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Résumé de l'article

Mortality forecasting is much needed for population projections and actuarial calculations. Forecasting mortality of males and females in an independent way leads in most of cases to some incoherence regarding the expected male-female mortality evolution. To avoid a possible unrealistic convergence/divergence in this sense, a coherent mortality forecasting is required. In this paper, we compare the performance of two coherent models, namely the model of Li and Lee (2005) and that of Hyndman et al. (2013) on forecasting male and female mortality of the Algerian population. Results show that the first model provides better goodness-of-fit but less coherence compared to the second one.

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COHERENT MORTALITY FORECASTING FOR THE ALGERIAN POPULATION

Farid FLICI[1]

ABSTRACT

Mortality forecasting is much needed for population projections and actuarial calculations. Forecasting mortality of males and females in an independent way leads in most of cases to some incoherence regarding the expected male-female mortality evolution. To avoid a possible unrealistic convergence/divergence in this sense, a coherent mortality forecasting is required. In this paper, we compare the performance of two coherent models, namely the model of Li and Lee (2005) and that of Hyndman et al. (2013) on forecasting male and female mortality of the Algerian population. Results show that the first model provides better goodness-of-fit but less coherence compared to the second one.

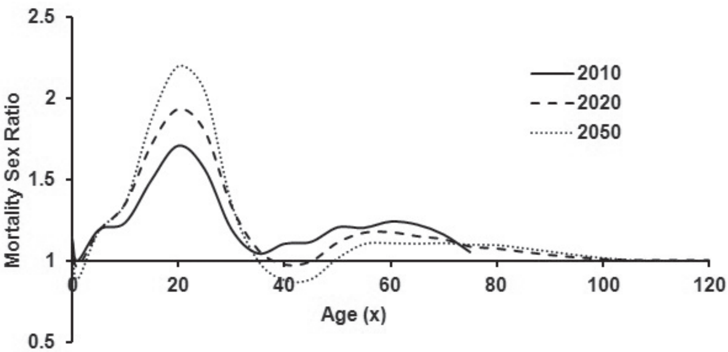
Key-words: Life expectancy, mortality, coherent forecast, sex ratio, Algeria.

1. INTRODUCTION

Mortality forecasting is much needed for population projections and actuarial calculations. Because of the gender-based difference in mortality, projections must be performed by sex. Prospective mortality models aim to summarize the historical mortality surfaces into a reduced number of parameters related to age, time (Cairns et al. 2006; Lee and Carter 1992) and cohort in some cases (Cairns et al. 2009; Currie 2004; Renshaw and Haberman 2006) by using data analysis techniques. Then, the projection process is reduced to project the time components in the future using time series models. When male and female mortality rates are independently projected, results can display some weaknesses regarding the male-female coherence. This latest can be translated by a crossover of the male and female projected life expectancies by the horizon of the projection, by an exaggerated divergence (Chen and Millosovich 2018), or by a female mortality excess at certain ages not observed on the historical data.

Flici (2016a) projected mortality in Algeria independently for males and females by using the Lee-Carter (LC) model (Lee and Carter 1992). The projected life expectancy for males and females haven't displayed any crossover or exaggerated diverging by the horizon of the projection. The comparison of the ASMRs by sex, however, showed a female mortality excess at some ages. Figure 1 shows the Mortality Sex Ratio (MSR) in 2020 and 2050 compared to 2010. The MSR at age x and time t is calculated as the ratio of the male on the female age-specific mortality rate corresponding to x and t.

FIGURE 1 *Mortality Sex Ratio (2010, 2020 and 2050)*



Source: Flici (2016a)

The increasing female mortality excess at around age 40 years can easily be observed on the projection results. Such imperfection is purely due to a statistical effect since historical data do not display similar effects.

Also, Flici (2016b) compared seven mortality models, i.e., M1 (Lee and Carter 1992), M2 (Rensaw and Haberman 2006), M3 (Currie 2006), M5 (Cairns et al. 2006), M6, M7, and M7* (Cairns et al. 2009), to project the ASMRs for the Algerian population, of males and females separately, for the ages 50 years and older, and based on the data of the period 1977 to 2013. All the evaluated models have led to a crossover of the male and female life expectancies by the horizon of the projection.

This paper aims to improve the male-female coherence of mortality projections for the Algerian population by comparing two coherent models: The model of Li and Lee (2005) and the model of Hyndman et al. (2013).

After presenting the two models, they will be calibrated on the Algerian mortality surface. This last is constituted by the annual life tables published by the Office of National Statistics (ONS) from 1977 to 2014 and completed by Flici (2014). The models' selection will be based on the goodness-of-fit and the male-female mortality ratio as a coherence criterion. The selected model will be used to project the ASMRs for males and females and to project the corresponding life expectancies.

2. Coherent Mortality forecasting methodology

In 1992, Roland Lee and Lawrence Carter designed a model allowing to forecast the age-specific death rates (ASDRs) in the future. Their main idea was to decompose the historical mortality surface into three components; two are related to age while the third one reflects a time evolution index. The initially proposed model had the following form:

$$\ln(\mu_{xt}) = \alpha_x + \beta_x * \kappa_t + \varepsilon_{xt} \qquad [1]$$

According to Equation [1], the logarithm of the observed ASDR (μ_{xt}) is decomposed into three components. The parameter α_x represents the average mortality age-pattern along the whole observation period in log scale, and can be estimated by $\hat{\alpha}_x = \frac{1}{T}\Sigma\ln(\mu_{xt})$ with T denoting the length of the observation period. κ_t is the mortality time variation index. β_x is the sensitivity of mortality at age x to the mortality time variation.

Once $\hat{\alpha}_x$ is estimated, the residual matrix $\ln(\mu_{xt}) - \hat{\alpha}_x$ is approximated by the product of two vectors: $\ln(\mu_{xt}) - \hat{\alpha}_x \approx \hat{\beta}_x * \hat{\kappa}_t$. In the initial contribution of Lee and Carter (1992), a Singular Values Decomposition (SVD) process was used before that many other decomposition techniques have been developed. Two constraints are imposed to ensure the uniqueness of the solution: $\Sigma\beta_x = 1$ and $\Sigma\kappa_t = 0$.

