## Geometrical Knowledge in Early Sri Lanka

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## Résumé de l'article

This article addresses on history of mathematics (specially one of its specific branch, geometry) in Sri Lanka. Despite the large amount of research on the history of mathematics in India, China and the Middle East, that on Sri Lanka still remains limited. Sri Lanka had close relations with all these regions from ancient times and knowledge of mathematics should not be an alien subject there. This article tries to address the paucity of research on the history of mathematics in Sri Lanka while emphasizing the local character of that ancient knowledge.

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# History of Science in South Asia 

A journal for the history of all forms of scientific thought and action, ancient and modern, in all regions of South Asia

# Geometrical Knowledge in Early Sri Lanka 

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# Geometrical Knowledge in Early Sri Lanka 

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The earliest records of mathematics show it arising in response to the practical needs of societies with regards to agriculture, business, manufacturing, and ritual practices. Depending on these needs, two important ideas were developed, i.e., the multiplicity of quantities and the demarcation of space. The first concept involved counting (of animal, people, days, etc.) and the second involved areas and volumes (of land, water, crop, yield, etc.), which in turn evolved into two important branches of mathematics, i.e., arithmetic and geometry. ${ }^{1}$

It is often said that early geometry, except that of the Greeks, was intuitive; it sought facts relating to measurement without attending to prove any theorem by deductive reasoning. ${ }^{2}$

The etymology of the word "geometry" itself comes from two Greek words meaning earth and measure, indicating that the subject has its origin in land surveying. Among the Egyptians, the geometry of surveying was considered to be the science of "rope-stretches" (harpedonap'tae). ${ }^{3}$ The Śulba-sūtra (ca.600-500 все), ${ }^{4}$ an Indian genre, grew out of the need to ensure strict conformation of the orientation, shape and areas of altars to prescriptions laid down in the Vedic scriptures. There, procedures for constructing altars in different shapes in accordance with accurate geometric methods were discussed. ${ }^{5}$ Chiu Chang Suan Shu (second or early third century се), one of the most influential Chinese mathematical books, covered land surveying among other subject fields like engineering and taxation in its nine chapters. ${ }^{6}$ Throughout the Islamic world, buildings decorated with intricate geometric designs can be found, depicting the space in decorative forms. These are a common feature of Islamic art, which has had an ornamental tradition, since the Islamic religion generally proscribes the portrayal of living things. ${ }^{7}$

[^1]Despite its archaeologically proven urbanized settlements since the seventh century все, ${ }^{8}$ and textually recorded past since the third century все, ${ }^{9}$, studies on the history of geometry (or any branch of mathematics for that matter) in Sri Lanka are far behind similar scholarship in India, China and the Middle East. Considering the fact that Sri Lanka has had close relations with all of these regions since pre-Christian times, some form of geometrical knowledge might not have been alien to early Sri Lankan society. This paper tries to fill this gap, at least in a limited manner, by addressing the geometry related materials in Buddhist Pāli commentaries (possibly fifth century ce), primary sources based on older Sinhalese works. ${ }^{10}$ The selection of these primary sources demarcates the time and space covered in this study. With regard to the time, the term "early" indicates the period from third century все to the fifth century $с е$, the time of the introduction of Buddhism to the Island and the time of the compilation of Buddhist Pāli Commentaries. References will be made to subsequent literature when further clarification of the commentarial contents is required. With regard to location, this study is specific as to Sri Lanka. It is commonly accepted that the current Buddhist Pāli Commentaries were translations of previously existing Sinhalese Commentaries (Helatuvā), implying that the information in the commentaries was well known to Sri Lankan society at that time. ${ }^{11}$ This is not to say that such information originated and evolved exclusively within Sri Lanka. Given the well established connectivity from far eastern China to western Rome, there was always a possibility of knowledge transfers among societies and the discussion in this paper is subject to that possibility. ${ }^{12}$

The above-mentioned primary sources were intrinsically ecclesiastical and not meant for describing secular details in society. As such, we cannot expect to find a dedicated and thematic terminology related to mathematics - or any other secular subject - within these sources. There are two main possible reasons for the non-specific nature of the mathematical terms found in these texts. First, and more obvious, such terms might have been used in parlance but did not appear in the sources because of their ecclesiastical nature. Second, such terms might not have been required, as mathematics was developed in Sri Lanka as an

[^2][^3]application-based subject, one that did not call for for subject rigour nor highly specific and dedicated terminologies.

