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Article abstract

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ABSTRACT

Cummins and Outreville (1987) suggested that the cyclical nature of insurance profits might result from the combination of insurance prices created in an environment of rational expectations with lags in information and in profit reporting. Their theoretical model predicts the autoregressive properties of underwriting profits. They interpret the model as an AR2 process, and from this, they derive the range of cycle lengths and conclude that the hypothesis is in general agreement with the empirical results they present. This paper looks in more detail at empirical tests of this "rational pricing with lags" model of insurance cycles. We find the predicted cycle lengths are often not in the same range as most of the observed cycle lengths. The predicted joint range of the two AR2 coefficients is fairly restricted and the empirical data seem to be away from this region. The values of R^2 predicted by the theory are lower than most of those observed empirically. We show that under the Cummins and Outreville hypothesis, the autocorrelation coefficients at lags of three or more should be zero. Data by line of insurance from the United States for the periods 1960 to 1980 and 1973 to 1997, from Europe for the years 1955 to 1979, and from Asia for various periods show that this consequence of the model is contrary to the empirical evidence.

Keywords: Insurance, underwriting cycles, time series, ARIMA processes.

RÉSUMÉ

Cummins et Outreville (1987) ont suggéré que les profits liés à la nature cyclique de l'assurance peuvent résulter de la fixation des prix d'assurance établie dans un environnement de prévisions rationnelles avec décalages dans l'information et dans la déclaration des profits. Ils ont rapporté que cette hypothèse se trouvait

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vérifiée par leurs résultats empiriques. Cet article examine en détail les tests empiriques du modèle « *rational pricing* » dans les cycles d'assurance. Nous montrons que, sous l'hypothèse de Cummins et Outreville, les coefficients d'auto-corrélation avec des décalages de trois ans ou plus doivent être de zéro. Les données américaines par ligne d'assurance pendant les périodes de 1960 à 1980 et de 1973 à 1997, celles de l'Europe pendant les années 1955 à 1979 et celles d'Asie pendant diverses périodes montrent que cette conséquence du modèle est contraire à l'évidence empirique.

Mots clés **:** Assurance, cycles de souscription, séries temporelles, processus Arima.

INTRODUCTION

Understanding what factors are involved in the fluctuations of underwriting profits is important. If the factors that cause these fluctuations were well understood, the magnitude and effect of these cycles on insurance capacity and insurer insolvency could be mitigated. If those fluctuations could be predicted, several important consequences would follow. Insurers would be in a better position to manage their capacity and this would reduce their costs of raising capital. Policyholders would be in a better position allocate risk bearing between self-insurance and market insurance. Regulators would be in a better position to oversee the solvency of insurance companies. It is not surprising that a substantial literature exists dealing with these fluctuations.

Cummins and Outreville (1987, CO hereafter) developed a rational expectations model with informational lags (we call it COREM hereafter) to show that insurance cycles exist due to information lags even if the pricing of insurance does not rely on statistically-based autoregressive forecasting methods such as those suggested by Venezian (1985). They postulate that insurers can infer from market signals the expected value of future losses. Under these conditions they show that if there are no lags in the system there will be no cycles. If the information is dated by one period (that is, if data for current year are not available when prices for next year have to be set) then a first order autocorrelation in profits is predicted. If a calendar year straddles two rate-making periods, the reported profits in a calendar year are the weighted average of the profits in a rate-making year. Under those circumstances, the results exhibit second order auto correlation[1].

In their empirical analysis, CO fits a second order autoregressive (AR2) model with deterministic trend to data on the ratio of premium to losses[2] in individual calendar years. They use the results to compute nominal cycle lengths and compare these results with the range of cycle lengths that would be expected from their theoretical analysis. They find that there is a general correspondence between the empirical and analytical cycle lengths. From this, they conclude that their hypothesis is enough to create insurance cycle such as those observed in practice.

COREM has been evaluated by Niehaus and Terry (1993), Lamm-Tennant and Weiss (1997), Fung, Lai, Patterson, and Witt (1998), Chen, Wong, and Lee (1999), and Meier (2001). The results are mixed but the studies agree that an AR2 model provides a good description of the results. The first study is ambiguous in endorsing the rational expectations hypothesis against other alternatives due to possible data bias. The second concludes that "the rational expectations/ institutional intervention hypothesis explains many aspects of the underwriting cycle", though it does not attempt to discriminate between this and other hypotheses. The third uses vector autoregression on individual lines of insurance and finds support for the extrapolation hypothesis from analysis several lines and support for the rational expectation hypothesis from an analysis of impulse response functions for premiums. Chen, Wong, and Lee (1999) and Meier (2001) adopted AR(2) processes to find cycle lengths and conclude that their results support COREM.

The purpose of this paper is to determine whether COREM is consistent with the empirical findings. In the next section, we present briefly COREM and some of its consequences[3]. The paper then compares the published empirical findings first with the COREM predictions of cycle lengths and then with the parameter values obtained from fitting the AR2 model imposed by CO methodology. Following this, we discuss issues of specification. We then propose a different method of testing COREM and present results based on both previously published and recent data. A discussion concludes the paper.

FEASIBLE REGION FOR THE CUMMINS AND OUTREVILLE MODEL

CO used a process in which basic time intervals are established by the rate-making process. If the loss process is stationary and the lag in information is one period, then the reported losses in calendar year t, Π_t^R, are a weighted average of the losses for the current rate-making period, Π_t, and those in the preceding rate-making period, Π_{t-1}, with weight α. That is,

$$\begin{aligned}\Pi_t^R &= \alpha\Pi_t + (1-\alpha)\Pi_{t-1} \\ &= \alpha(\varepsilon_t + \varepsilon_{t-1} + \mu_t) + (1-\alpha)(\varepsilon_{t-1} + \varepsilon_{t-2} + \mu_{t-1})\end{aligned} \quad (1)$$

where ε_t is a permanent random component which becomes incorporated into future losses while μ_t is a temporary random element that affects only the losses during that rate-making period. The random variables ε_t and μ_t are stationary, independent, and not serially correlated.

