
|------------|-------------|---------------|---------|-----------|---------|--------------------|-----------|---------|
| | | R_v , % | B , % | R_e , % | B , % | R_v , % | R_e , % | B , % |
| 157rat783 | 23.6 | 3.2 | 11 | 9.8 | 5 | 3.1 | 8.4 | 36 |
| 200dsj1000 | 100.3 | 0.8 | 47 | 9.6 | 36 | 0.8 | 9.4 | 42 |
| 201pr1002 | 54.9 | 1.9 | 12 | 9.2 | 22 | 1.9 | 8.7 | 43 |
| 207si1032 | 21.3 | 8.2 | 3 | 12.1 | -1 | 8.2 | 10.2 | 24 |
| 212u1060 | 88.8 | 3.3 | 8 | 14.4 | 35 | 3.3 | 11.2 | 42 |
| 217vm1084 | 78.1 | 22.2 | 49 | 24.0 | -2 | 22.2 | 8.9 | 57 |
| 235pcb1173 | 107.9 | 0.9 | 5 | 8.2 | 30 | 0.9 | 8.2 | 32 |
| 259d1291 | 169.4 | 3.7 | 9 | 12.4 | 25 | 3.7 | 9.8 | 26 |
| 261rl1304 | 140.4 | 1.5 | 9 | 7.9 | 47 | 1.4 | 7.2 | 66 |
| 265rl1323 | 132.6 | 1.8 | 20 | 7.8 | 20 | 1.7 | 7.0 | 32 |
| 276nrw1379 | 111.5 | 0.8 | 4 | 7.4 | 22 | 0.7 | 7.1 | 46 |
| Average | | 4.4 | 16.1 | 11.2 | 21.7 | 4.4 | 8.7 | 40.5 |

Table 8

Time benefit for GK.

| Instance | T_0 , sec | Vertices Red. | | Edge Red. | | Combined Reduction | | |
|----------------|-------------|---------------|---------|-----------|---------|--------------------|-----------|---------|
| | | R_v , % | B , % | R_e , % | B , % | R_v , % | R_e , % | B , % |
| 89pcb442.gtsp | 3.43 | 5.4 | 16 | 12.0 | -2 | 5.4 | 9.8 | 7 |
| 99d493.gtsp | 6.36 | 0.8 | 2 | 17.9 | 0 | 0.8 | 19.4 | 2 |
| 107att532.gtsp | 5.96 | 3.9 | 7 | 20.6 | 10 | 3.9 | 18.1 | 11 |
| 107si535.gtsp | 4.52 | 17.9 | 14 | 26.5 | 8 | 17.9 | 18.0 | 15 |
| 107ali535.gtsp | 8.91 | 5.4 | 17 | 16.6 | 19 | 5.4 | 14.3 | 25 |
| 113pa561.gtsp | 6.86 | 26.2 | 20 | 31.3 | 6 | 26.2 | 22.6 | 23 |
| 115u574.gtsp | 7.43 | 1.9 | -2 | 14.4 | -6 | 1.9 | 14.0 | -1 |
| 115rat575.gtsp | 7.29 | 3.1 | 0 | 11.3 | 0 | 3.1 | 10.9 | 2 |
| 131p654.gtsp | 5.47 | 13.5 | 11 | 32.7 | 2 | 13.5 | 28.3 | 13 |
| | | 8.7 | 9 | 20.4 | 4 | 8.7 | 17.3 | 11 |

$I_{\text{cur}} \geq \max(1.5I_{\text{max}}, 0.05M + 5)$, see [8]).

The modified algorithm does not reproduce exactly the results of the initial GK heuristic; it gives a little bit better solution quality at the cost of slightly larger running times. However, one can see (Table 8) that all the Reduction Algorithms proposed in this paper influence the modified GK algorithm positively.

Different reductions have different degree of success for different solvers. The Edge Reduction is more efficient than the Vertex Reduction for GKK and SD; in other cases the Vertex Reduction is more successful. For every solver except SG the Combined Technique is preferred to separate reductions.

Preprocessing is called to reduce the solution time. On the other hand, there is no guaranty that the outcome of the preprocessing will be noticeable. Thus, it is important to ensure at least that the preprocessing time is significantly shorter than the solution time.