Once the model parameters are all estimated, the time variation index needs to be forecasted in the future to reconstitute the projected mortality surfaces.

When first proposed, the LC model has been applied to the US both sexes mortality data. Some incoherence arose when it was implemented separately for males and females. The parametric forecasting leads, in some cases, to a kind of incoherence between the projected mortality age-pattern for males and females. In order to enhance the forecast coherence, Li and Lee (2005) proposed to include a common age-time component in the gender-specific LC models. Their main idea was to model the mortality of each sex by the classical LC model while adding a common component ensuring the long run converging. The suggested model had the following form:

For males: $$\ln(\mu_{xt}^{m}) = \alpha_x^m + \beta_x^m * \kappa_t^m + \beta_x * \kappa_t + \varepsilon_{xt}^m \quad [2]$$

For females: $$\ln(\mu_{xt}^{f}) = \alpha_x^f + \beta_x^f * \kappa_t^f + \beta_x * \kappa_t + \varepsilon_{xt}^f \quad [3]$$

With $\Sigma\beta_x^m = \Sigma\beta_x^f = \Sigma\beta_x = 1$ and $\Sigma\kappa_t^m = \Sigma\kappa_t^f = \Sigma\kappa_t = 0$.

The parameters α, β and κ have the same interpretation like in the LC model; α represents the average mortality age pattern during the observation period; κ is the time trend of general mortality reduction; and β reflects the age-specific sensitivity to general mortality trend. With α_x^m, β_x^m and κ_t^m are specific for males; α_x^f, β_x^f and κ_t^f are specific for females; β_x and κ_t are common factors.

Despite of the large number of parameters in the model, the male-female coherence is not guaranteed in all cases. Since the three time-parameters κ_t^m, κ_t^f, and κ_t are independently projected, some incoherence can be displayed in long run projections.

Hyndman et al. (2013) proposed a new approach for coherent forecasting with a lesser number of parameters. This method is well known under the Product-Ratio Method (PRM). The main idea of the authors was to consider the MSR age-pattern into account. Introducing such a component in the model has to avoid any unrealistic divergence or crossover for long run projections. The PRM aims to reshape the historical mortality surfaces, of males and females, into two new components: a joint mortality function and a differential mortality component, both are to be projected in the future.

According to Hyndman et al. (2013), male and female mortality surfaces need to be joined in one surface. A geometrical average can be used for this issue. If we let μ_{xt}^m and μ_{xt}^f to be the ASDRs at age x and year t, respectively, for males and females, the average ASDR, noted μ_{xt}^*, can be calculated as:

$$\mu_{xt}^* = \sqrt{\mu_{xt}^m * \mu_{xt}^f}$$

The second component, the differential mortality function, is calculated by:

$$R_{xt} = \sqrt{\mu_{xt}^{m} / \mu_{xt}^{f}}$$

The MSR is usually determined by dividing the male death rate on the female one. Introducing the root aims to facilitate reconstituting the male and female mortality surfaces simply by: $\mu_{xt}^{m} = \mu_{xt}^{*} * R_{xt}$ and $\mu_{xt}^{f} = \frac{\mu_{xt}^{*}}{R_{xt}}$.

In final, μ_{xt}^{*} and R_{xt} are separately projected using the LC model. To forecast the joint mortality surface, the LC model, as described in equation [1], can be used.

$$\ln(\mu_{xt}^{*}) = \alpha_{x}^{*} + \beta_{x}^{*}\kappa_{t}^{*} + \varepsilon_{xt}^{*} \quad [4]$$

To forecast R_{xt}, we use the following formulation:

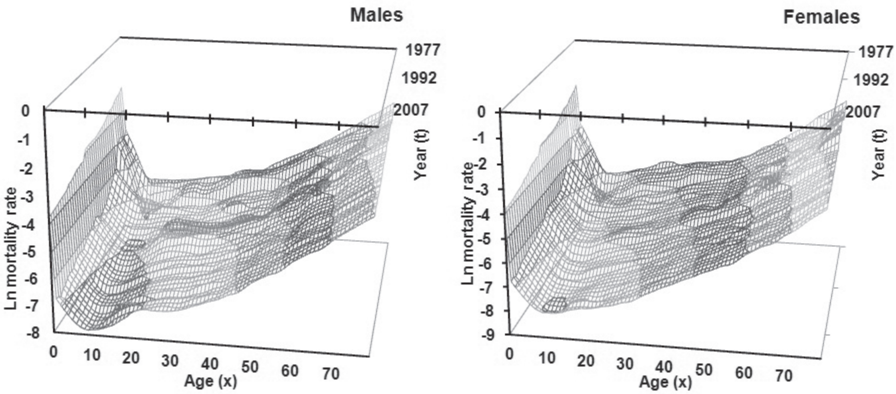
$$R_{xt} = A_{x} + B_{x} * K_{t} + \zeta_{xt} \quad [5]$$

With A, B, and K having respectively the same interpretations as α, β, and κ; ζ_{xt} is the specific error of the age x and year t and is supposed to be normally distributed. Two constraints are imposed to each model to ensure solution uniqueness: $\Sigma\beta_{x}^{*} = \Sigma B_{x} = 1$ and $\Sigma\kappa_{t}^{*} = \Sigma K_{t} = 0$.

3. Data

The data used in this paper is issued from the mortality data provided by the annual demographic publications of ONS. This data was made available, almost annually, starting from 1977. Mortality rates are published following five-age intervals from age 0 until the closure age which has been varying between 70 and 85 years. Missing data were estimated by Flici (2014). In most of the ONS publications, age-specific mortality is principally expressed by five-age mortality rates rather than death rates. First, the detailed age's data is interpolated following the methodology presented in Flici (2017). The obtained mortality surfaces expressed in detailed ages from age 0 to 79 years and for the period going from 1977 to 2014 are given in Figure 2.

■ FIGURE 2 *Male and Female Mortality surfaces (Mortality rates): 1977-2014*



Source: ONS annual statistics and Flici (2014) for missing data.