CO then fit the losses defined by Equation (1) to an AR2 specification :

$$\Pi_t^R = a_0 + a_1\Pi_{t-1}^R + a_2\Pi_{t-2}^R + \zeta_t \quad (2)$$

the estimators of the coefficients will be given by :

$$\begin{bmatrix}\hat{a}_1 \\ \hat{a}_2\end{bmatrix} = \begin{bmatrix}\Sigma_{00} & \Sigma_{01} \\ \Sigma_{01} & \Sigma_{11}\end{bmatrix}^{-1}\begin{bmatrix}\Sigma_{01} \\ \Sigma_{02}\end{bmatrix} = \begin{bmatrix}\dfrac{\Sigma_{00}\Sigma_{01} - \Sigma_{01}\Sigma_{02}}{D} \\ \dfrac{\Sigma_{00}\Sigma_{02} - \Sigma_{01}^2}{D}\end{bmatrix} \quad (3)[4]$$

$$R^2 = \frac{\hat{a}_1\Sigma_{01} + \hat{a}_2\Sigma_{02}}{\Sigma_{00}}$$

where

$$\Sigma_{ij} = \left(\sum_{t=1}^{T}\Pi_{t-i}^R - \frac{\sum_{t=1}^{T}\Pi_{t-i}^R}{T}\right)\left(\sum_{t=1}^{T}\Pi_{t-j}^R - \frac{\sum_{t=1}^{T}\Pi_{t-j}^R}{T}\right) \quad (4)$$

and

$$D = \Sigma_{00}\Sigma_{11} - {\Sigma_{01}}^2$$

If we define $\theta = \alpha(1\text{-}\alpha)$ and $\eta = \sigma_\mu^2 / \sigma_\varepsilon^2$ and combine Equations (1) and (4), the corresponding elements of the variance-covariance matrix can be written as :

$$p\lim\Sigma_{00} = p\lim\Sigma_{11} = \sigma_\varepsilon^2[2(1-\theta) + \eta(1-2\theta)]$$

$$p\lim\Sigma_{01} = \sigma_\varepsilon^2[1 + \eta\theta]$$

$$p\lim \Sigma_{02} = \sigma_\varepsilon^2 \theta$$

$$p\lim \Sigma_{0k} = 0 \text{ for } k \geq 3 \quad (5)$$

We can rewrite $p\lim D = \sigma_\varepsilon^4 \left\{ \left[2(1-\theta) + \eta(1-2\theta) \right]^2 - (1+\eta\theta)^2 \right\} = \sigma_\varepsilon^4 D^*$, and Equation (3) as :

$$p\lim \begin{pmatrix} \hat{a}_1 \\ \hat{a}_2 \end{pmatrix} = \frac{1}{D^*} \begin{pmatrix} 2(1-\theta)+\eta(1-2\theta) & -(1+\eta\theta) \\ -(1+\eta\theta) & 2(1-\theta)+\eta(1-2\theta) \end{pmatrix} \times \begin{pmatrix} 1+\eta\theta \\ \theta \end{pmatrix}$$

The quantities a_1, a_2, and R^2 are jointly determined by two key parameters : 0 and 2. The cycle length is a function of a_1 and a_2 and is therefore determined uniquely by the same two parameters. Since $0 \leq \alpha \leq 1 \Rightarrow 0 \leq \theta \leq 0.25$ and $\sigma_\varepsilon^2 \geq 0, \sigma_\mu^2 \geq 0 \Rightarrow \eta \geq 0$, the COREM places the combinations of a_1, a_2, R^2 and cycle lengths in a fairly restricted feasible region.[5] As shown in Leng (2001), the regression coefficients a_1, and a_2 are not affected if a trend is included in regression, whereas the value of R^2 depends on how a trend is included in the analysis. Performing the AR2 regressions on detrended data preserves the relationships described above.

COMPARISON OF THE RESULTS WITH EMPIRICAL DATA

Preliminary Comparison

In this section, we discuss of the relation between the empirical results and theoretical range of the cycle length and R^2 based on COREM. The empirical data for this preliminary comparison[6] come from three sources : Venezian (1985) and CO (1987) and Chen, Wong and Lee (1999).[7] Venezian analyzed data from the United States on underwriting profit margins for 13 separate lines of insurance and for all the lines combined. CO analyzed the ratio of premiums to claims for all lines combined in 12 countries[8] and for automobile insurance in 6 countries. Chen *et al.* used data for four lines from 5 Asian countries and also analyzed all lines results. A summary of these results is given in Table 1. A large fraction of the empirical results are not second order autoregressive[9], a term that we identify with coefficients of the second lag significant at the 5 percent level.

Cycle lengths, according to COREM, should be between 6 and 8 years, but the observations fall outside the predicted range about as often as the fall inside it.

TABLE 1
SUMMARY OF THE EMPIRICAL RESULTS ON AUTOREGRESSIVE NATURE AND CYCLE LENGTH

	Venezian	CO		Chen et al.		Total
	by line	all lines	auto	all lines	by line	
Not AR(2)*	5	9	1	5	16	**36**
AR2 but not cyclical	0	0	0	0	1	**1**
Subtotal	5	9	1	5	17	**37**
Cycle length						
<=6	5	0	2	0	0	**7**
6-8	3	2	1	0	3	**9**
>=8	0	1	2	0	0	**3**
Subtotal	8	3	5	0	3	**19**
Total	**13**	**12**	**6**	**5**	**20**	**56**

*A result is classified as "AR2" if the coefficient of lag two significant at 5 percent level and as "not AR2" otherwise.