One of the earliest geometrical descriptions appearing in the Sri Lankan chronicles explains the laying of the Mahāthūpa, the great memorial mound (cetiya) built during the reign of Dutṭhagāminiabhaya (161-137 все) at the then capital of the island Anuradhapura. ${ }^{13}$

According to the Mahāvamsa (fifth century ce), the circumference of the cetiya was marked by using a cord fixed to a golden pole;

He [the king], commanded that the pure turning staff (for tracing the circular boundary), made of silver and secured (by means of a rope) to a post of gold, be grasped by a minister of noble birth, being resolved to allot a great space for the cetiya, he ordered him to walk around (with the turning staff in his hands) along the ground already prepared. ${ }^{14}$

The Mahāvamsa further detailed this incident which suggests a geometrical visualization linking the circumference of the base with overall volume of the structure. Realizing that the cetiya could not be constructed during the lifetime of the king, one of the saints (arahants) present at the occasion advised the reduction of the circumference, which was executed. After the laying of the base, some architects (vaddhaki) were consulted for the possible shape of the cetiya. The Mahāvaṃsa reference continued:

He [the king] questioned him saying: "In what form wilt thou make the cetiya?" At that moment Vissakamma entered (and possessed) him. When the master-builder had had a golden bowl filled with water, he took water in his hand and let it fall on the surface of the water. A great bubble rose up like unto a half-globe of crystal. He said: "thus will I make it". ${ }^{15}$

This description of the laying of the base and visualization of the shape expresses spatial concepts in the context of the Mahāthūpa construction. Considering its original height of 90 m , construction procedures would not have been possible without such considerations. Although not mentioned in the literary sources, the

[^4]in size either in Sri Lanka or in the whole of the Buddhist world (Paranavitana 1988:7).
14 Mahāvaṃsa xxix: 49-51, translation Geiger (Mahāvamsa: 195). Unless otherwise indicated, as is done here, any translations appearing in this paper are the author's own. 15 Mahāvaṃsa xxx: 11-12, translation Geiger (Mahāvaṃsa: 199).


Figure 1: The Mahāthūpa as seen today from (a) an elevation view showing the spherical shape of the dome (Lankapic 2007) and from (b) an aerial view showing the square shape of the terrace, approximately aligned along four cardinal directions (Google 2020)
terrace of the Mahāthūpa is a perfect square, aligned along the four cardinal directions, again a geometrical application in its construction, approximately 2100 years ago.

We have already mentioned the origins of geometry in land-surveying and the professionals involved in Egypt (harpedonap'tae) and among specialists in ritual layout (śulbavid) in India. A similar rank is denoted in Sri Lankan Buddhist Pāli Commentaries with the expression "the minister who bears the ropes" (rajjugāhaka-amaccho). ${ }^{16}$ Such practical applications would have provided initiative for geometrical propositions, which are depicted through doctrinal and secular details like, "a circle is drawn by turning," "perpendicular is ascertained by the plumb line," "horizontal is ascertained by a water surface," "flatness is denoted by stretched out ox hide" and "roundness is denoted by water bubble." Familiarity with these propositions is illustrated in the references below, where geometric concepts are linked with general observations.

The relationship between the "applied thought" (vitakka) and "sustained thought" (vicāra) is explained as an analogy; when one is drawing a circle, the pin that stays fixed down in the centre is like the "applied thought," which directs on to the object, and the pin that
revolves around is like "sustained thought," which continuously presses. ${ }^{17}$
When a cetiya was constructed [during the time of Kassapa Buddha], the size of the base was limited saying that the cetiya should be feasible to be renovated in the future, and the periphery was marked by a rope. ${ }^{18}$
Realizing that the fruits were thrown by the hunter who was hiding on the tree, the deer addressed the tree [pretending] and said, "You, in the past, were dropping your fruits straight down like a plumb line (olambakam.). But today, that "law of tree" is violated [in the most suspicious manner], hence I will go to another tree for my meals."19
In order to prove the strength and fineness of the needles he made, the craftsman placed one needle on an anvil and a water vessel underneath the anvil, and struck the needle. The needle piercing the anvil lay across the surface of the water not moving a hair's breadth up or down. ${ }^{20}$

At one stage, the "area of concentration" appears to the meditating bhikkhu like an ox hide stretched out with a hundred pegs over the earth ridges and hollows, river ravines, tracks of scrubs and thorns, and rocky inequalities in area to which it has been extended. ${ }^{21}$
An extremely beautiful woman is termed as a "bindutthan $\bar{\imath}$, " which means the "women who have the bubble-like breasts". ${ }^{22}$

Geometry is the study of spatial forms involving the recognition, visualization and drawing of shapes, and the description of their features and properties. Citations from commentaries that discuss geometric terms found in circa fifth century ce Sri Lankan Pāli source texts, such as the Samantapāsādik $\bar{a}$ (on the Vinayapiṭaka), Papañcasūdanī (on the Majjhimanikāya), Sāratthappakāsin̄̄ (on the Sam̀yuttanikāya), Atthasālini (on the Abhidhammapiṭaka) and Jātakaṭ̣hakathā (on the Jätakas), illustrate this phenomenon:
"kataka" is meant for the stand used when feet are washed. That could be circular (vatta) or quadrilateral (caturassa) in shape and resembles to the pericarp of a lotus (padumakaṇnikākāra). ${ }^{23}$