Five GTSP solvers are considered in this paper. The first solver, **Exact**, is an exact one and, thus, it is clear that its time complexity is larger than $\Theta(N^{2.6})$ (see Section 4.1.) or even the upper bound $O(N^3)$. The time complexities of the other four solvers were estimated experimentally, i.e., experiments were conducted for problems of different size obtained from TSPLIB [16] and then an approximation for “solution time”/“instance

Table 9

| <i>SD work time estimation.</i> | | |
|---------------------------------|-------------------------|-----------------|
| Instance name | Real solution time, sec | Estimation, sec |
| 45ts225 | 0.6 | 0.72 |
| 45tsp225 | 0.5 | 0.72 |
| 46pr226 | 0.7 | 0.73 |
| 46gr229 | 0.8 | 0.76 |
| 53gil262 | 0.9 | 1.14 |
| 53pr264 | 1.2 | 1.17 |
| 60pr299 | 1.3 | 1.69 |
| 64lin318 | 1.8 | 2.04 |
| 64linhp318 | 1.6 | 2.04 |
| 80rd400 | 3.5 | 4.05 |
| 84fl417 | 3.5 | 4.59 |
| 87gr431 | 3.7 | 5.07 |
| 88pr439 | 4.7 | 5.36 |
| 89pcb442 | 5.5 | 5.47 |
| 107si535 | 5.7 | 9.70 |
| 113pa561 | 6.7 | 11.18 |
| 115u574 | 13.2 | 11.97 |
| 115rat575 | 11.1 | 12.04 |
| 131p654 | 10.2 | 17.71 |
| 134gr666 | 14.0 | 18.70 |
| 145u724 | 27.9 | 24.03 |
| 157rat783 | 23.6 | 30.40 |
| 200dsj1000 | 100.3 | 63.32 |
| 201pr1002 | 54.9 | 63.70 |
| 207si1032 | 21.3 | 69.59 |
| 212u1060 | 88.8 | 75.41 |
| 217vm1084 | 78.1 | 80.65 |
| 235pcb1173 | 107.9 | 102.19 |
| 259d1291 | 169.4 | 136.24 |
| 261rl1304 | 140.4 | 140.40 |
| 265rl1323 | 132.6 | 146.63 |
| 276nrw1379 | 111.5 | 166.05 |

size” dependence was found. The experimental complexity of SD is about $\Theta(N^3)$ and it is about $\Theta(N^{3.5})$ for GK, SG and GKK. Table 9 demonstrates the quality of our estimate for SD (here $T_{\text{estimate}}(N) = 6.3319 \cdot 10^{-8} \cdot N^3$).

Having the solvers time complexities, we can conclude that the preprocessing time is significantly smaller than the solution time for arbitrary large instances as the experimental complexity of preprocessing is smaller than the complexity of even the fastest of the considered solvers.

5. Conclusion

The GTSP reduction techniques allow one to significantly decrease the problem complexity at a very low cost. Experiments show that the Combined Reduction is often the most powerful among the presented algorithms and takes even less time than the single Edge Reduction. While the Vertex Reduction yields very natural problems and is successful with every considered solver, the Edge Reduction changes some edge weights to infinity values and, thus, not every solver benefits from it. However, in this paper, it is shown that a solver can be modified to process such problems well.

In this paper we consider the symmetric case only, i.e., $\text{dist}(x, y) = \text{dist}(y, x)$ for every pair of vertices x and y . Other vertex and edge reduction algorithms that can be immediately derived from Definitions 1 and 2 exist for the asymmetric case, and their time complexity is $O(N^3)$. Recall that N is the total number of problem vertices.