The predicted range for values of R^2 extends from zero to 0.583. All of Venezian's results for specific lines are in this range. Most of the results of CO and half of those of Chen *et al.* have R^2 values above the feasible range, but this may well be due to the fact that these values include the variance removed by a linear trend[10].

□ More Detailed Comparisons

COREM limits the possible combinations of the regression coefficients. The preliminary test is not very demanding because it is possible to obtain cycle lengths of between 6 and 8 years from parameter values that are not consistent with COREM.[11] A more detailed comparison of the combinations of the regression coefficients would be useful in assessing whether the data are consistent with COREM. In conduction, with such a comparison, we need to take into account two potentially important effects. One is that the theory only provides results that are valid in the limit of very large number of observations.

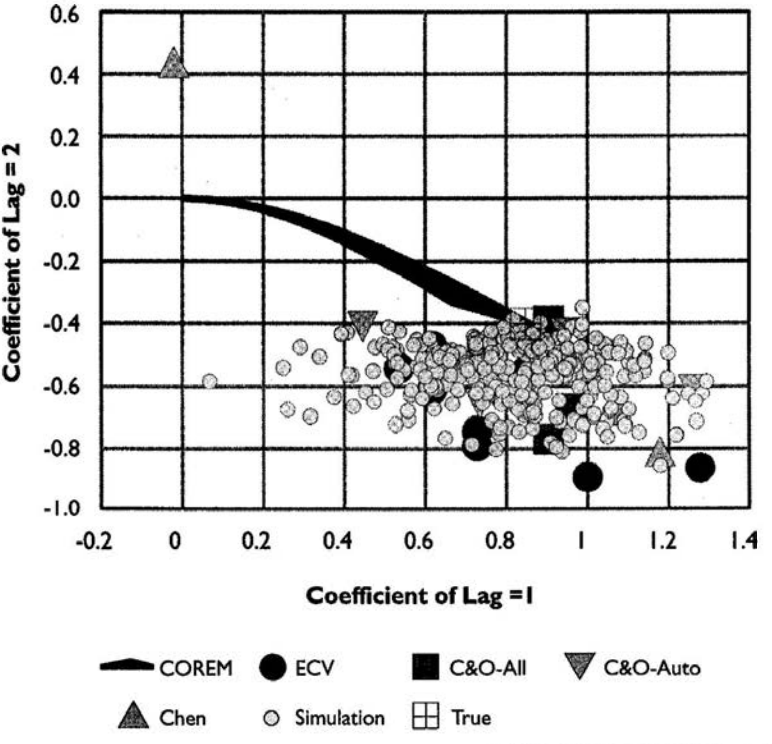
The second is that the published literature will not necessarily contain all the data; a preference for submission for publication of results that are statistically significant may be expected.

To take these factors into account, we rely not merely on the theoretically feasible regions determined by COREM, but on actual simulations[12] of Equation (1). Simulations were conducted of Equation 1, using normally distributed random variables for both the permanent and temporary random elements. The resulting series were then analyzed as AR2 processes in accordance with Equation (2) to obtain the results that CO would have obtained. The simulations used 15 to 100 points and 500 replications with a wide range of parameter values, both fixed and randomly distributed over their possible ranges. The simulation results that exhibited an a_2 coefficient significant at the five percent level were plotted on grids to show their relation to both the feasible regions and the empirical points censored in the same way[13]. Those presented in this section used a single set of parameter values and 25 points, but the results are typical of all the simulations conducted.

Figure 1 shows the feasible region for COREM, in the a_1, a_2 plane, along with the empirical and simulated observations. Only 3 of the empirical points fall in the feasible region. The distortion that arises from requiring a significant coefficient for the second lag is evident from the departure of the simulated points from the feasible region. The effect of the small number of observations is clear from the scatter of the points. These 2 effects can be separated by simulating the results with different numbers of observations at given significance levels and at different significance levels for given numbers of points [14]. It is noteworthy that the simulated points do not cover all the empirical points; this suggests that the data are not consistent with the postulated model.

Figures 2 and 3 show the feasible regions in the space defined by R^2 and one of the two coefficients. Only the points from Venezian (1985) are shown in these two figures because of inflated R^2 by the inclusion of a time trend in the other two papers. In Figure 2, the empirical points fall below and to the right the feasible region and below the cluster of simulated points, suggesting that the empirical values of R^2 are below the values required by COREM if compared at equal values of the coefficient of the first lag. In Figure 3, the points are below and to the left of the COREM feasible region and of the cluster of simulated points. This, again, suggests that the observed values of R^2 are below those required by the model. Thus, the empirical points do not conform with the model even after the adjustment for small samples and selection bias.