[^5][^6]When the $\operatorname{sim} \bar{a}$ is a circle, the length should be measured from the mid point, $1^{112}$ yojana in four directions thus 3 yojanas in a direction. For the sima of quadrilateral (caturassam), the length should be 3 yojanas from side to side. For a sīmā of a triangle (tikonaṃ), the length should be 3 yojanas from vertex to vertex. ${ }^{24}$
Taking that there is a life inside body, [one] wonders whether the shape of that living matter is long (dīga), short (rassa), quadrilateral (caturaṃsa), hexagonal (chalamsa), octagonal (atṭhamsa) or hexadecagon (solasamsa). ${ }^{55}$

Taking there is a life inside body, [one] wanders whether the shape of that living matter is long (dīga), short (rassa), quadrilateral, pentagonal, hexagonal, heptagonal, octagonal, decagonal or hexadecagonal (caturassa pañcachasatta aṭthadasasolasa sādīnaṃ).. ${ }^{26}$
Round shape is called "vatṭa" (circle). Egg shape is called parimandala. Shape having four sides is called "caturassa" (quadrilateral). This is the case of hexagon (chalamsa), etc. ${ }^{27}$

The snake danced, taking the shapes of a circle (vatṭa) or a quadrilateral (caturassa), according to the wish of snake charmer. ${ }^{28}$

Examples similar to those addressing the circle (vaṭa), triangle (tunäs) and quadrilateral (sivuräs) also appear in subsequent Sinhalese works. ${ }^{29}$ An association of the concept of shape with the presentation of an unnown object is evidenced in these Sinhala works. For example, the Pansiyapaṇasjātaka (fourth century $\mathbf{C E}$ ) describes the lie-telling of king Cetiya, a figure described as an incorrigible liar in Buddhist legends and in Sinhalese folklore. In the story, people are described as wondering about the shape of a lie as a physical object, and speculating whether a lie has a conical, circular or rectangular shape. ${ }^{30}$ These types of associations are not evident in the earlier Jātakaṭthakath $\bar{a}$, which presents a narrative parallel to the Pansiyapaṇasjātaka.

Because of its properties, the circle, as a shape, is associated with basic doctrinal issues in Buddhist philosophy. Triangles, quadrilaterals, hexagons, octagons and hexadecagons are formed by intervals on lines, but a circle is the curved

24 Samantapāsādik $\bar{a}$ III: 780 . These shapes imply a circle with a diameter of 3 yojanas, a square with a side of 3 yojanas and an equilateral triangle with a side of 3 yojanas. Here, sima means the boundaries of the buildings where the bhikkhus conduct their ecclesiastical acts (uposatha). Cūlavamsa details the process of demarcating $\operatorname{sim} \bar{a}$ of the uposatha-house named

[^7]line formed by loci of a point moving at a constant distance from a fixed point. A circle is symmetrical around its own centre, in that every rotation of the circle around its centre rotates the circle onto itself. If AOB (figure 2) is the diameter of the circle with centre $O$, then a reflection on the line $A O B$ would reflect the circle onto itself, thus every diameter of a circle (like $\mathrm{A}^{\prime} \mathrm{OB}^{\prime}$ ) is an axis of symmetry. As a result of these symmetries, any point $P$ on the circle can be moved to any other point $\mathrm{P}^{\prime}$, making every point on the circle essentially the same as every other point - a feature no other plane geometric shape possesses. The following examples illustrate the power of these properties of the circle in illustrating the entwined Buddhist doctrines of 'dependent origination' (pațiccasamuppāda) and 'cycle of rebirth' (samsāra).


Figure 2: Circle as a shape presenting every possible rotation symmetry.

The exposition of the "round" has been given with one of two things as the starting point: either ignorance, as it is said, "No first beginning of ignorance is made known, bhikkhus, before which there was no ignorance, and after which there came to be ignorance. And while it is said thus, bhikkhus, nevertheless it is made known that ignorance has its specific conditions. [same description for craving also]. But why does the Buddha give the exposition of the "round" with these two things as starting points? Because they are the outstanding cause of kamma that leads to happy and unhappy destinies". ${ }^{31}$
Here, formation and becoming are the round of kamma. Ignorance, craving and clinging are the round of defilements. Consciousness, mentality-materiality, the sixfold base, contact and feeling are the round of result. So, this wheel of becoming, having a triple round

[^8]with these three rounds should be understood to spin, revolving again and again, forever, for the conditions are not cut off as long as the round of defilements is not cut off. ${ }^{32}$

The conceptions of geometrical shapes discussed above provide a basis for exploring the knowledge of properties of shapes, including dimension, perimeter, area, similarity, symmetry, etc., as referenced in commentarial literature. For example, properties like perimeter and area are derived from the dimensions of a shape (i.e., the perimeter of a circle is the multiplication of its diameter by a fixed ratio, $\pi$ ). In some cases the dimension of a shape implicates a mathematical relationship (i.e., the hypotenuse of a right angle triangle is equal to the sum of the squares of the other two sides). Knowledge of mathematical relationships such as these are found in commentarial material, along with correct and approximately correct formulae to identify such relationships.