Mortality models, LC among others, were initially conceived on the central mortality rates or the ASDRs when a piecewise mortality constancy assumption is adopted. For the Algerian data, we remind that these two last mortality measures are not published in official statistics except for some years of the period 1977-1985 and 1995. Usually, the ASDRs (μ_x) can be estimated from ASMRs (q_x) by using Kimball's approximation (Kimball 1960):

$$\mu_x = \frac{2 * q_x}{2 - q_x}$$

This approximation is only applicable when the observed deaths are uniformly distributed along the year. This assumption is not verified at lower ages, infant and child deaths are usually concentrated at the beginning of the age intervals (Bourgeois 1946; Coale and Demeny 1966). Estimating the mortality rates for such age categories needs more detailed data, and more precisely the average age at death in the age categories 0 and [1,5[years. Unfortunately, this data is not available. Therefore, we prefer to orient all models to fit the mortality surfaces based on ASMRs rather than ASDRs.

4. Application

Recall that this paper aims to compare the two models proposed by Li and Lee (2005) and by Hyndman et al. (2013). We will use as a fitting criterion the Mean Absolute Relative Error (MARE) between the observed and the fitted ASMRs. Given the difference between the numbers of parameters to be estimated under each model, we use the Akaike Information Criterion (AIC) (Akaike 1973) and the Bayesian Information Criterion (BIC) (Schwartz 1978) for model comparison. The Li-Lee model, as presented in equations [2] and [3], is based on p parameters, $p = 2 * (2X + T) + 2X + T = 5X + 3T$ with X and T representing respectively the number of the age intervals and the length of the observation period. The model of Hyndman is based on two separated LC models. So, it is based on p′ parameters, p′ = $2 * (2X + T)$. This huge difference in terms of the number of parameters has to affect significantly the comparison of the two models if based on simple comparison criteria.

4.1. The Li and Lee Model

We remind that the Li and Lee model is based on the following formulation:

For males: $\ln(q_{xt}^{m}) = \alpha_x^m + \beta_x^m * \kappa_t^m + \beta_x * \kappa_t + \varepsilon_{xt}^m$

For females: $\ln(q_{xt}^{f}) = \alpha_x^f + \beta_x^f * \kappa_t^f + \beta_x * \kappa_t + \varepsilon_{xt}^f$

With $\Sigma\beta_x^m = \Sigma\beta_x^f = \Sigma\beta_x = 1$ and $\Sigma\kappa_t^m = \Sigma\kappa_t^f = \Sigma\kappa_t = 0$.

The parameters α_x^m and α_x^f are estimated as follows:

$$\hat{\alpha}_x^m = \frac{1}{38}\sum_{t=1977}^{2014} \ln(q_{xt}^m)$$

And

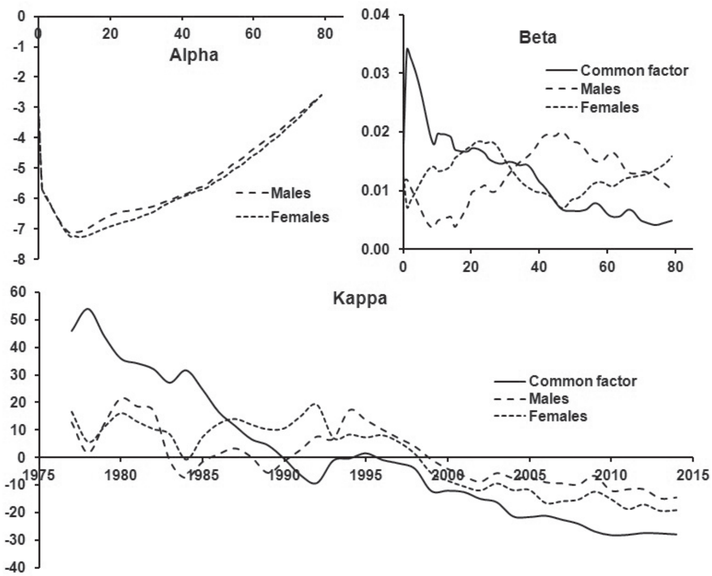
$$\hat{\alpha}_x^f = \frac{1}{38}\sum_{t=1977}^{2014} \ln(q_{xt}^f)$$

Then, the residual matrices are decomposed into six vectors: $\hat{\beta}_x$, $\hat{\beta}_x^m$, $\hat{\beta}_x^f$, $\hat{\kappa}_t$, $\hat{\kappa}_t^m$, $\hat{\kappa}_t^f$. Here, we use the linear programming techniques under EXCEL-Solver to estimate the models' parameters. The minimization problem will be oriented to minimize the MARE. Since we are working on two populations (males and females), the fitting criterion was adapted as follows:

$$\min \text{MARE} = \frac{1}{2}\left(\frac{1}{37*80}\sum_{x=0,\,t=1977}^{79,\,2014}\left|\frac{q_{xt}^{m} - (\hat{\alpha}_x^m + \hat{\beta}_x^m * \hat{\kappa}_t^m + \hat{\beta}_x * \hat{\kappa}_t)}{q_{xt}^{m}}\right| + \frac{1}{37*80}\sum_{x=0,\,t=1977}^{79,\,2014}\left|\frac{q_{xt}^{f} - (\hat{\alpha}_x^f + \hat{\beta}_x^f * \hat{\kappa}_t^f + \hat{\beta}_x * \hat{\kappa}_t)}{q_{xt}^{f}}\right| \right)$$

Some constraints are imposed on the estimation process to ensure the uniqueness of the solutions, i.e, $\Sigma\beta_x^m = \Sigma\beta_x^f = \Sigma\beta_x = 1$ and $\Sigma\kappa_t^m = \Sigma\kappa_t^f = \Sigma\kappa_t = 0$. The estimated parameters are shown in Figure 3.