FIGURE 1
COMPARISON OF EMPIRICAL OBSERVATIONS, COREM'S THEORETICAL REGION, AND SIMULATION RESULTS IN THE a_1 -a_2 PLANE

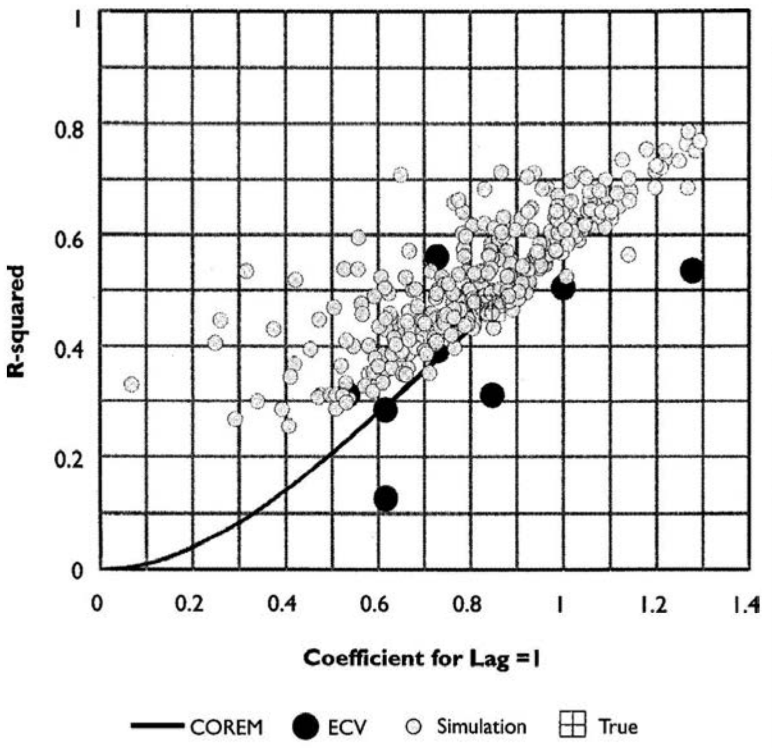


☐ Issues of Specification

Three issues arise with respect to the specification to be used in empirical analysis. The first issue relates to the completeness of the model[15], the second issue relates to the relation between the data and the model, and the third issue relates to the correspondence of the model and the specification used for analysis. We will discuss these briefly.

Since our interest is in the internal consistency of COREM, the first issue is tangential. It is worthwhile to point out, however, that any test must assume that the model is complete. If insurance companies pay attention to the magnitude of their reserves, as suggested by capacity constraint models[16], for example, COREM would have the omitted variables which could vitiate the results of any empiri-

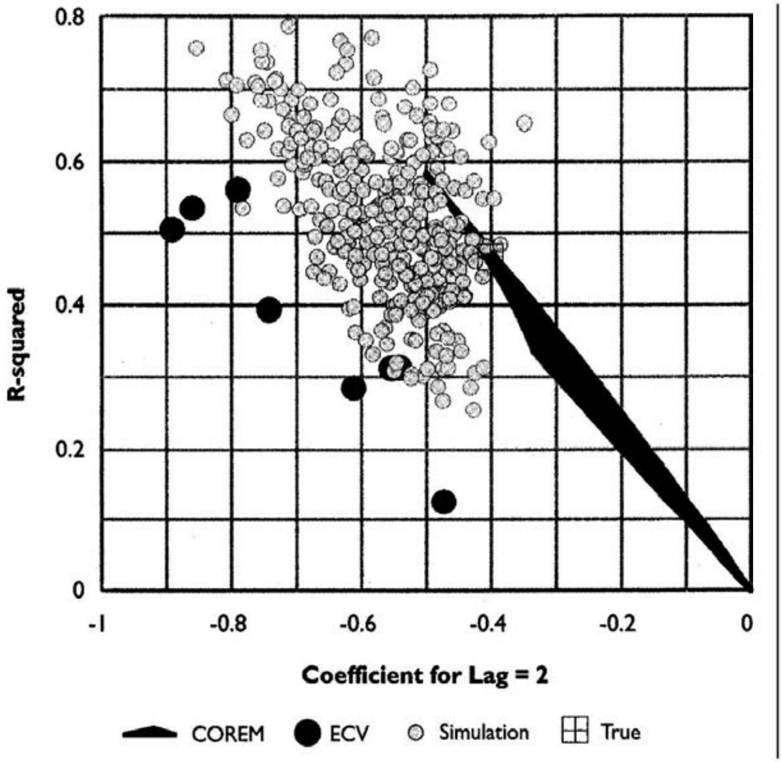
FIGURE 2
COMPARISON OF EMPIRICAL OBSERVATIONS, COREM'S THEORETICAL REGION, AND SIMULATION RESULTS IN THE R^2 - a_1 PLANE



cal analysis (Snedecor and Cochran, 1968; Theil, 1978). The same holds true if the model fails to account for the effect of changes in interest rates on the behavior of insurance companies or for the relationship between the premium to surplus ratio, the interest rate and the financially stable underwriting profit, which would be a case of misspecification (Maddala, 1977). Yet another area of potential misspecification is the effect of competition. The model assumes that there is a collective adjustment to rationality. If the market works through a series of adjustments, with individual companies reacting to the actions of their competitors, then this would have to be modeled explicitly in order to avoid specification errors.

The second issue deals with the relationship between the model and the data used. The theoretical analysis of CO is appropriate if the

FIGURE 3
COMPARISON OF EMPIRICAL OBSERVATIONS, COREM'S THEORETICAL REGION, AND SIMULATION RESULTS IN THE R^2 - a_2 PLANE



variable modeled is the aggregate profit for a fixed number of policies or the profit per policy. The variable actually analyzed empirically is neither of these. Actually, CO used the ratio of premiums[17] to claims in most of the regressions. In this case, some of the randomness enters through the denominator. If we re-derive Equation (1) in terms of the ratio of losses to premiums we find :

$$\Lambda_t = \frac{L_t}{P_t} = \frac{E(L_t)+\varepsilon_t+\mu_t}{E(L_{t-1})+\varepsilon_{t-1}} = \frac{E(L_t)}{E(L_{t-1})}\frac{(1+\hat{\varepsilon}_t+\hat{\mu}_t)}{(1+\hat{\varepsilon}_{t-1})}$$

where $\hat{\varepsilon}_t = \varepsilon_t / E(L_t)$ and $\hat{\mu}_t = \mu_t / E(L_t)$.