The relationships between the three sides of a right triangle, popularly known as the Pythagoras theorem, was known in several cultures well before the time of Pythagoras. ${ }^{33}$ In a much later period, around the fifth century Ce commentarial material in Sri Lanka also evidence traces of these ideas while describing the size of a Majjhimapadesa (or Madhya mandala, the region where the Buddhas of the past and the future are born.

Majjhimapadesa is of 300 yojana in length, 250 yojana in width and 900 yojana in perimeter. ${ }^{34}$
Madhya maṇ̦dala is of 300 yodun in length, 250 yodun in width and 900 yodun in perimeter. ${ }^{35}$

[^9]> attribution of the theorem to Pythagoras in the Greek sources themselves comes very late; the first mention is found in Cicero's writing (circa 50 BCE) The earliest proof of the theorem in the Greek tradition is no later than Euclid's (circa 300 to 400 BCE) Goonatilake 1998:119. In the Chinese arithmetical classic Chou Pei Suan Ching ( 300 to 400 BCE) the numerical relationship 4,3 and 5 is given Bose et al. (1971:149).
> 34 Papañcasūdanı̄ II: 167. The same information appears in Sumañgalavilāsin̄̄ I: 286, Papañcasūdan̄̄IV: 116, Jätakaṭthakathā I: 48.
> 35 Pūjāvaliya: 117, Pansiyapaṇasjātakapota I: 24 .

Madhya mandala is of 1200 gavu in length, 1000 gavu in width and 3600 gavu in perimeter. ${ }^{36}$

How should one interpret the shapes illustrated by these dimensions? At the very least do they suggest a unique geometrical shape? The criterion that I have adopted here is to assume a fitting shape on an intuitive basis, and then to prove the consistency of the given dimensions. I have done this by attributing those dimensions to the assumed shape based on the possible contemporaneous geometrical knowledge. Thus, I have inferred that the shape given by above dimensions is an isosceles triangle as shown in figure $3 .{ }^{37}$


Figure 3: Isosceles triangle depicting the shape of the Majjhimapadesa as per Sumañgalavilāsin̄̄, Papañcasūdan̄̄, Jātakaṭthakathā, Pūjāvaliya and Pansiyapaṇasjātakapota.

The given dimensions are, $B D=300, A C=250$ and perimeter $A C D=900$ Assuming the relationship known as the Pythagoras theorem was available,

$$
\begin{aligned}
A D^{2} & =A B^{2}+B D^{2} \\
& =(A C / 2)^{2}+B D^{2} \\
& =125^{2}+300^{2}
\end{aligned}
$$

Gives, $A D=325$

36 Saddharmaratnāvaliya: 1174. This is same information as appears in Pūjāvaliya also. The measures are left undisturbed, but the
units changed with the standard conversion rate, 4 gavu $=1$ yodun.
37 Jayawardana 2017: 73.

Therefore,

$$
A C+C D+D A=900
$$

Hence, if $B D=300$ and $A C=250$, then the perimeter $A C D(=A C+C D+$ $D A)=900$ is proved, based on the relationship known as the Pythagoras theorem, suggesting that such knowledge is attested in the commentarial passages cited above. ${ }^{88}$

The shape of the Majjhimapadesa is presented in an alternative manner in some commentaries, for example in the following example which equates its shape with that of a drum mudinga.

Majjhimapadesa is of the shape of mudinga. In straight one place is of 80 yojanas, one place is of 100 yojanas and one place is of 200 yojanas.
In the centre, it is 300 . In perimeter, it is approximately 900 yojana. 39
Despite the fact that this description is in reference to the same object discussed in the previous example, there are some notable differences in presentation. First, the shape of the Majjhimapadesa has been equated to that of another object "mudingga" and several dimensions have been noted. However, the meanings of these measurements are only possible to understand after identifying the shape of the object of comparison. Generally, the term "mudinga" refers to type of a drum, and here, the given measurements are compatible with the key dimensions of a drum. Further, in Visuddhimagga the "shape of the bloated carcass of a dog" is equated to the "shape of the mudinga drum"40, implying that this is a cylindrical object with tapering ends. With that background information, I follow the criterion adopted in the previous example and assume the shape to be a polygon as figure $4.4^{11}$

Here, $B E$ represents the centre line of the figure and the given dimensions are, $A B=100 / 2=50, H C=G D=200 / 2=100, F E=80 / 2=40, B E=300$ and half perimeter $B A H G F E=900 / 2=450$ (approximately).

$$
\begin{aligned}
A H^{2}=A J^{2}+J H^{2} & =B C^{2}+(H C-J C)^{2} \\
& =B C^{2}+(H C-A B)^{2} \\
& =B C^{2}+50^{2}
\end{aligned}
$$

[^10]
## Atthasālinī: 122

41 See also Jayawardana 2017: 74. This is a conjecture, but such conjectures are unavoidable when attempting to depict commentarial data in geometrical forms. There may be several alternative shapes that match with the given data, and this conjecture only presents one of these possible shapes.