FIGURE 3 *Parameters estimation—Li and Lee coherent model*



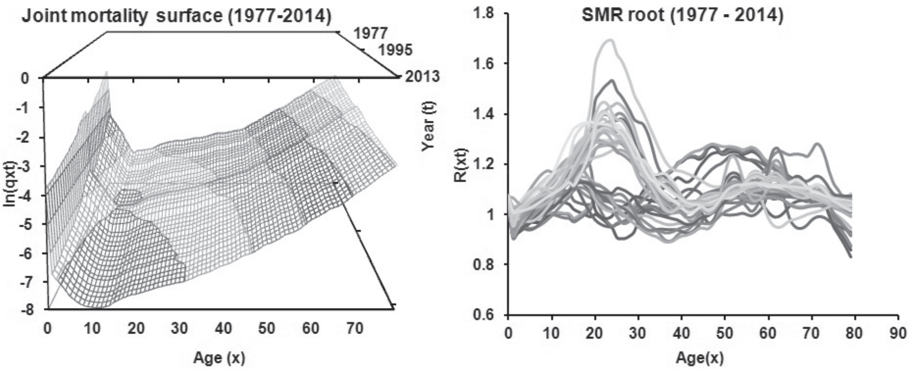
4.2. The Product-Ratio method

We recall that the Product Ratio method aims to forecast, independently, the joint mortality surface and the differential mortality function. Also, we note that, here in our case, the sex ratio is calculated on the mortality rates rather than on the death rates:

$$R_{xt} = \sqrt{\frac{q_{xt}^{m}}{q_{xt}^{f}}}$$

Figure 4 shows the observed joint mortality surface and the observed root of the MSR. Each component will, first, be fitted by a specific LC model. Then, the time components will be projected using time series models.

FIGURE 4 *Joint mortality surface and Mortality sex ratio*



Source: Calculated from data in Figure 2.

4.2.1. Joint mortality forecasting

To forecast the joint mortality surface, we use a formulation of the LC model based on the ASMRs. It gives:

$$\ln(q^*_{xt}) = \alpha^*_x + \beta^*_x \kappa^*_t + \varepsilon^*_{xt}$$

With $\Sigma\beta^*_x = 1$ and $\Sigma\kappa^*_t = 0$.

First, we estimate α^*_x as the average over time of the log of the ASMRs:

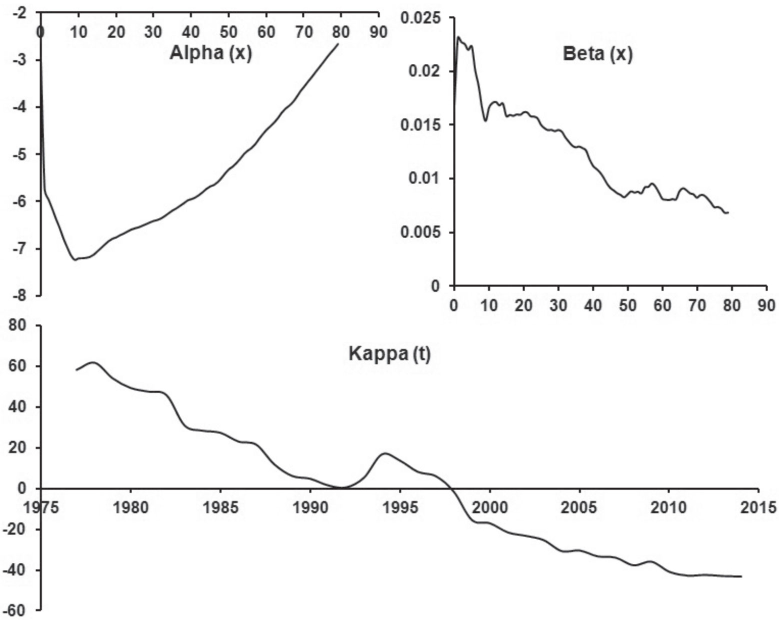
$$\hat{\alpha}^*_x = \frac{1}{38}\sum_{t=1977}^{2014} \ln(q^*_{xt})$$

Then, the residual matrix $\ln(q^*_{x,t}) - \hat{\alpha}^*_x$ is decomposed into two vectors $\hat{\beta}^*_x$ and $\hat{\kappa}^*_t$. In the original paper of Lee and Carter (1992), the authors proposed to use the SVD techniques to estimate the two parameters. Here, we use EXCEL-Solver (Flici 2016c) to estimate the parameters of the model while minimizing the MARE between the observed and the fitted ASMRs. We can write:

$$\min \text{ MARE } (2) = \frac{1}{x * t}\sum_{x=0,\, t=1977}^{79,\, 2014} \left|\frac{q^*_{xt} - \hat{q}^*_{xt}}{q^*_{xt}}\right|$$

In a third step, we re-estimate the three parameters $\hat{\alpha}_x^*$, $\hat{\beta}_x^*$ and $\hat{\kappa}_t^*$ within a common process to improve the goodness-of-fit further (Renshaw and Haberman 2003). The obtained results are shown in Figure 5.

FIGURE 5 *Parameters estimation of the LC Model—the Joint mortality surface*



The LC model allows reducing all the forecasting process to only projecting the mortality time index using time series models. The stability of this last is a significant determinant of the projection quality. Additionally, the LC model has the disadvantage to consider a fixed average mortality age-pattern over time. Usually, the shape of the alpha parameter changes continually during the observation period. Hence, it seems inappropriate to adopt a fixed alpha pattern for the future (Koissi and Shapiro 2008). Such a hypothesis can distort the robustness of the forecasting results. To avoid such an inconvenient, Bongaarts (2004) proposed to adjust the alpha parameter to fit the recent years' data. That assumes that the recently observed trend is more likely to be sustained in the future than the old trend.

4.2.2. Differential mortality

After having estimated the parameters of the LC model for the joint mortality function, the second step consists of estimating the LC model for the sex differential mortality function. For this purpose, we will fit the historical age-specific MSR by the model:

$$R_{xt} = A_x + B_x * K_t + \zeta_{xt}$$

With $\Sigma B_x = 1$ and $\Sigma K_t = 0$.

The parameter A_x is estimated by the average over time of R_{xt} for each age x:

$$\hat{A}_x = \frac{1}{38} \sum_{t=1977}^{2014} R_{xt}$$

Then, the residual matrix $R_{xt} - \hat{A}_x$ must be decomposed into two vectors:

$$R_{xt} - \hat{A}_x \approx \hat{B}_x * \hat{K}_t$$

We use the MARE between the observed age-specific MSR and the values expected by the LC model as a quantity to minimize in the estimation process:

$$\text{min MARE (3)} = \frac{1}{x * t} \sum_{x=0,\, t=1977}^{79,\, 2014} \left| \frac{R_{xt} - \hat{R}_{xt}}{R_{xt}} \right|$$

As shown in Figure 6, the MSR age-pattern which is expressed by the parameter A_x marks two prominent bumps; the first one is around the age of 20 and the second is around age 60. The B_x parameter shows a relatively high sensitivity of the ages from 15 to nearly 40 years to time variations represented by K_t. This later (K_t) has shown a relatively high variability until the mid-90s and displayed more relative stability afterward. As we can see in Figure 7, the age group [15, 30[has witnessed an increasing MSR starting from the mid-90s.