Elements of randomness affect the denominator and a Taylor expansion involves the change of sign on one of the variables. The analysis would need to be carried out in terms of this, rather than the original model.

The third issue relates to the correspondence of the model and the empirical framework. CO stated that Equation (1) exhibits second order autocorrelation and concluded that it follows an AR2 process. While the statement is correct, the conclusion is not : the equation does not represent an AR2 process. In fact, we saw in Equation (5), the co-variances at lags of three or more are identically zero. Box and Jenkins (1976) and Hamilton (1994) point out that this is characteristic of MA2 processes, whereas for AR2 processes, it is the partial, rather than the total, autocorrelation coefficients that vanish at lags of three or more. The impossibility of formulating Equation (1) as an AR2 process can be shown by simple substitution. If the process were indeed an AR2 process, then using Equation (2) to form the function :

$$\xi_t = \Pi_t^R - \left(a_0 + a_1\Pi_{t-1}^R + a_2\Pi_{t-2}^R\right)$$

would lead to values of a_0, a_1, and a_2 which will give a pure error term relating only to the current time t[18]. By substitution we find that :

$$\begin{aligned}\xi_t &= -a_0 + \alpha\varepsilon_t + (1-a_1\alpha)\varepsilon_{t-1} + (1-\alpha-a_1-a_2\alpha)\varepsilon_{t-2} \\ &\quad -\left[a_1(1-\alpha)+a_2\right]\varepsilon_{t-3} - a_2(1-\alpha)\varepsilon_{t-4} + \alpha\mu_t + (1-\alpha-a_1\alpha)\mu_{t-1} \\ &\quad -\left[a_1(1-\alpha)+a_2\right]\mu_{t-2} - a_2(1-\alpha)\mu_{t-3}\end{aligned}$$

For this expression to be a pure error term at time t the coefficients of all terms other than constants and the contemporary error terms must vanish identically, that is, we need to have the following seven expressions satisfied simultaneously :

$$1 : (1-a_1\alpha)=0$$

$$2 : (1-\alpha-a_1-a_2\alpha)=0$$

$$3 : \left[a_1(1-\alpha)+a_2\right]=0$$

$$4 : a_2(1-\alpha)=0$$

$$5 : (1-\alpha-a_1\alpha)=0$$

$$6 : \left[a_1(1-\alpha)+a_2\right]=0$$

$$7 : a_2(1-\alpha)=0$$

It is impossible to satisfy those conditions simultaneously.[19] Therefore, we conclude that error term in COREM is not equal to zero.

Actually if the values of Π_t^R were given by the COREM, then all autocorrelation coefficients beyond lag two would be zero. This leads to a very different pattern of autocorrelation coefficients than would be obtained from an AR2 process, for which the sequence of autocorrelation coefficients "is a damped sinusoidal function" (Hamilton, 1994, page 58). Thus the model does not correspond to a second order autoregressive process and analysis the data as such involves serious errors in specification; hence even if the analysis did not reject the model, this would be a poor indication of the model's validity.

A MORE DIRECT TEST OF COREM

So far, we have discussed the relation between the CO model and the data on the basis of tests much like the ones that CO applied to it. The tests are based on a mis-fitting of the basic equation and hence may not have much validity. Even so, they lend less support to the hypothesis than one would hope for. The COREM theoretical framework does suggest a more direct test of the model. Equation (1) predicts that the autocorrelation function of the profit will be exactly zero for three or more lags. Therefore, we test directly whether the autocorrelation coefficients at lags of three or more are significantly different from zero.

To test this hypothesis, we used two sets of data from the United States, one from Europe and one from Asia : data on profit margins from Venezian (1985) on 13 lines of insurance[20] in the United States, for the years 1960 to 1980, a more recent set of data on combined ratios for 20 lines of insurance[21] in the United States from 1973 to 1997[22], a set of 25 series of loss ratios for various lines in various European countries, obtained from Outreville (1984)[23], and data from Chen, Wong and Lee (1999)[24] for 4 lines of insurance in 5 Asian countries. The data were detrended to ensure that any small secular change during the period did not bias the correlation coefficients. The correlation coefficients at lags of k were then computed from the relation :

$$r_k = \frac{\sum_{i=0}^{N-k} \Pi_i \Pi_{i+k} - \frac{\sum_{i=0}^{N-k} \Pi_i \sum_{i=0}^{N-k} \Pi_{i+k}}{N-k}}{\sqrt{\sum_{i=0}^{N-k} \Pi_i \Pi_i - \frac{\sum_{i=0}^{N-k} \Pi_i \sum_{i=0}^{N-k} \Pi_i}{N-k} * \sum_{i=0}^{N-k} \Pi_{i+k} \Pi_{i+k} - \frac{\sum_{i=0}^{N-k} \Pi_{i+k} \sum_{i=0}^{N-k} \Pi_{i+k}}{N-k}}}$$

This definition corresponds exactly to the covariance between two independent variables rather than to the approximations usually adopted in time series analysis. It has the virtue that the small-sample statistics can be taken from standard regression methodology rather than approximated. The t-value corresponding to a value of r_k is given by :

$$t(r_k) = r_k \sqrt{\frac{(N-k-2)}{1-r_k^2}}$$

with N-k-2 degrees of freedom (Snedecor and Cochran, 1968).