Figure 4: Polygon depicting the shape of the Majjhimapadesa as per Manorathapūranī.

$$
\begin{aligned}
G F^{2} & =G K^{2}+K F^{2}=(G D-K D)^{2}+K F^{2} \\
& =(G D-F E)^{2}+D E^{2} \\
& =60^{2}+D E^{2}
\end{aligned}
$$

$$
B C+C D+D E=300
$$

Taking $B C=120, C D=169$, (intuitively) and $D E=11$,

$$
A H^{2}=120^{2}+50^{2}
$$

gives $A H=130$

$$
G F^{2}=60^{2}+11^{2}
$$

gives $G F=61$
Therefore,

$$
B A+A H+H G+G F+F E=50+130+169+61+40=450
$$

This proves that the half perimeter is BAHGFE $(=B A+A H+H G+G F+F E)=$ 450, as attested by the relationship known as Pythagoras theorem. In the passage,
the perimeter is given as "approximately 900," which might be an approximation of the actual shape of a mutinga, an object with curved edges, using a hypothetical model of a polygon with straight sides to which contemporaneous geometrical knowledge could be applied.

Here, it is interesting to note that the same object (the Majjhimapadesa) has been depicted as having different shapes in different texts which belong to the same literary tradition, and even in texts having the same authorship. $4^{2}$ The reasons for these variations is beyond the scope of this article, as an interdisciplinary study on contemporaneous geometrical, geographical and cartographical knowledges needs to be undertaken before arriving at any conclusion. However, a rudimentary guess is that that term as encountered in the first narrative may refer to the Indian subcontinent, where Buddhist doctrine originated, and the second narrative may refer to Sri Lanka, where the Buddhist Commentarial tradition, in its present form, originated. This is based on a consideration of the attempts of map making in contemporaneous times.

One of the earliest attempts at mapmaking was made by Claudius Ptolemy around 150 все, a period during which much information assimilated into the Buddhist Pāli Commentaries. ${ }^{43}$ In this attempt, Ptolemy extensively used existing information available to him in Alexandria as well as information received from sources like ships' captains, seafarers and travelers. Based on such information, he recorded the co-ordinates of 49 points in Sri Lanka (Taprobane, as per Ptolemy) giving their meridian (longitude) and parallel (latitude) references. However, these co-ordinates had two inherent errors: first, the radius of the earth and hence its circumference were given incorrect dimensions, and second, the observation of the sun Azimuthal from the different points in the island were assumed to be independent of time, when in fact they were not. After correcting these errors in accordance with present known values, Fernando noted that the relative positions of those 49 points were $100 \%$ correct, while their absolute positions were within 10 miles from the actual locations. ${ }^{44}$ This level of accuracy and the fact that such accuracy was based on the information provided by well-travelled sources implies that the Ptolemy's work could potentially be used as an indicator of the geographical knowledge of other societies. A world map produced by Byzantium scribes, based on Ptolemy's co-ordinates, represents the Indian subcontinent and Sri Lanka as having shapes similar to the shapes derived in the figures above 3 and 4 , respectively; see figure 5 .

The above discussion attempts to link two knowledge spheres, one related to geometrical shapes and the other to geographical spaces. Geographical spaces

42 A range of connotations of the term $M a$ jjimapadesa as used in Buddhist literature has been discussed in Jayawardana 2017: 6387.

43 Jayawardana 2017: 76.
44 Fernando 1985,Fernando 1986-7: 83-84, Fernando 1991: 123-131, Fernando 1995: 109127.


Figure 5: The Indian subcontinent and Sri Lanka as depicted in world map from Ptolemy's Geography. Photo excerpted from the illustrated manuscript by di Antonio del Chierico (1450-75).
have been presented through examples using the demarcation of shapes and presentation of the fixed characteristic of those shapes. An example of this is the circle (shape) for which the ratio between circumference and the diameter is a fixed characteristic. To draw together these observations about the representation of geometrical shapes and geographical spaces I turn to an example from the Manorathapūraṇi. This example is prefaced by a discussion of a variety of ways that this circle was understood and measured in the ancient world. We discussed the understanding of a circle as a geometrical shape in the Visuddhimagga evidenced by the citation, "when one is drawing a circle, the pin that stays fixed down in the centre is like "applied thought," which directs on the object, and the pin that revolves round is like "sustained thought," which continuously presses". ${ }^{45}$ In this example, the process of drawing a circle is used as an example to explain a doctrinal issue, which suggests the familiarity of the process among the readership. The practical aspect of such notation is evident in the Mahāvamsa description of the process of laying out the ground profile of Mahāthūpa, as detailed above. ${ }^{46}$