This increase reflects an increasing gap between male and female mortality. Even if mortality is decreasing for both sexes, young males lose some opportunities to improve their life expectancy because of alcohol, car accidents, their risky and unhealthy behavior (Kruger and Nesse 2004).

FIGURE 6 *Parameters estimation of the LC model—The differential mortality function*

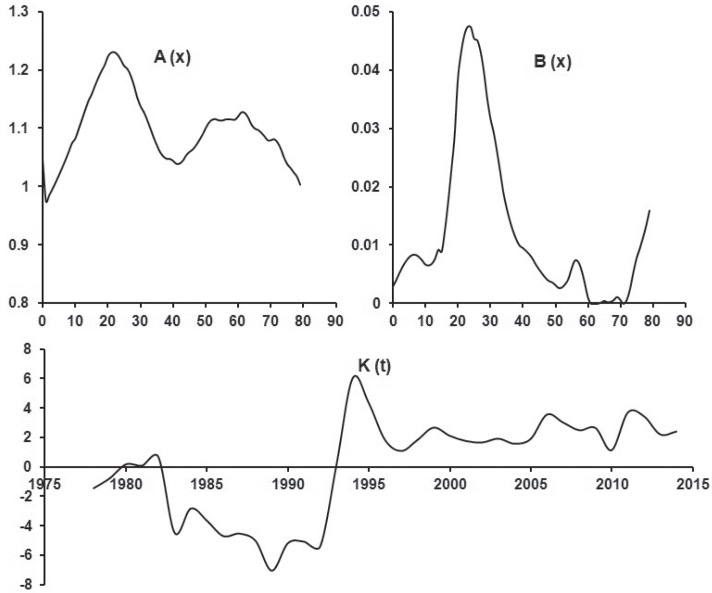
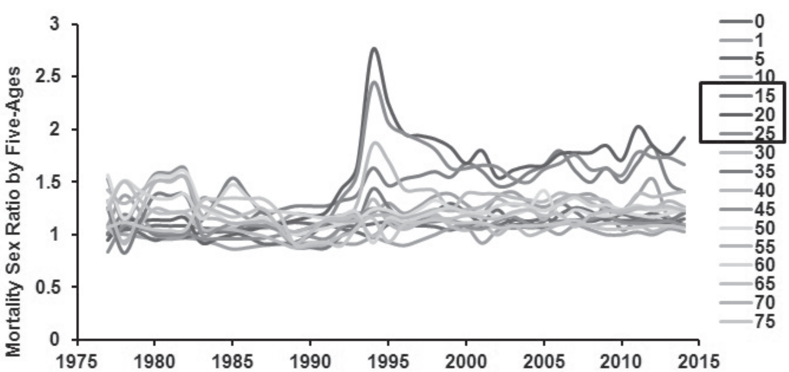


FIGURE 7 *MSR time evolution by five-age intervals*



4.3. Model selection

Model selection is based on comparing the Goodness-of-fit of the two models. We remind that the models' calibration was based on the MARE. The comparison must consider the goodness-of-fit in regards to the number of parameters of each model. The use of the AIC and the BIC allows such a comparison. In their original versions, these criteria were conceived to suit the case of Likelihood estimations. Later, some authors proposed an adapted version to the Least Squared Errors (LSE) method (Burnhan and Anderson 1998; Hansen 2007). If we consider m as the number of observations, p as the number of the parameters of the model and MSE to be the Mean Squared Error, AIC and BIC can be calculated as follows:

$$AIC = 2 * p + m * \ln(MSE)$$

$$BIC = m * \ln(MSE) + p * \ln(m)$$

Table 1 compares the goodness-of-fit of the two models. Accordingly, the model of Li and Lee has better fitting quality than the model of Hyndman et al according to all criteria.

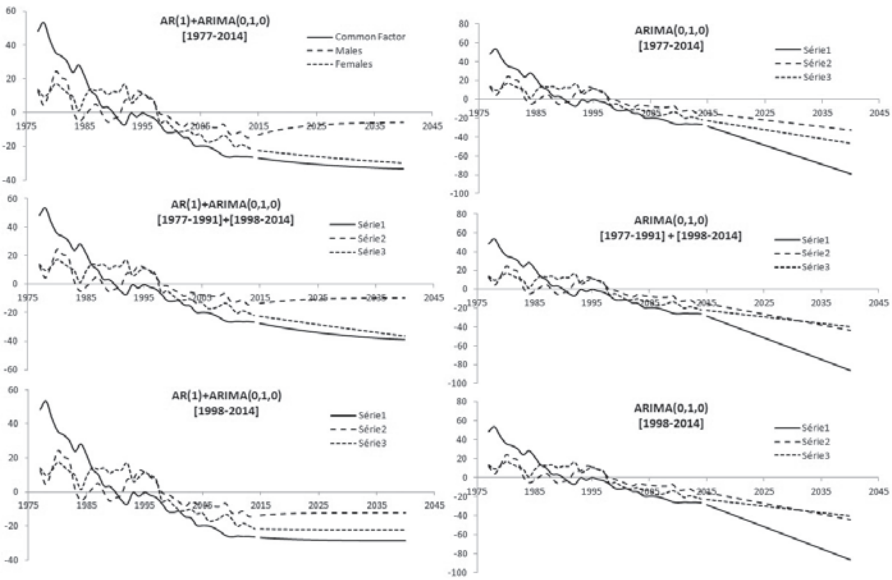
TABLE 1 *Models comparison—goodness-of-fit*

MODEL	MARE	MSE	*p*	*m*	AIC	BIC
Li and Lee Model	0.07511	0.01481	514	6080	–24,583.93	–21,133.57
Hyndman et al. Model	0.08735	0.01761	396	6080	–23,766.08	–21,107.83

The comparison was mainly based on goodness-of-fit. The assessment of the results using other qualitative and quantitative criteria is highly recommended (Cairns et al. 2009). The predictive capacity and the coherence of the forecast regarding the MSR can be considered for such an issue. The Li-Lee model is based on three time indices to be independently projected in the future. Hence, coherence is again not guaranteed in all situations. In other words, even if the Li and Lee model was conceived to avoid the incoherence between male and female projections by adding some common factors, results can be adverse in some cases.