The autocorrelation coefficients were evaluated for lags of 1 through 8. A range of probabilities was used because with the limited data set the power of tests will be low at conventional levels. For example, with 20 years of data and at a lag of 8 years, the correlation coefficient would have to be greater than 0.55 in absolute value to attain significance at the one percent level. Table 2 shows the number of significant coefficients observed in the 4 data sets, along with the probability that the number observed would have been equaled or exceeded under the null hypothesis of zero autocorrelation. The probabilities less than 0.05 are shown in bold face.

At lags of one and two are clearly significant in all four data sets. At lags of three, the number of significant coefficients is in excess for the first, third, and fourth data sets, but the excess is not clear for the most recent data set for the United States. It would appear that at lags of four and five, there is a significant excess of significant autocorrelation coefficients in all data sets, which reflects badly on the null hypothesis. At higher lags, there is a significant excess in all but the most recent U.S. data.

To ensure this result is not caused by the many comparisons being performed, we aggregated all the instances of significance for lags 3 through 8. The probability of a regression coefficient being significant, given the null hypothesis, is equal to the significance level selected for the test. The number of trials is six (for the number of lags tested) times the number of lines. An exact probability was determined from the binomial distribution with these parameters. The comparison was made for various significance levels on the t-test. The results are shown in Table 3. The results for all four data sets reject the null hypothesis at all the significance levels[25].

TABLE 2 – NUMBER OF SIGNIFICANT AUTOCORRELATION COEFFICIENTS AT INDIVIDUAL LAGS

a. Data from 1960-1980 for 13 lines of insurance in the United States

Lag	Level of Significance on the Autocorrelation Coefficient							
	0.025		0.050		0.075		0.100	
	N	P	N	P	N	P	N	P
1	12	**0.0000**	13	**0.0000**	13	**0.0000**	13	**0.0000**
2	5	**0.0000**	5	**0.0003**	5	**0.0018**	6	**0.0009**
3	5	**0.0000**	6	**0.0000**	6	**0.0002**	7	**0.0001**
4	1	0.2805	3	**0.0245**	3	0.0684	5	**0.0065**
5	1	0.2805	3	**0.0245**	3	0.0684	5	**0.0065**
6	1	0.2805	1	0.4867	4	**0.0130**	4	**0.0342**
7	0	1.0000	3	**0.0245**	4	**0.0130**	5	**0.0065**
8	1	0.2805	3	**0.0245**	4	**0.0130**	8	**0.0000**

b. Data from 1973-1997 for 20 lines of insurance in the United States

Lag	Level of Significance on the Autocorrelation Coefficient							
	0.025		0.05		0.075		0.1	
	N	P	N	P	N	P	N	P
1	12	**0.0000**	14	**0.0000**	14	**0.0000**	16	**0.0000**
2	2	**0.0882**	4	**0.0159**	4	0.0582	5	**0.0432**
3	0	1.0000	0	1.0000	0	1.0000	1	0.8784
4	1	0.3973	3	0.0755	6	**0.0027**	6	**0.0113**
5	2	0.0882	4	**0.0159**	5	**0.0142**	7	**0.0024**
6	1	0.3973	2	0.2642	4	0.0582	6	**0.0113**
7	0	1.0000	0	1.0000	1	0.7897	2	0.6083
8	0	1.0000	1	0.6415	1	0.7897	2	0.6083

c. Data from 1955-1979 for 25 line-country combinations in Europe

Lag	Level of Significance on the Autocorrelation Coefficient							
	0.025		0.05		0.075		0.1	
	N	P	N	P	N	P	N	P
1	10	**0.0000**	10	**0.0000**	11	**0.0000**	12	**0.0000**
2	2	0.1286	5	**0.0072**	6	**0.0091**	6	**0.0334**
3	3	**0.0238**	4	**0.0341**	5	**0.0356**	5	0.0980
4	4	**0.0032**	5	**0.0072**	5	**0.0356**	5	0.0980
5	4	**0.0032**	4	**0.0341**	7	**0.0019**	8	**0.0023**
6	3	**0.0238**	3	0.1271	6	**0.0091**	7	**0.0095**
7	3	**0.0238**	4	**0.0341**	6	**0.0091**	7	**0.0095**
8	4	**0.0032**	4	**0.0341**	6	**0.0091**	6	**0.0334**

d. Data from 4 lines in 5 Asian countries (Chen et al.)

Lag	Level of Significance on the Autocorrelation Coefficient							
	0.025		0.05		0.075		0.1	
	N	P	N	P	N	P	N	P
1	14	**0.0000**	15	**0.0000**	15	**0.0000**	15	**0.0000**
2	4	**0.0014**	6	**0.0003**	6	**0.0027**	7	**0.0024**
3	2	0.0882	4	**0.0159**	4	0.0582	6	**0.0113**
4	3	**0.0130**	3	0.0755	5	**0.0142**	6	**0.0113**
5	3	**0.0130**	4	**0.0159**	5	**0.0142**	5	**0.0432**
6	2	0.0882	2	0.2642	2	0.4487	3	0.3231
7	3	**0.0130**	3	0.0755	4	0.0582	6	**0.0113**
8	5	**0.0001**	7	**0.0000**	7	**0.0004**	8	**0.0004**