A crucial element in the geometry of the circle is the ratio of its circumference to its diameter, generally denoted as $\pi$. Various approximate values of $\pi$ had been in use in many early cultures, 3.1604 and 3.125 by the ancient Egyptians and old Babylonians, 3.1416 by Indians (Āryabhaṭa's Ganitapāda, fifth century се) and $355 / 113$ by Chinese (Tsu Ch'ung-chih, fourth century ce). However, the commonest practice in ancient times was to take the value of $\pi$ as 3 , the nearest natural number. ${ }^{47}$ Details found in commentaries as well as post commentarial
literature indicate that this is also the case in the Sri Lankan history of geometry. The following examples provide evidence of this approximation.

He [the deity Vissakamma disguised as a smith] took the gold and fashioning it with his hand, by his supernatural power, made a golden vase 9 cubits in circumference, 3 cubits in diameter. ${ }^{48}$

$$
9: 3=3
$$

The mansion named Sudharmā, having a length of 300 yojanas in breadth and width and 900 yojanas in circumference. $4^{4}$

$$
900: 300=3
$$

One cakkavāla is 1,203,450 yojanas in breadth and width, and $3,610,350$ yojanas in circumference. ${ }^{\circ}$

$$
3,610,350: 1,203,450=3
$$

Despite the fact that Egyptian, Babylonian, Indian and Chinese traditions sought to find more accurate values for $\pi$, it seems the Sri Lankan tradition as depicted in commentarial and later Sinhalese works continues to treat value of $\pi$ as 3, the nearest natural number.

Apart from the ratio between circumference and diameter, another important aspect of circle geometry is the area. Different formulas have been formulated to calculate area of a circle, which is closely associated with the value of $\pi$. In the subcontinent and parallel to the time of commentaries, A Aryabhaṭa ( 476 CE ) gave $1 / 2 \mathrm{cr}$ for calculating area of a circle, where $c$ is the circumference and $r$ is the radius. Brahmagupta ( 598 CE ) gave $3(d / 2)^{2}$ for "gross" and $\sqrt{10}(d / 2)^{2}$ for "neat" values. ${ }^{51}$ However, a different approach can be found in older works, like Ahmes papyrus ( 1650 все) in Egypt. There, the area of a circle was determined as the square of $8 / 9$ of its diameter. ${ }^{52}$ Many explanations have been put forward to understand the criterion adopted in arriving this solution. Vogel formulated a conjecture using a semi-regular octagon which approximates the circle (figure 6). 53 The area of the octagon ABCDEFGH is equal to $7 / 9$ of the square of the diameter. To Engles, the conjecture of the semi-regular octagon seemed too complicated and he suggested a simple alternative of a circle in an orthogonal net (figure 7). ${ }^{54}$ The area of the circle was to be understood as equal to the area of the square $A B C D$. Engles argued that $a=8 d / 9$ where $a$ denotes the length of the side of square $A B C D .55$


Figure 6: Vogel's hypothesis on Egyptian way of measuring area of a circle (after Gerdes 1985: 262).


Figure 7: Engels' hypothesis on the Egyptian way of measuring area of a circle (after Gerdes 1985: 262).

Gerdes suggested that the Egyptian criterion would have been based on an attempt to find a square and a circle such that same number of small circles could be packed in to each as tightly as possible (Figure 8). ${ }^{56}$


Figure 8: Egyptian way of measuring area of a circle (after Gerdes 1985: 267).
Segregating a circle into smaller parts for the calculation of area is also common in ancient Chinese mathematics. An illustration from a Taoist work of the Sung (perhaps eleventh century) attributed to Hsiao Tao-Tshun, the Hsiu Chen Thai Chi Hun Yuan Thu, shows rectangles inscribed in a circle. It may be that here, in the context of Taoist and Neo-Confucian expression, we see one of the germs of the idea of filling up a circular area by inscribing it with rectangles. 57

This historical sketch of calculating area of a circle gives some background to understand a narration found in Manorathap $\bar{u} r a n ̄ \bar{\imath}$, a commentary on the Anguttaranik $\bar{a} y a, ~ p o s s i b l y ~ c o m p i l e d ~ i n ~ t h e ~ f i f t h ~ c e n t u r y ~ c e . ~$

Magadha has three types of land allotments, small, medium and large. The small type is of 40 usabha from this point and 40 usabhas from this point, hence 1 gā̃vuta. The medium type is of 1 gā̃vuta from this point and 1 gāvuta from this point, hence half a yojana. The large type is of $11 / 2$ gāvutas from this point and $11 / 2$ gāvutas from this point,

[^11]56 Gerdes 1985:261-8; This explanation was based on material evidence, related to a board game which was played on a board with three rows of fourteen hollows with counters that were spherical objects. It was assumed that the exigencies of the game prompted experimentation to find a square and a circle such that the same number of spherical counters could be packed in to each as tightly as possible.
57 Ronan 1981: 60.