In figure 8, we show an illustration confirming this statement. To forecast the three time-components (κ_t, κ_t^m, κ_t^f), we compare two time-series models and several time intervals as a basis of the forecast. The chosen time series models are the two most commonly used models to forecast the time index in LC model which are: the random walk with drift model ARIMA(0,1,0) (Dowd et al. 2011; Lee and Carter 1992; Zhou et al. 2013) and a 1st order autoregressive model for which a drift term is added AR(1)+ARIMA(0,1,0). These models were calibrated at different time intervals. The predictive capacity of a model is widely related to the quality and the regularity of the historical data series. Occasionally, some years must be excluded from the initial dataset (Bell 1997) to improve its regularity. The effects of the terrorist events on mortality evolution in Algeria during the period from 1992 to 1997 are significant (Flici 2020a) and can be considered from this point of view. We propose to calibrate the models on different periods, i.e., 1977-2014, 1977-1991 & 1998-2014, and 1998-2014.

FIGURE 8 *Time indexes forecasting in Li and Lee Model*



The left column shows the results obtained using an AR(1) model for which a drift term is added. The right one shows the results obtained with ARIMA (0,1,0) model. In lines, we show respectively results in function of the time range used to calibrate models: (1977-2014), (1977-1991; 1998-2014) and (1998-2014).

As illustrated in Figure 8, the three time-factors evolve in an incoherent way in all situations. Being conscious of such an imperfection, we prefer using the model of Hyndman et al. which ensures better results regarding the male-female coherence.

5. FORECASTING

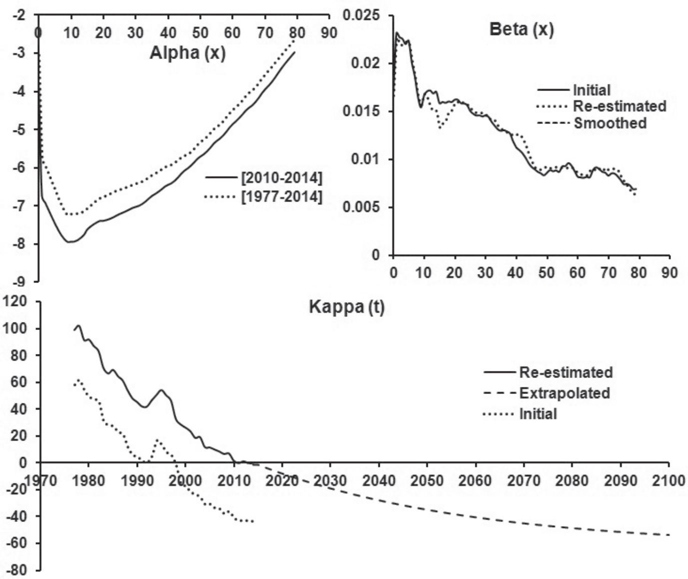
Since it was first proposed in 1992, many generalized versions have been proposed to the LC model to improve its fitting quality and predictive capacity. Other evaluation criteria have been introduced later as the regularity of the projected mortality surfaces. The projected mortality surfaces can display evident irregularities resulting from the high variability of the LC parameters. Thus, Currie et al. (2004) proposed to smooth the estimated parameters before to perform any forecast. On the other side, the LC model leads to a kind of deceleration of the projected life expectancy (Debonneuil et al. 2015). To deal with such imperfection, Boongarts (2004) proposed to re-estimate alpha to the recently observed data and to readjust the other parameters to fit the change of alpha. Accordingly, the LC model in this paper will be estimated with fitted beta parameter and using only the recently observed data to estimate alpha.

5.1. Joint Mortality forecasting

Several time series techniques can be used to project the mortality time index in the future. Lee and Carter (1992) used a random walk with drift model, i.e., ARIMA(0,1,0).

Later, other time series models have been proposed to improve fitting and forecasting qualities. Flici (2016b) compared a set of models to forecast κ_t for the case of Algeria. It was turned out from this comparison that the 1st order autoregressive model AR(1) fits well the historical trend and leads to an acceptable long-run forecast quality either the period 1992-1997 is included or not. Here, we use the same model to forecast κ_t. Estimation results are shown in Figure 9.

FIGURE 9 *Estimation and forecasting results for the joint mortality function*

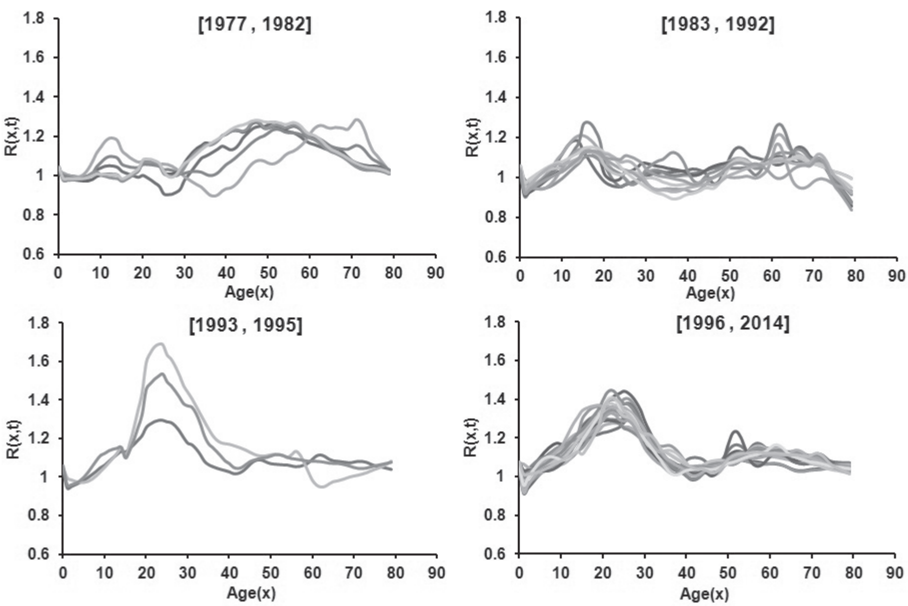


5.2. Sex Differential mortality forecasting

The LC model is based on a time-invariant age mortality pattern represented by the parameter A_x. This later is averaged on the whole observation history and assumed to keep constant in the future. On the other side, K_t summarizes the time evolution of mortality at all ages while age-specific variations are represented by B_x. One of the biggest inconvenient of the LC model is that it doesn't capture all the changes occurred in the age schemes during the observation period. According to Koissi and Shapiro (2008), changes in the age-mortality pattern must be considered when projecting mortality in the future.