TABLE 3
OBSERVED AND EXPECTED NUMBERS OF SIGNIFICANT AUTOCORRELATION AT LAGS OF 3 THROUGH 8 YEARS

a. Data from 1960-1980 for 13 lines of insurance in the United States

Significance Level	Number Observed	Number Expected	Probability
0.010	4	0.78	**0.007933**
0.025	9	1.95	**0.000147**
0.050	19	3.90	**0.000000**
0.075	24	5.85	**0.000000**
0.100	34	7.80	**0.000000**
0.125	39	9.75	**0.000000**
0.150	41	11.70	**0.000000**
0.200	45	15.60	**0.000000**
0.300	56	23.40	**0.000000**
0.500	66	39.00	**0.000000**

b. Data from 1973-1997 for 20 lines of insurance in the United States

Significance Level	Number Observed	Number Expected	Probability
0.010	10	1.2	**0.000000**
0.025	16	3	**0.000000**
0.050	23	6	**0.000000**
0.075	29	9	**0.000000**
0.100	42	12	**0.000000**
0.125	45	15	**0.000000**
0.150	54	18	**0.000000**
0.200	64	24	**0.000000**
0.300	76	36	**0.000000**
0.500	87	60	**0.000000**

c. Data from 1955-1979 for 25 line-country combinations in Europe

Significance Level	Number Observed	Number Expected	Probability
0.010	11	1.50	**0.000000**
0.025	21	3.75	**0.000000**
0.050	24	7.50	**0.000000**
0.075	35	11.25	**0.000000**
0.100	38	15.00	**0.000000**
0.125	47	18.75	**0.000000**
0.150	53	22.50	**0.000000**
0.200	67	30.00	**0.000000**
0.250	74	37.50	**0.000000**
0.300	82	45.00	**0.000000**
0.500	103	75.00	**0.000003**

d. Data from 4 lines in 5 Asian countries (Chen et al.)

Significance Level	Number Observed	Number Expected	Probability
0.010	8	1.2	**0.000031**
0.025	18	3	**0.000000**
0.050	23	6	**0.000000**
0.075	27	9	**0.000000**
0.100	34	12	**0.000000**
0.125	39	15	**0.000000**
0.150	47	18	**0.000000**
0.200	54	24	**0.000000**
0.300	66	36	**0.000000**
0.500	84	60	**0.000007**
0.750	103	90	**0.002792**

CONCLUSION

It is certain that some of the elements of COREM are at play in reality. Certainly the calendar year results are a mixture of results from different ratemaking periods[26]. On the whole, however, the COREM does not account well for the observed results of fitting an AR2 process by ordinary least squares. The model can fit the range of cycle lengths about half of the time but does poorly at predicting the pairs of coefficient values from which the cycle length is calculated unless the parameter values are restricted. The range of R^2 values predicted by the model is close to the range of the empirical data, but if we view the parameter values to R^2 from the second order autoregressive model, then the apparent fit breaks down. Small sample size and bias from the selection of significant results do not account for the difference between theory and empirical observations. An important consideration is that COREM leads to a profit generating process that is more closely related to an MA2 process than to the AR2 process which has been used by CO (1987) and others in interpreting the data.

The autoregressive process of order two has *partial* autocorrelation coefficients of zero at lags of three or more, but the *total* autocorrelation coefficients are not zero. COREM has *total* autocorrelation coefficients equal to zero at lags of three or more. The empirical evidence shows that autocorrelation coefficients at lags of three or more are significantly different from zero, thus leading to a rejection of COREM. This evidence may be challenged since there is evidence that the time series characteristics of combined ratios in the United States were not stable in the period 1958-1997.[27] Such a challenge might be valid for the data from the United States for the period 1973-1997, since Leng (2001) documents such problems. For the other sets, however, there is no evidence of such instability.

There is a possibility that these results are influenced by outliers, notably in the United States during 1984 to 1986.[28] More relevantly, it is possible that the disturbances over this period were not independent, contrary to the assumption in COREM. The existence of outliers is recognized in COREM by the existence of temporary random noise and the model predicts zero autocorrelation for order three and higher. Moreover, the import of such a challenge is reduced by observing that the data for the period from 1960 to 1980, which do not include these outliers, reject COREM just as strongly as the data which include this potentially abnormal period.

It is worth pointing out that even though our analysis shows that empirical results are not consistent with COREM, it does not imply that underwriting cycles are caused by irrational behavior from insurers. It merely suggests that other features need to be incorporated in the model.

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☐ Notes

1. This is appropriate when the loss distribution has a stationary mean as assumed in COREM. If the losses are not stationary then the last several months of each ratemaking year would lead to underwriting losses and the first several months of the rate-making year would result in profits. This would require a different formulation of the results reported for a calendar year which overlaps two rate-making years.

2. In some cases CO used the underwriting profit margin and loss ratio but, in most cases, they used the ratio of premiums to losses because of missing information on expenses.

3. A detailed analysis of the relationship between the model and the empirical methods used in its evaluation is available in Leng (2001).

4. Note that $\Pi_u^R \Pi_v^R = (-\Pi_u^R)(-\Pi_v^R)$. Hence in the following relations it does, it does not matter whether we interpret Π_t^R as the reported net losses at time *t* or as the profit at time *t*.

5. Two papers discuss the possible range of η. Nelson and Plosser (1982) argued that the variance of the permanent component should be greater than that of the transitory component, that is $0 \le \eta \le 1$. Under these conditions, we would have $0.375 \le a_1 \le 1$, $-0.5 \le a_2 \le -0.125$, $6.21 \le CL \le 8$, and $0.125 \le R^2 \le 0.5833$. On the other hand, Hodrick and Prescott (1980) argued that the transitory component should have the larger variance, that is $\eta \ge 1$. Under these conditions, we would have $0 \le a_1 \le 0.8974$, $-0.436 \le a_2 \le 0$, $6 \le CL \le 7.6297$, and $0 \le R^2 \le 0.5064$.

6. In order to determine whether the data are consistent with theory, it is necessary, as a minimum, to have the variance covariance matrix for the regression coefficients. These are not available in the original papers and the data appear to be irretrievably lost. We have dealt with this problem, along with some others, in the next section.