Figure 9: Rectangles inscribed within a circle from Hsiu Chen Thai Chi Hun Yuan Thu (Ronan 1981: 80).
hence 3 gā̃utas (tigāvutaṃ). His [a certain being with an astral body] body is having 3 small allotments, a medium and a small allotment, as such his body is of 3 gãvutas (tigā̃utaṃ). ${ }^{58}$

The information contained in this narration is mathematically vague, hence several assumptions need to be made in order to convert them into a comprehensible format. I have made the following assumptions:

- Land allotments and the shape of the astral body correspond to geometrically definable shapes.
- Land allotments are square in shape, measured from a mid-point with equally distanced sides from that mid-point.
- The shape of the astral body is not a square, as it is not given in terms of a large allotment with the same dimensions. In the original narration in the Anguttaranikāya, it is said that this particular astral body doesn't harm the one who possesses it nor does it harm others (nevattānaṃ no paraṃvyäbādheti). 59 This may imply a figure without any edges, the simplest being the circle. Therefore, the shape of the astral body is circular in shape.
- The comparison between land allotments and an astral body is area-based.

58 Manorathapūran̄̄̄ II: 616; Here the standard conversion rates should be considered,

8o usabhas $=1$ gāvuta, 4 gāvutas $=1$ yojana . 59 Añguttaranikāya III: 197.

- Three small allotments, one medium allotment and one small allotment denote separate parts of the astral body, otherwise they could have been noted cumulatively as four small allotments and one medium allotment.

Analyzed on the basis of these assumptions, the narration can be read as a description of an effort to explain the area of a circle in terms of squares, by inscribing square objects into a circle. This process would have been an attempt to 'exhaust' the residual area between the outside of the squares and the inside of the circumference of the circle. The segments - three small allotments, a medium and another small allotment - are named in this process of 'exhausting'. The relevant calculation could be illustrated as follows.

Let's take area of the small allotment as $A_{S}$ and area of the medium allotment as $A_{M}$. The astral body, figure 10 b , consists of three types of components, $P$, $Q$ and $R$. If areas of small, medium and large land allotments are $A_{S}$ and $A_{M}$ respectively, then,

$$
\begin{aligned}
\text { Area of } P & =A_{M} \\
\text { Area of } Q & =\frac{3}{4} A_{S} \\
\text { Area of } R & =\frac{1}{8} A_{S}
\end{aligned}
$$

$$
\begin{aligned}
\text { Area of the astral body }= & P+4 Q+8 R \\
= & A_{M}+4 \times\left(\frac{3}{4} A_{S}\right)+8 \times\left(\frac{1}{8} A_{S}\right) \\
= & A_{M}+3 A_{S}+A_{S} \\
= & \text { Medium allotment }+3 \text { Small allotments } \\
& + \text { Small allotment }
\end{aligned}
$$

In conclusion, this study has investigated the geometrical knowledge that prevailed in ancient in Sri Lanka by surveying the Buddhist Pāli Commentaries, a locally generated literary tradition widely attributed to the fifth century се. The examples presented here have shown that geometrical propositions such as horizontality, verticality, flatness, and roundness are described by being associated with their occurrences in nature. We have also examined the representation of geometrical properties such as shapes, dimensions, perimeters, areas, similarities and symmetries. The quantification of those properties in the form of the perimeters and areas of shapes, as well as ratios such as $\pi$, illustrate that the geometrical knowledge expressed in these texts transcended mere mental visualizations and had pragmatic applications.


Squares depicting the shape of the land allotments as per Manorathapūranī: (a) small, (b) medium and (c) large


Area of the circle is commensurate with the area of squares, where we find (a) the residual area between circle and the squares were exhausted by segmenting the squares and (b) the final presentation of the area of a circle is in terms of squares.

Figure 10

The Buddhist Pāli Commentaries, the primary source of this study, are neither exclusively dedicated to the subject of mathematics nor are they solely an indication of Sri Lankan historical developments. They were compiled as exegetical material elaborating the original Buddhist doctrine. As such, findings in this article are subject to the limitations intrinsic to this type of primary sources. These limitations suggest the need for further study. Firstly, the find-
ings need to be placed in broader context of knowledge transfers, a phenomenon among Asian societies since pre-Christian times. Secondly, the social context of mathematical developments needs to be investigated. According to scholars like Struik, Restivo and Goonatilake, social factors have deeply influenced the development of mathematics in individual societies as well as the subject as a whole. ${ }^{60}$

Broadly speaking, however, these limitations could be avoided to a certain degree by expanding the primary source base to the historiographical, archaeological and anthropological fields, but these types of sources have not yet been investigated in relation to geometry.