Figure 10 reveals that $R_{x,t}$ has marked many changes between 1977 and 2014. Four different patterns can easily be distinguished corresponding to the periods 1977-1982, 1983-1992, 1993-1995, and 1996-2014.

FIGURE 10 *$R_{x,t}$ changes during the period 1977-2014*



Here, we assume that the recent data should be more reliable compared to the old ones and that the recently observed trend is more expected to be reproduced in the future compared to the oldest one (Boongarts 2004). In contrast to the mortality time index which is supposed to fall in the future, the stagnation of K_t is very expected as a scenario.

On the other side, we suppose that the observed sudden changes in the MSR age-pattern from a period to another to not be due to natural variations. Using an average pattern of the whole period as a basis to deduce the projected pattern has to deteriorate the quality of the projection. Our idea is to use only the recent period 1996-2014 to calibrate the model. Accordingly, A_x is estimated on the period 1996-2014 rather than on the period 1977-2014. The new estimated parameter is noted A'_x. Then, we decompose the residual matrix into two vectors B'_x and K'_t. To improve the regularity of the forecast, B'_x is smoothed using a quadratic function, that we note B''_x as suggested by Currie et al. (2004). The parameters of the quadratic smoothing function are estimated as well as the parameters A'_x and K'_t by the same optimization process as suggested by Renshaw and Haberman (2003). The new estimated parameters are noted A''_x and K''_t.

Next, K''_t needs to be projected in the future. The stability of this parameter along the period 1996-2014 allows obtaining robust results when a 1st order autoregressive model AR(1) is used. As the length of the considered period is relatively short, we use a linear function to perform the projection as suggested by Planchet and Ueller (2007). Figure 11 shows the re-estimated parameters compared to those in Figure 6. Also, the figure shows the projected time index. Figure 12 shows the expected evolution of the MSR age pattern in 2020 and 2050.

FIGURE 11 *Estimation and forecasting results for the sex differential mortality function*

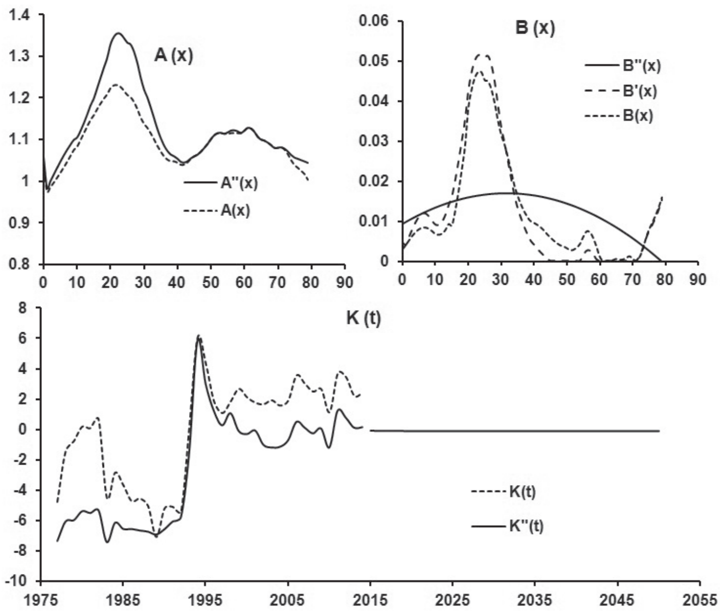
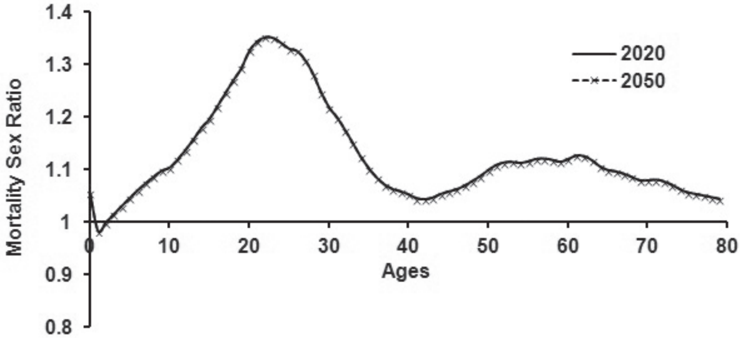


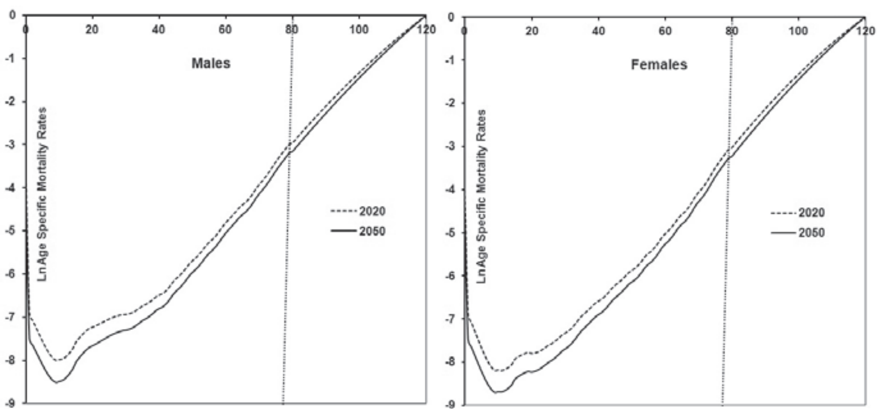
FIGURE 12 *Expected evolution of the MSR Age pattern (2020/2050)*



6. RESULTS

After projecting the joint mortality surface and the differential mortality function in the future, the ASMRs can be deduced by $q^m_{xt} = q^*_{xt} * R_{xt}$ and $q^f_{xt} = \frac{q^*_{xt}}{R_{xt}}$. Figure 13 shows a comparison of the male and female mortality age-pattern by 2020 and 2050.