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References

- [1] D. Applegate, R. E. Bixby, V. Chvátal, and W. J. Cook. Concorde TSP Solver. Available at <http://www.tsp.gatech.edu>.
- [2] D. Ben-Arieh, G. Gutin, M. Penn, A. Yeo, and A. Zverovitch. Transformations of generalized ATSP into ATSP, *Operations Research Letters* 31 (2003), 357–365.
- [3] M. Fischetti, J. J. S. González, and P. Toth. A Branch-and-Cut algorithm for the symmetric generalized traveling salesman problem. *Operations Research* 45 (3) (1997), 378–394.
- [4] M. Fischetti, J. J. S. González, and P. Toth. The Generalized Traveling Salesman and Orienteering Problems. In *The Traveling Salesman Problem and its Variations* (G. Gutin and A. Punnen, eds.), Kluwer, Dordrecht, 2002.
- [5] R. Fourer, D. M. Gay. Experience with a Primal Presolve Algorithm. *Large Scale Optimization: State of the Art*. 1994. 135–154.
- [6] G. Gutin. Traveling Salesman Problems. In *Handbook of Graph Theory* (J. Gross, and J. Yellen, eds.), CRC Press, 2003.
- [7] G. Gutin, D. Karapetyan, and N. Krasnogor. A memetic algorithm for the generalized traveling salesman problem. NICSO’07, In *Studies in Computational Intelligence*, Vol. 129, pp. 199-210, 2008.

- [8] G. Gutin, and D. Karapetyan. A memetic algorithm for the generalized traveling salesman problem. *Natural Computing*, 2009, to appear.
- [9] P. Gutman, and I. Ioslovich. On the generalized Wolf problem: Preprocessing of nonnegative large-scale linear programming problems with group constraints. *Automation and Remote Control*, Volume 68, Number 8, August 2007, 1401–1409.
- [10] H. Huang, X. Yang, Z. Hao, C. Wu, Y. Liang, and X. Zhao. Hybrid Chromosome Genetic Algorithm for Generalized Traveling Salesman Problems. *Phys. Rev. E*, American Physical Society, 2004, 70.
- [11] G. Laporte, A. Asef-Vaziri, and C. Sriskandarajah. Some applications of the generalized travelling salesman problem. *Journal of the Operational Research Society* 47 (12) (1996), 1461–1467.
- [12] G. Laporte, H. Mercure, and Y. Nobert. A Branch and Bound Algorithm for a Class of Asymmetrical Vehicle Routing Problems. *The Journal of the Operational Research Society*, Vol. 43, No. 5, *Mathematical Programming in Honour of Ailsa Land*. (May, 1992), pp. 469–481.
- [13] G. Laporte, and F. Semet. Computational evaluation of a transformation procedure for the symmetric generalized traveling salesman problem, *INFOR* 37 (1999) (2), pp. 114–120.
- [14] Y. N. Lien, E. Ma, and B. W.-S. Wah (1993). Transformation of the Generalized Traveling-Salesman Problem into the Standard Traveling-Salesman Problem. *Information Sciences* 74, 177–189.
- [15] C. E. Noon, and J. C. Bean (1993). An Efficient Transformation of the Generalized Traveling Salesman Problem. *INFOR* 31, 39–44.
- [16] G. Reinelt. TSPLIB—A traveling salesman problem library. *ORSA Journal on Computing*, 3 (1991), 376–384, <http://www.crpc.rice.edu/softlib/tsplib/>.
- [17] M.W.P. Savelsbergh. Preprocessing and probing techniques for mixed integer programming problems. *ORSA Journal on Computing*, 6:445–454, 1994.
- [18] J. Silberholz, B. Golden. The Generalized Traveling Salesman Problem: a new Genetic Algorithm approach. *Extending the Horizons: Advances in Computing, Optimization, and Decision Technologies*, 2007, 165–181.
- [19] L. V. Snyder, and M. S. Daskin. A random-key genetic algorithm for the generalized traveling salesman problem. *European Journal of Operational Research* 174 (2006), 38–53.
- [20] M. F. Tasgetiren, P. N. Suganthan, and Q.-K. Pan. A discrete particle swarm optimization algorithm for the generalized traveling salesman problem. *GECCO '07: Proceedings of the 9th annual conference on Genetic and evolutionary computation*, 2007, 158–167.
- [21] X. Zhao, J. Lin, H. Huang, and Z. Hao. Void Vertex Genetic Algorithm for Production-Distribution Supply Chain GTSP Model. *Journal of Information and Computing Science*, Volume 1, Number 5, December 2006, 259–265.