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[^12]| Papañcasūdanī | Pannasara, D. S. (1987), Papañcasūdan̄̄ (Colombo: Simon Hewavitarne Bequest). |
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[^1]:    1 S. S. Alam and S. N. Alam 2015: 6-43.
    2 Ronan 1981:39-40.
    3 Bose et al. 1971:149.
    5 Joseph 1992: 228.

    4 Pingree 1981: 4.

[^2]:    8 Deraniyagala 1990: 252-292.
    9 Mahāvamsa, "The Great Chronicle," compiled in the fifth century CE , records historical events in the island since the third century все.
    10 Sinhala is the major language in Sri Lanka with its origins in circa the fifth to sixth centuries BCE. For the evolution of the Buddhist Pāli Commentaries see Malalasekera 1928-94; Law 1933; Barua 1945-86; Adikaram 1946-2009; Godakum-

[^3]:    bura 1955; Bapat 1956-76; Rahula 1956; Mori 1984; 1989.
    11 For more details on the relation between Pāli Commentaries and early Sri Lankan society, see Jayawardana and Bohingamuwa. 2021.

    12 For Sri Lanka's foreign relations during the contemporary times of the Buddhist Pāli Commentaries, see Tennent 1860; Mendis 1983; Gunawardana 1987; Weerasinghe 1995.

[^4]:    13 The original height of the cetiya was $90 m$. The diameter of the present monument at its base is 89.4 m . There is archaeological evidence to prove that later restorations have not increased its original dimensions. The monument was justifiably called the Mahāthūpa ("The Great Stūpa"), as there was no other shrine of its class rivalling it

[^5]:    17 Visuddhimagga:105; Atthasālinī:122; Nāṇamoli: 137. For more details of the terms "applied thought," "sustained thought" and the "first absorption," see Nyanatiloka 1952-2004:242.
    18 Sāratthappakāsinī II: 174.
    19 Jātakaṭthakathā I: 154 .

[^6]:    20 Jātakațthakathā III: 200.
    21 Visuiddhimagga: 112; Ñāṇamoli: 145. 22 Jātakaṭthakathā VI: 89.
    23 Samantapāsādik $\bar{a}$ IV: 894. Here the geometrical notion may transcend the peripheral shape as the "pericarp of lotus" should indicate the internal convex form.

[^7]:    Bhaddhasīmāmāḷaka; see Cūlavaṃsa 78: 5570.

    25 Papañcasūdanī I: 65.
    26 Sāratthappakāsin̄̄ II: 32.
    27 Atthasālinī: 274.
    28 Jātakaṭthakathā V: 137.
    29 Pansiyapaṇasjātakapota V: 1745.
    30 Ibid: 1266.

[^8]:    31 Visuddhimagga: 392; Ñāṇamoli 2010: 542.

[^9]:    32 Visuddhimagga: 437; Ñāṇamoli 2010: 603.
    33 Goonatilake 1998:119; Bose et al. (1971:149); Cultures such as the ancient Egyptian, that fed the Greeks, had used this mathematical relationship for several centuries previous to Pythagoras. The Kahun papyrus (circa 2000 BCE) records the relation $4^{2}+3^{2}=5^{2}$ Goonatilake 1998:119. Vedic literature, Śulba-sūtra (circa 6oo500 BCE) which codified the construction of altars, also evidences knowledge of this relation in a series of statements which take an arithmetical approach, in contrast to the geometrical proof-based approach of Greek mathematics Bose et al. (1971:149). The

[^10]:    38 This may be a topographical representation of the Indian subcontinent as the actual shape resembles it to some extent. If that is the case then the importance of this reference transcends that of a Pythagorean triple (125:300:325) and marks an instance of contemporary cartographical knowledge appearing in early Buddhist literary works.
    39 Manorathapūrañ̄ I: 265.
    40 Visuddhimagga: 184; Nāṇamoli 2010: 245;

[^11]:    49 Sumañgalavilāsin̄̄ II: 458. The same figures are also repeated in Pūjāvaliya: 485 .
    50 Visuddhimagga: 151; Ñānamoli: 199; Samantapāsādikā I: 72; Paramatthajotik $\bar{a}: 366$.
    51 Bose et al. 1971: 188.
    52 Joseph 1992: 83
    53 Gillings 1972: 142.
    54 Engels 1977: 139.
    55 In fact, $a$ is not exactly equal to $8 d / 9$, but $2 d / \sqrt{5}$. However, the relative error is less than $0.62 \%$.

[^12]:    60 Struick 1986, Restivo 1992, and Goonatilake
    1998: 116-282.