FIGURE 13 *Age mortality pattern for males and females—projection results (2020-2050)*



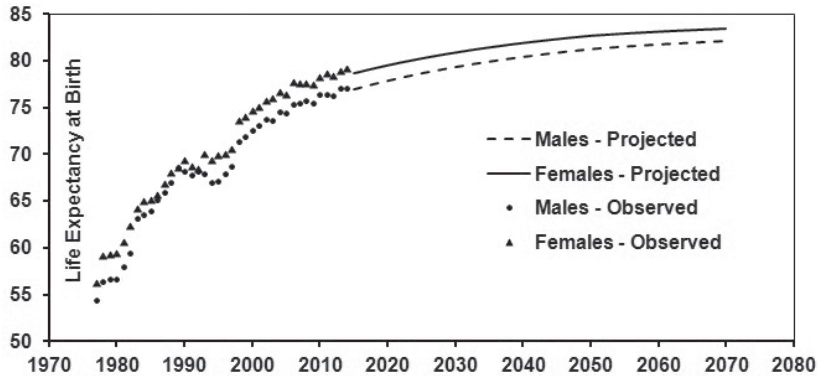
We note that mortality rates beyond the age of 80 were estimated by the quadratic model proposed by Denuit and Goderniaux (2005):

$$\ln(\hat{q}_x) = a + bx + c^2$$

The model was calibrated on the age range [60,79] by imposing a closure constraint at age 120: $\hat{q}_{120} = 1$ as it was suggested by Flici (2020b).

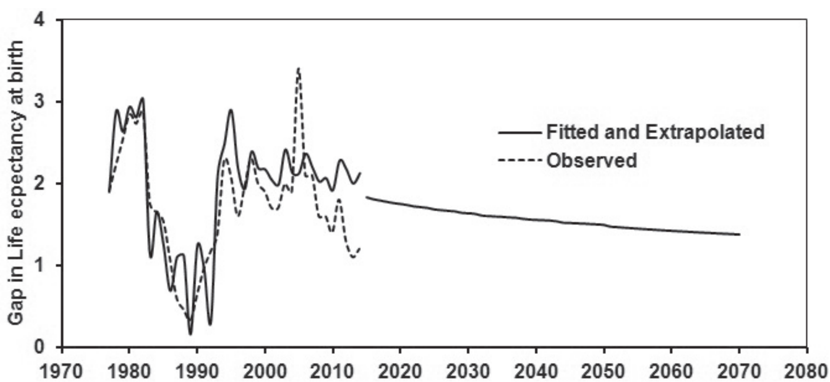
The obtained results allowed expecting the future evolution of the life expectancy at birth for males and females as it is shown in Figure 14.

FIGURE 14 *Projected Life Expectancy at Birth for males and females*



According to the results, the life expectancy at birth for men is expected to rise from 76.9 in 2015 to 79.3 in 2030 and 81.2 in 2050. For women, it will pass from 78.8 in 2015 to 80.9 in 2030 and 82.7 in 2050. The male-female gap in life expectancy at birth (Figure 15) is expected to fall slightly during the upcoming years passing from 1.8 in 2015 to nearly 1.5 in 2050.

FIGURE 15 *Expected evolution of the Gap in Life Expectancy at birth*



7. Conclusion

Starting with the contribution of Lee and Carter (1992), several models have been developed to facilitate mortality forecasting. The prospective mortality models made forecasting mortality rates at different ages very easier and with a lesser number of parameters. The main idea was to decompose the historical mortality surface into age and time components. Then the forecasting process consists only to project the time component in the future by using the conventional time series models.

In addition to the goodness-of-fit, mortality projections need to be evaluated regarding complementary criteria. The male-female coherence can be taken in that framework. In some cases, mathematical models can lead to incoherent evolution of the projected life expectancy by sex. It can be displayed by either a crossover or an unrealistic diverging between males and females. In some cases, a slight female mortality excess can be observed at certain ages by the horizon of the forecast only due to a statistical effect.

In this sense, Flici (2016a) has shown that independent mortality forecasts for males and females using the LC model result in a female mortality excess at around age 40. Accordingly, this paper aimed to improve the quality of the projections regarding male-female coherence. Two models were evaluated and compared on the Algerian mortality data of the period 1977-2014. It concerns the coherent models proposed by Li and Lee (2005) and by Hyndman et al. (2013).

First, the two models have been compared according to the goodness-of-fit. According to the obtained results, the Li and Lee model fits the historical surfaces better than the Hyndman et al. model according to all criteria, i.e., MSE, AIC, and BIC. However, even if the Li and Lee model was conceived as a coherent mortality model, the male-female coherence is not guaranteed in all situations. The inconvenient of this model relates to the fact of being based on three time-components to be forecasted independently. A difference in the historical trend of these three parameters is supposed to lead to incoherent forecasts. As we have seen in this paper, incoherent results have been obtained with the Li and Lee model. For that, we preferred to use the second model to project the male and female mortality rates in the future.

According to the final results, the life expectancy in Algeria is supposed to evolve from 76.9 in 2015 to 81.2 in 2050 for males and from 78.8 in 2015 to 82.7 in 2050 for females. The gap in life expectancy at birth is supposed to decrease slightly from around 1.8 in 2015 to 1.5 in 2050.

The use of mortality forecasting, in actuarial calculations and public planning, needs it to be based on robust projections. In this sense, attention must be given to specific age categories, e.g., infants, child, and young adult ages. For example, when we compare the increase of male and female mortality in Algeria we can observe a significant difference in terms of the pace of improvement at specific ages. Men aged between 15 to 30 years benefited less than corresponding women from the general mortality improvement. Unfortunately, our projections did not address such a point, and we observed that the sex ratio age pattern observed during the period 1996-2014 is supposed to keep constant during the coming decades. This element must be considered with more attention in other works in order to improve the quality of the projections.

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