7. Those studies are included because they reported coefficients of AR2, significance level, cycle lengths, and R^2.

8. This excludes Italy which has two different time periods in CO.

9. A caveat is in order. COREM is based on a stationary process. The regressions used by CO and by Chen *et al.* include a time trend. The additional variable would affect the value of the regression coefficients if it is partially collinear with the included variables. CO, who had information about the collinearity, compared the cycle lengths based on the empiri-

cal coefficients with those predicted by their model. We assume this implies the effect of multicollinearity was negligible.

10. The loss process underlying COREM is a unit root process. The inclusion of a deterministic linear trend is an ad hoc assumption made by CO and is not consistent with COREM.

11. As an example, the pair $a_1 = 1$, $a_2 = -0.9$ is not within the region permissible under COREM, but the computed cycle length is 6.2 years which is within the range allowed by COREM.

12. The program was implemented in Quattro Pro for Windows. The parameter values used in two figures were : 0 = 0.3 and 2 = 0.1875. This last value corresponds to ∀ = 0.25. The "True" value shown in the figures refers to the coordinate values that correspond to these parameter values.

13. Censoring on the basis of significant values of a_2 restricts the results to those that may be deemed to be cyclical.

14. The simulation program allows observations up to 100 points and allows censoring at any significance level between 0 and 100% of the coefficient of the second lag. Simulations with 15 to 100 points showed that the scatter of the simulated results decreases substantially as the number of points increases, regardless of the level of significance demanded for the coefficient of a_2. For any given number of observations, the departure of the simulated results from the feasible region decreased as the significance level demanded of the a_2 coefficient was changed from 5% to 10% and eventually to 100%.

15. The completeness of a model is usually discussed in terms of "omitted variables." Technically this phrase has come to mean that the functional description needs additional terms of the form of a coefficient times an independent variable. A model may be incomplete in other ways. For example, it may include a term with a constant times and interest rate when it ought to contain a term of a constant times an interest rate times a premium to surplus ratio. We use 'completeness' to emphasize the misspecification may go beyond the omission of variables.

16. Models including capacity constraints have been discussed by Winter (1988), Gron (1994), and Doherty and Garven (1995). The validation of these models leaves much to be desired. None of these models includes the interaction of taxes and investment income. Such an interaction brings into the relations the product of the premium to surplus ratio, the risk free interest rate, and a function of the tax rates, as shown by Hill and Modigliani (1981). Thus the tests of capacity constraint models are all based on incomplete models in which one of the variables omitted is directly correlated with capacity. Given the omission, the finding that the coefficient of capacity is significant would not be surprising even if capacity constraints did not exist.

17. There is no indication whether the data relate to premiums written or premiums earned. The empirical results may therefore be distorted by uneven growth from year to year.

18. If the value of $>_t$ depends on lagged errors, then $>_t$ must exhibit serial autocorrelation and the process will not be AR2.

19. Expressions 4 and 7 are identical and they dictate that either ∀ = 1 or $a_2 = 0$ or both. If ∀ = 1, $a_1 = 1$ to satisfy Expression 1, then we would need $a_2 = -1$ to satisfy Expression 2, which is contradicted to $a_2 = 0$ from Expression 4. Expression 3, the same as Expression 6, would then require that ∀ = 0, inconsistent with the premise that ∀ = 1. If $a_2 = 0$ the process would be AR1 rather than AR2.

20. The lines included were nine non-auto lines and four auto lines. The non-auto lines were : fire, allied lines, homeowners multiple peril, commercial multiple peril, ocean marine, inland marine, workers compensation, property damage liability, bodily injury liability. The auto lines were : bodily injury liability, property damage liability, collision, and fire, theft, and comprehensive.

21. The lines were fire, farm owners, earthquake, homeowners multiple peril, commercial multiple peril, ocean marine, inland marine, workers compensation, other liability, medical malpractice, aircraft, private passenger auto liability, commercial auto liability, private passenger physical damage liability, commercial auto physical damage liability, fidelity, surety,

crime, boiler and machinery, and reinsurance. The source of the data is reported by A.M. Best Co.

22. Medical malpractice was separated from general liability in 1975, so for medical malpractice and other liability the period is 1975 to 1997. Reinsurance was listed only after 1977.

23. The authors are grateful to Dr. Outreville for making a copy of the data available. The data used included 25 line-country combinations. For Germany : accidents, liability, motor cars, fire, and transport; for Italy : aviation, automobile, cattle, theft, fire, accidents, automobile liability, other liability, and transport; for Sweden : property-liability, automobile liability, automobile accidents, cattle, and transport; for Switzerland : accidents, liability, fire, transport, automobiles, and other. The data were from 1955 to 1979 except for Italy, in which case, the series ended in 1978.

24. The authors are grateful to Professor Chen for making available the spreadsheets containing his data and analysis. The data used related to MAT, fire, automobile, and miscellaneous insurance in Japan, Korea, Malaysia, Singapore, and Taiwan. The data from Malaysia are suspect because loss ratios are most frequently above one and average 1.7, 3.6, 0.9, and 2.5. Eliminating this data would not affect the conclusions.

25. The same calculations were repeated for the first data set using the less conservative tests normally used in time series analysis. The results reject the null hypothesis much more strongly than the conservative results shown here.

26. The model may, however, be very different from that discussed by CO. Losses are subject to trends, both in frequency and in severity. If the net trend is upward, insurers who set their premiums so that they will achieve a reasonable profit over a full ratemaking period will experience higher than average profits during the early part of the ratemaking period and losses or lower than average profits during the late part of that period. The weighted average is probably inappropriate when the losses are not stationary.

27. See, for example, Leng (2000) and Leng, Powers, and Venezian (2002).

28. We would like to thank anonymous referee for pointing out this possibility.